10

Wave Optics

4.1 Introduction

We have noted in the previous semester in the chapter 'Ray Optics' that various theories have been put forward to understand the nature of light. Ray optics or geometric optics has limitations in explaining certain optical phenomena such as interference, diffraction, polarization, transmission, holography, etc. In 1678, Huygen proposed a wave theory of light. According to this theory, light energy is supposed to be transferred from one point to another in the form of waves. He, based on his wave theory, could explain the laws of reflection and refraction. Later, in 1801, Thomas Young could explain the phenomenon of interference of light. Augustin Fresnel in 1815 had developed the wave theory to explain rectilinear propagation of light. The polarization phenomenon, as discovered by Malus in 1808, remained an unsolved problem to Huygen's wave theory. Huygen's wave theory assumes light waves as longitudinal, while the polarization effect can be observed only for transverse waves. As longitudinal waves always require elastic medium for propagation, Young and Fresnel assumed presence of luminiferous ether in entire universe.

Later Young realized that light is transverse waves, though he was still believing in the presence of omnipresent ether. It was Faraday who showed that the polarization of light was affected by a strong magnetic field. This was the first hint about electromagnetic nature of light. Clerk Maxwell unified the empirical laws of electricity and magnetism into a coherent theory of electromagnetism. As studied in the previous chapter, Maxwell made the prediction that light is a high frequency electromagnetic waves. Theoretical prediction of Maxwell was confirmed by Hertz by producing and detecting electromagnetic waves. In 1887, Michelson-Morely performed the famous ether-drift experiment, and concluded that ether does not exist. Hence, light waves are high frequency non-mechanical transverse electromagnetic waves, comprising of oscillating electric and magnetic field vectors.

However, the simple wave theory capable of explaining reflection, refraction, interference, diffraction etc. is described by a single scalar function. This is known as Wave Optics or precisely Scalar Wave Optics.

In this chapter, we shall study propagation of light and related optical phenomena using the ideas of wave optics.

4.2 Wavefront and Huygen's Principle

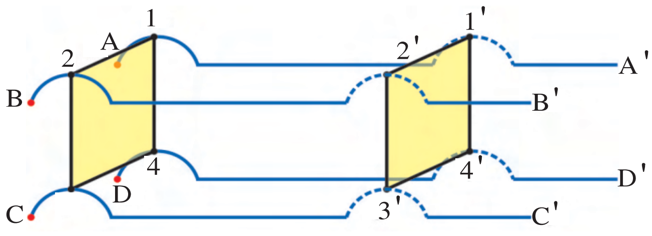


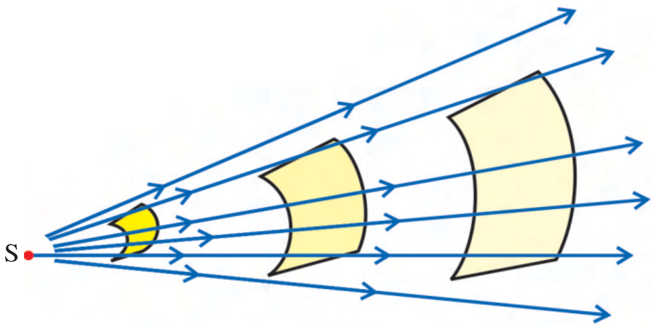
Figure 4.1 Construction of Wavefront

Propagation of disturbance in the medium (space) is known as wave. Thus, waves start from a source (origin) and spread out to new regions of medium (space). To understand, wave propagation, the concept of wavefront is used. As shown in figure 4.1, on identical four mutually parallel strings AA', BB', CC' and DD' four identical crests are created at points 1, 2, 3 and 4 respectively. All the particles located at the crest will begin the same state of oscillation and hence they are in the same phase. An imaginary surface passing through particles (rectangular-plane 1234 as shown in the figure 4.1) oscillating with same phase is known as a wavefront.

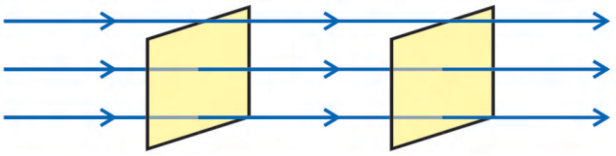
Since the shape of wavefront 1234 is a plane surface, it is known as a plane wavefront. Wavefront can be of various shapes.

Waves originating from a point like source and propagating in three dimensional homogeneous and isotropic medium have spherical wavefronts, while in the case of water ripples and due to linear source they are circular and cylindrical, respectively. Although, at considerably large distance (theoretically infinite) wavefronts are locally plane (See figure 4.2).

As shown in the figure 4.1, if we observe the strings after sometime crests have reached to particles 1', 2', 3' and 4'. However, their phase of oscillations remain same. Here, also we can imagine a plane wavefront 1'2'3'4'. In this way, as wave propagates ahead in the medium or space, wavefronts also move along with the wave. Thus, the propagation of wave can be visualized in the form of advancing wavefronts.



(*a*) Spherical Wavefront



(*a*) Plane Wavefront

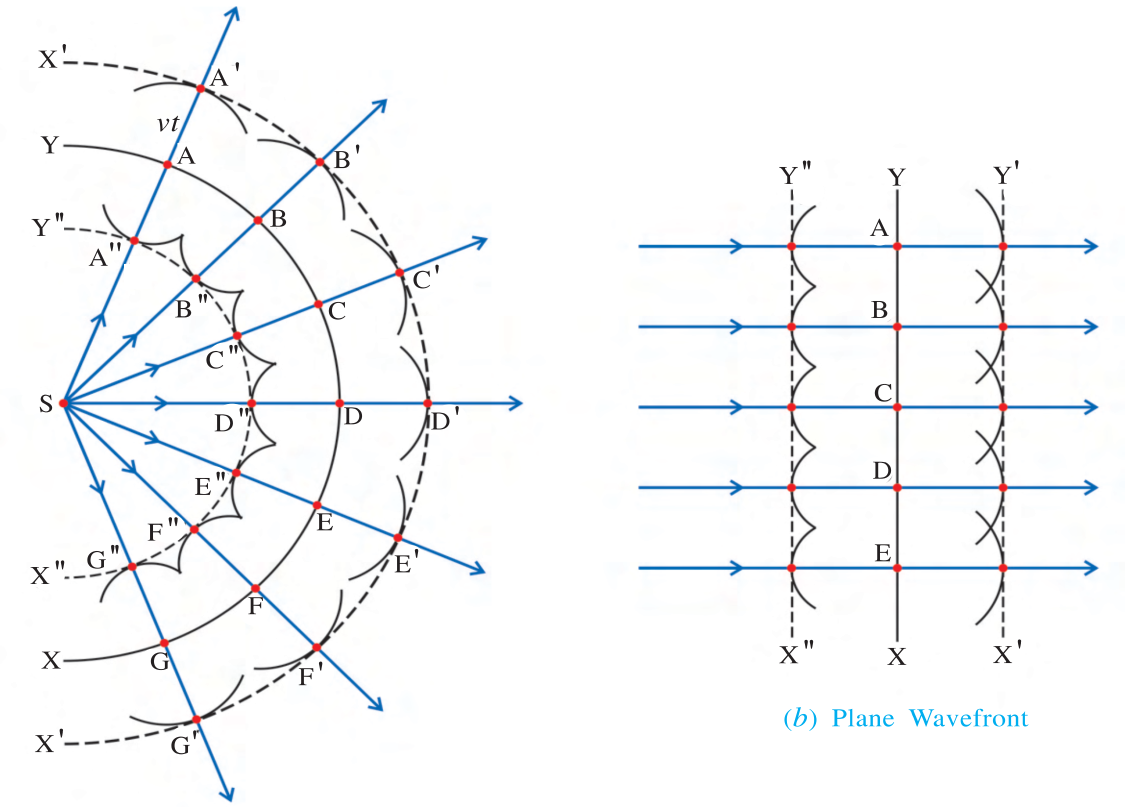
Figure 4.2 Different Shapes of Wavefronts

Lines perpendicular to the wavefront and indicating the direction of propagation of the wave are called rays. Remember that ray is just a geometrical concept.

Having noted that along with the wave, wavefront also propagates, a natural question which we may ask is. How is a new wavefront formed after a very small time interval Δt ? This question can be answered by Huygen's principle.

Huygen's Principle : "Every point or particle of a wavefront behaves as an independent secondary source, and emits by itself secondary spherical waves. After a very small time interval the surface tangential to all such secondary spherical wavelets gives the position and shape of new wavefront."

As shown in figure 4.3 (a), part of corss section of spherical or cylindrical wavefront at a particular instant of time (t) is shown as XY. According to Huygen's principle, all particles of this wavefront (i.e. A,B,C ….. etc.) behave as secondary sources and emit spherical waves. If the velocity of wave is v, then we can draw spheres of radii $v\Delta t$ with these particles as centers. Now, we can imagine a surface touching these spheres as a new wavefront at later time $t + \Delta t$. In the figure 4.2 such two surfaces $X'Y'$ and $X''Y''$ are shown. This means that from the wavefront XY, light propagates in both forward and backward directions ! Of course, this is never experienced in day to day life. A satisfactory explanation to this apparent paradox was given by scientists named by Voigt and Kirchoff. They showed that the intensity of secondary wave, making an angle θ with the direction of propagation is proportional to a factor $\cos^2\left(\frac{\theta}{2}\right)$. For the direction of propagation of wave (i.e. forward direction) $\theta = 0$ and hence the intensity is maximum. Whereas for the backward direction ($\theta = \pi$) intensity becomes zero. Hence, the effect due to the secondary waves at $X''Y''$ is zero or in other words, there is no back radiation of energy. Figure 4.3 (b) explains the wavefront formation for plane wave.



(b) Plane Wavefront

(a) Spherical Wavefront

Figure 4.3 Propagation of Wavefront

For the isotropic medium new wavefront maintains its original shape.

4.3 Reflection of Light through the Concept of Wavefront

To understand the phenomenon of reflection of light using the concept of wavefront, consider a plane wavefront PQ in figure 4.4. It is incident on reflecting surface AB such that point P of wavefront just touches the reflecting surface AB at $t = 0$. So, at time $t = 0$, point P starts emitting secondary spherical waves. As time passes, one by one all the points between P and Q gradually touch the surface AB, and start emitting secondary waves.

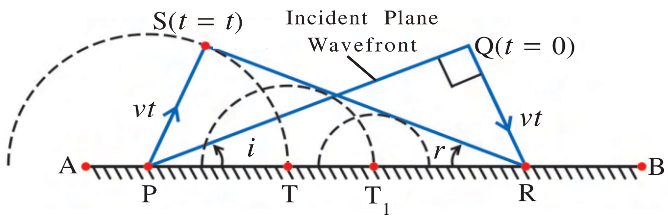


Figure 4.4 Reflection of Wavefront

Let the point Q touch the surface AB at later time t. That is, at time t, point R just starts emitting its secondary waves. During this time interval, a secondary wavefront produced by point P at time $t = 0$ has travelled a distance vt, where v is the speed of light wave in the medium. The corresponding wavefront is shown by dashed line. One such wavefront due to point T is also shown in the figure. According to Huygen's principle a common tangent drawn to such spherical wavefronts (SR in the figure) gives the new wavefront at time $t = t$.

Suppose incident and reflected wavefronts make angle i and r with reflecting surface AB, respectively. From the figure, in ΔPSR and ΔPQR, PR is common side.

$$\angle PSR = \angle PQR = \frac{\pi}{2}$$

Also, PS = vt = QR ($\because$ incident and reflected waves travel in the same medium having 2
speed v.)

These facts show that ΔPSR and ΔPQR are congruent.

$\therefore \angle QPR = \angle SRP$

i.e., $i = r$

Thus, the law of reflection (Angle of incidence = Angle of reflection) can also be proved by Huygen's wave theory.

4.4 Refraction of Light Through the Concept of Wavefront

Consider a plane wavefront PQ incident from a medium with refractive index n_1 on a transparent medium having refractive index n_2 (see figure 4.5). In the present discussion, we consider only transmitted wavefronts going into the medium-2. Let at time $t = 0$, the point P just touches the surface separating two media called an interface, and starts emitting secondary waves at $t = 0$ in the medium-2.

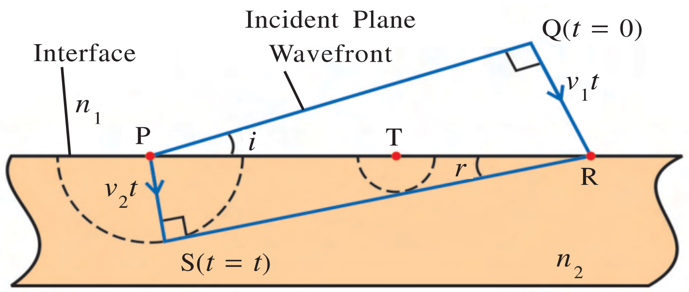


Figure 4.5 Refraction of Wavefront

Now, if speed of light wave in medium-2 is v_2, then secondary wavefront produced from point P travels a distance v_2t in the medium-2. The corresponding wavefront is shown by dashed line in the figure. Further, we assume that during this time ($t = t$), a wavefront produced from point Q has travelled a distance v_1t, and just touched the interface at point R. Here, v_1 is the speed of light in medium-1. According to Huygen's principle, a new wavefront in the medium-2 at time $t = t$ can be formed by drawing a common tangent to such spherical wavefronts (SR in the figure 4.5).

Using the geometry of the figure, angle of incidence (i.e., angle made by incident wavefront with the interface) is i and angle of refraction is r.

Also, PS = v_2t, QR = v_1t and PR is common side to ΔPQR.

From ΔPQR, $\sin i, \dfrac{QR}{PR} = \dfrac{v_1t}{PR}$

and from ΔPSR, $\sin r = \frac{PS}{PR} = \frac{v_2 t}{PR}$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} \tag{4.4.1}$$

But, $\frac{v_1}{v_2} = n_{21} = \frac{n_2}{n_1}$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \tag{4.4.2}$$

or

$$n_1 \sin i = n_2 \sin r \tag{4.4.3}$$

Equation (4.4.2) or (4.4.3) is nothing but the Snell's law for refraction.

4.5 Interference

As the disturbance produced at one point in a medium (space in case of non-mechanical wave) propagates, the particles (points in case of non-mechanical wave) coming in its way oscillate according to the type of the disturbance. Now, if a particle comes under the effect of more than one wave, what will be its displacement ? What kind of situation arise? To answer such questions, we should first study principle of superposition.

Principle of Superposition : "When a particle of the medium oscillates under the effect of two or more then two waves superposing at the given particle, according to the principle of superposition the resultant displacement of the particle is equal to the vector sum of the independent displacements due to each wave."

For example, if the displacement due to one wave superposing at a point is 1 cm in upward direction, and that due to other wave is 3 cm in the same direction, the resultant displacement due to both waves will be 1 + 3 = 4 cm in upward direction. But if the displacement due to second wave is 2 cm in downward direction, the resultant displacement at a point will be 1 + (–2) = –1 cm in downward direction.

Thus, superposition principle describes a situation when more than one waves superpose (i.e., interfere) at a point.

"The effect produced by superposition of two or more waves is called interference."

4.5 (a) Interference Due to Two Waves : Suppose two harmonic waves having initial phases ϕ_1 and ϕ_2 are emitted from two point like sources S_1 and S_2 respectively. They superimpose simultaneously (i.e. at the same time t) at a point P, as shown in the figure 4.6.

We have studied in the previous chapter that electromagnetic wave is represented by oscillating electric and magnetic field vectors. However, the effect of light (i.e. visible perception) is produced only by electric field, and therefore, in the present case we write light waves produced by source S_1 and S_2 interms of electric fields ($\vec{e}$) only.

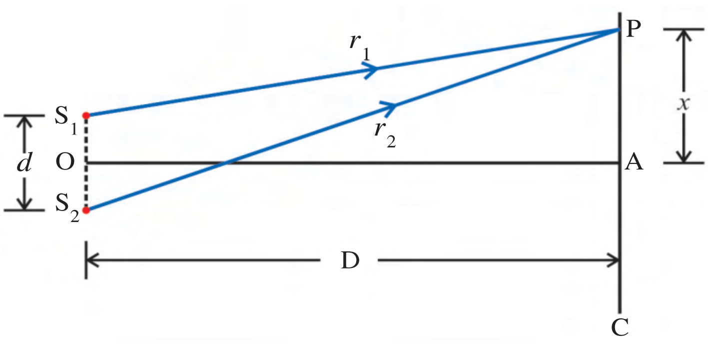


Figure 4.6 Superposition of Waves

Due to S_1 source,

$$\vec{e}_1 = \vec{E}_1 \sin(\omega_1 t - k_1 r_1 + \phi_1) \tag{4.5.1}$$

and that due to source S_2 source,

$$\vec{e}_2 = \vec{E}_2 \sin(\omega_2 t - k_2 r_2 + \phi_2) \tag{4.5.2}$$

Here, $\vec{E}_1$ and $\vec{E}_2$ represent amplitudes of electric fields, ω_1 and ω_2 denote angular frequencies of waves, and k_1 and k_2 are wave vectors. Arguments of sine function is known as phase of two waves.

Let, $\omega_1 t - k_1 r_1 + \phi_1 = \delta_1$ (4.5.3)

and $\omega_2 t - k_2 r_2 + \phi_2 = \delta_2$ (4.5.4)

Then, $\vec{e}_1 = \vec{E}_1 \sin\delta_1$ (4.5.5)

and $\vec{e}_2 = \vec{E}_2 \sin\delta_2$ (4.5.6)

Now, according to the principle of superposition, the resultant displacement at point P is,

$$\vec{e} = \vec{e}_1 + \vec{e}_2 \tag{4.5.7}$$

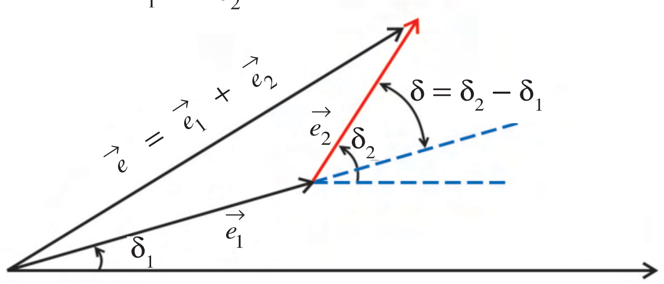


Figure 4.7 Phasor Diagram

To obtain the sum in equation (4.5.7), we use the method of phasor. (See figure 4.7)

$$\therefore\ e^2 = e_1^2 + e_2^2 + 2\vec{e}_1 \cdot \vec{e}_2$$

$$\therefore\ E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos(\delta_2 - \delta_1) \tag{4.5.8}$$

Where $\delta_2 - \delta_1 = \delta$ = angle between two vectors $\vec{e}_1$ and $\vec{e}_2$, and E is the resultant amplitude.

But, the average intensity of light is proportional to the square of amplitude, i.e. $I \propto E^2$.

Thus, equation (4.5.8) becomes.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\langle\cos(\delta_2 - \delta_1)\rangle \tag{4.5.9}$$

In equation (4.5.9), I_1 and I_2 are the average intensities due to each wave. They are independent of time. The last term in above equation is known as the interference term which depends on time.

$$\text{Now,}\quad \langle\cos(\delta_2 - \delta_1)\rangle = \frac{1}{T}\int_{t=0}^{t=T} \cos(\delta_2 - \delta_1)\,dt$$

$$= \frac{1}{T}\int_0^T \cos\{(\omega_2 t - \omega_1 t) + (k_1 r_1 - k_2 r_2) + (\phi_2 - \phi_1)\}\,dt \tag{4.5.10}$$

Here, T is the period of electric field oscillation.

Case I : Incoherent Sources : If two waves have different angular frequencies, i.e. $\omega_1 \neq \omega_2$. In this case, the phase difference, $\delta = (\delta_2 - \delta_1)$ between two waves is a function of time i.e. $\delta(t)$. Now, equation (4.5.10) becomes,

$$\langle \cos(\delta_2 - \delta_1) \rangle = \frac{1}{T}\int_0^T \cos(\delta(t))\,dt \tag{4.5.11}$$

But integration of cosine or sine function over its period is zero. Thus, in this situation last term in equation (4.5.9) is zero, and superposed two waves produce the average intensity $I_1 + I_2$ at point P.

The sources producing light waves with different frequencies (i.e., $\omega_1 \neq \omega_2$) are known as Incoherent Sources.

Case II : Coherent Sources : If two waves have same angular frequencies, i.e. $\omega_1 = \omega_2$.

Since two waves have same frequencies, they vibrate in such a way that the initial phase difference $\phi_2 - \phi_1$ remains same (or it can also be set to zero.) Light sources having same angular frequences and having constant initial phase difference are called Coherant Source. Here, we take $\phi_2 = \phi_1$. Also, since both waves are travelling in the same medium, their speed will be equal. Therefore, using the relation, $v = f\lambda = \frac{\omega}{k}$, we have $k_1 = k_2 = k$ ($\because \omega_1 = \omega_2$).

Thus, equation (4.5.10)

$$\langle \cos(\delta_2 - \delta_1) \rangle = \frac{1}{T}\int_0^T \cos\{k(r_1 - r_2)\}\,dt$$

$$= \frac{1}{T}\cos\{k(r_2 - r_1)\}\int_0^T dt \qquad (\because \cos(-\theta) = \cos\theta)$$

$$= \cos\{k(r_2 - r_1)\} \tag{4.5.12}$$

Putting the value of equation (4.5.12) in equation (4.5.9), and also by assuming that amplitude of both waves is equal, i.e. $I_1 = I_2 = I'$ (say) then,

$$I = I' + I' + 2\sqrt{I'I'}\ \cos k(r_2 - r_1)$$

$$= 2I'\{1 + \cos k(r_2 - r_1)\}$$

$$= 4I'\cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\} \qquad \left[\because (1 + \cos\theta) = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$

$$I = I_0\cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\} \qquad \text{Where, } 4I' = I_0 = \text{maximum intensity.} \tag{4.5.13}$$

Here, $k(r_2 - r_1)$ is known as the phase difference between superposing waves.

Special Cases :

Case I : When $\frac{k(r_2 - r_1)}{2} = n\pi$ or $k(r_2 - r_1) = 2n\pi$ (4.5.14)

Where $= n = 0, 1, 2, \ldots\ldots$

Then intensity, $I = I_0 =$ maximum ($\because \cos^2 n\pi = 1$)

"If the phase difference between the superposing waves is $2n\pi$ ($n = 0, 1, 2,$........), intensity at a superposing point is maximum. This interference is called constructive interference."

Substituting $k = \frac{2\pi}{\lambda}$ in equation (4.5.14)

$$\frac{2\pi}{\lambda}(r_2 - r_1) = 2n\pi$$

$\therefore$ The difference, $(r_2 - r_1) = n\lambda$ with $n = 0, 1, 2, 3,$ ...... (4.5.15)

"If the path difference between superposing waves is $n\lambda$ ($n = 0, 1, 2,$ ......) intensity at a superposing point is maximum. Such interference is called constructive interference."

Case II : When $\frac{k(r_2 - r_1)}{2} = (2n - 1)\frac{\pi}{2}$ or $k(r_2 - r_1) = (2n - 1)\pi$ (4.5.16)

where, $n = 1, 2, 3,$ ......

Then intensity, $I = 0 =$ minimum ($\because \cos\left(\frac{(2n - 1)\pi}{2}\right) = 0$)

"If the phase difference between superposing waves is $(2n - 1)\pi$, ($n = 1, 2, 3,$ ......), intensity at a superposing point is minimum. This interference is called destructive interference."

Corresponding path difference, $(r_2 - r_1) = (2n - 1)\frac{\lambda}{2}$ where $n = 1, 2,$ .... (4.5.17)

"If path difference between superposing waves is $(2n - 1)\frac{\lambda}{2}$ (where, $n = 1, 2,$ .....), intensity at superposed point is minimum. Such interference is known as destructive interference."

4.5 (b) Intensity Distribution : In principle, using equation (4.5.13), intensity distribution at different points P, P_m, P_{m+1} etc., can be found (see figure 4.8).

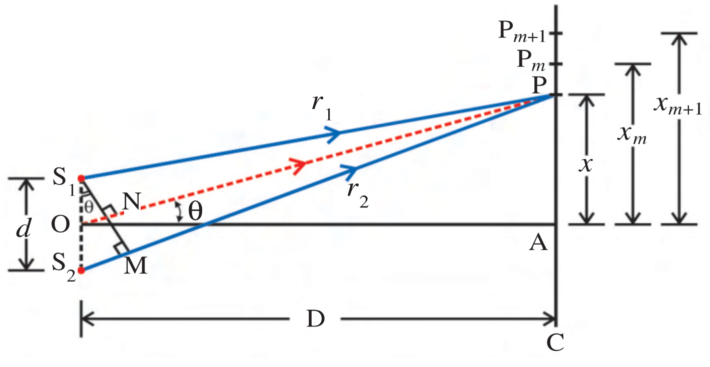


Figure 4.8 Interference of Waves

However, practically it is difficult to find path difference $(r_2 - r_1)$ directly. Therefore, we first convert equation (4.5.13) into such a form that experimentally path difference can be found. As shown in the figure 4.8, let point A on a screen is lying on perpendicular bisector of S_1S_2.

Also, suppose $S_1S_2 = d$, $OA = D$ and position of point P from point A, $AP = x$ and $\angle AOP = \theta$. To measure path difference, draw a perpendicular S_1M on S_2P from S_1. From the geometry of the figure,

path difference $r_2 - r_1 = S_2P - S_1P = S_2M$ (4.5.18)

In actual experiment S_1S_2 is of the order of 0.1 mm and distance D is of the order of meter. Hence, near S_1S_2, segment S_2M and ON may be considered parallel. Also, $\angle S_1NO = 90^\circ$.

$\therefore\ \angle POA = \angle S_2S_1M = \theta$ and $\sin\theta = \dfrac{S_2M}{S_1S_2}$

$\therefore\ S_2M = S_1S_2\sin\theta = d\sin\theta$

Using equation (4.5.18),

path difference $r_2 - r_1 = d\sin\theta$ (4.5.19)

Since S_1 and S_2 are very close to each other, θ (in rad) is very small.

$\therefore\ \sin\theta \approx \theta \approx \tan\theta$

$\therefore\ (r_2 - r_1) = d\tan\theta$ (4.5.20)

From ΔPOA, $\tan\theta = \dfrac{PA}{OA} = \dfrac{x}{D}$

$\therefore\ (r_2 - r_1) = \dfrac{xd}{D}$ (4.5.21)

Using equations (4.5.20) and (4.5.21) in (4.5.13), respectively, we get equation for intensity at point P.

$$I_P = I_0\cos^2\left\{\frac{k\,d\tan\theta}{2}\right\} \quad (4.5.22)$$

and

$$I_P = I_0\cos^2\left\{\frac{k\,xd}{2D}\right\} \quad (4.5.23)$$

Using this equation intensity at any point at a distance x or at an angle θ from point A can be found, which is shown in the figure 4.9. It is evident from equation (4.5.22) or (4.5.23) that intensity at any point does not change with time. This type of interference is known as stationary interference.

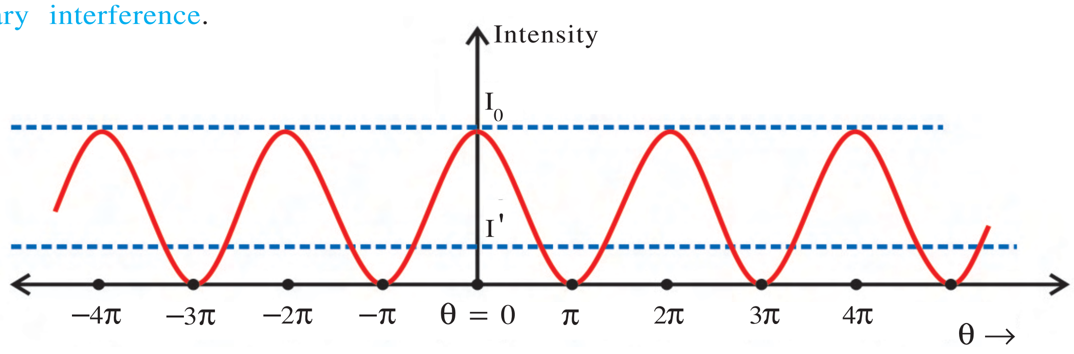


Figure 4.9 Intensity Distribution on the Screen

For the case of $\omega_1 \neq \omega_2$, waves oscillate with different frequencies. Therefore, their phase difference changes continuously. Thus, interference intensity at a point is no longer constant and it will be equal to the sum of average intensity due to both waves. For example, in the case of ordinary electric bulb, electrons transit randomly in the filament, producing waves of various frequencies. Hence, with an ordinary bulb, stationary interference pattern cannot be obtained. Thus, special techniques are required to obtain coherent sources for stationary interference pattern. They are classified into two categories : (i) division of wavefront and (ii) division of amplitude. In first type of method only narrow source is required, while for the latter, an extended source is necessary. We shall study only one method due to Young for obtaining coherent sources by using the method of division of wavefront.

For constructive interference, maximum intensity due to interference of two waves is written as,

$$I = I_0 = 4I'$$

$$= 2^2 I'$$

where $I' = I_1 = I_2$ is the intensity due to individual waves. This equation is a special case of N-source (wave) experiment as $I = N^2 I'$.

Distance Between Two Consecutive Bright Fringes : As shown in the figure 4.8, at point P_m and P_{m+1}, m^{th} and $(m+1)^{th}$ bright fringes are produced. Using the expression for path difference, $r_2 - r_1 = \frac{xd}{D}$,

Path difference at point P_m is

$$\frac{x_m d}{D} = m\lambda \tag{4.5.24}$$

Similarly at P_{m+1} path difference is

$$\frac{x_{m+1} d}{D} = (m+1)\lambda \tag{4.5.25}$$

$\therefore$ distance between these consecutive bright fringes is,

$$(x_{m+1} - x_m)\frac{d}{D} = \{(m+1) - m\}\lambda = \lambda \tag{4.5.26}$$

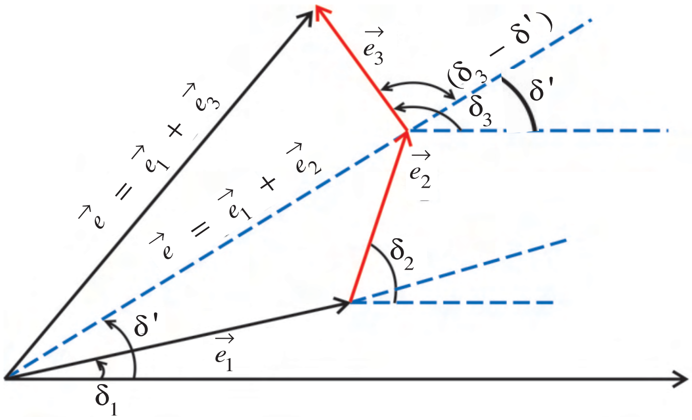
Denoting $x_{m+1} - x_m = \bar{x}$,

$$\bar{x} = \frac{\lambda D}{d} \tag{4.5.27}$$

We can similarly prove that even for two consecutive dark fringes also the distance remains same, i.e. $\bar{x}$.

Further, it can be seen from equation (4.5.27) that the distance between two consecutive bright or dark fringes does not depend on the order of the fringes. That is, all the fringes are of equal width. It is also evident from equation (4.5.27) or (4.5.23) that all bright fringes are equally bright.

Illustration 1 : Using the method of phasor diagram, prove that for constructive interference due to equally intense three waves from coherent sources, the maximum intensity is given by, $I = 3^2 I'$. Here, I' is the maximum intensity of individual waves.



Solution : As shown in the figure, first we add two vectors $\vec{e}_1$ and $\vec{e}_2$, and then $\vec{e}_3$ to their sum, using an equation (4.5.12) for coherent sources resultant intensity due to $\vec{e}_1$ and $\vec{e}_2$ is given by,

$$I_1' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta_1 - \delta_2) \tag{1}$$

Resultant intensity due to all waves is,

$$I = I'_1 + I_3 + 2\sqrt{I'_1 I_3}\cos(\delta' - \delta_3) \qquad (2)$$

But for constructive interference, phase difference will be in multiple of $2n\pi$. Therefore, all $\cos\delta$ terms will be unity. Using equations (1) in (2), we get,

$$I = I_1 + I_2 + I_3 + 2\sqrt{I_1 I_2} + 2\sqrt{(I_1 + I_2 + 2\sqrt{I_1 I_2})I_3}$$

But, $I_1 = I_2 = I_3 = I'$ (given),

$$\therefore\ I = I' + I' + I' + 2\sqrt{I'I'} + 2\sqrt{I'I' + I'I' + (2\sqrt{I'I'})\,I'}$$

$$= 5\,I' + 2 \times 2\,I' = 9\,I'$$

$$\therefore\ I = 3^2\,I'$$

4.5 (c) Young's Double Slit Experiment : As early as in 1665, Grimaldi attempted to produce interference using sunlight into a dark room through two pinholes in a screen. Unfortunately, he could see only an average uniform illumination. The reason is now clear, as described in the previous section.

Later in 1801, British physician Thomas Young made a special arrangement to obtain two coherent sources by the method of division of a wavefront. An experimental arrangement of Young's experiment is shown in the figure 4.10.

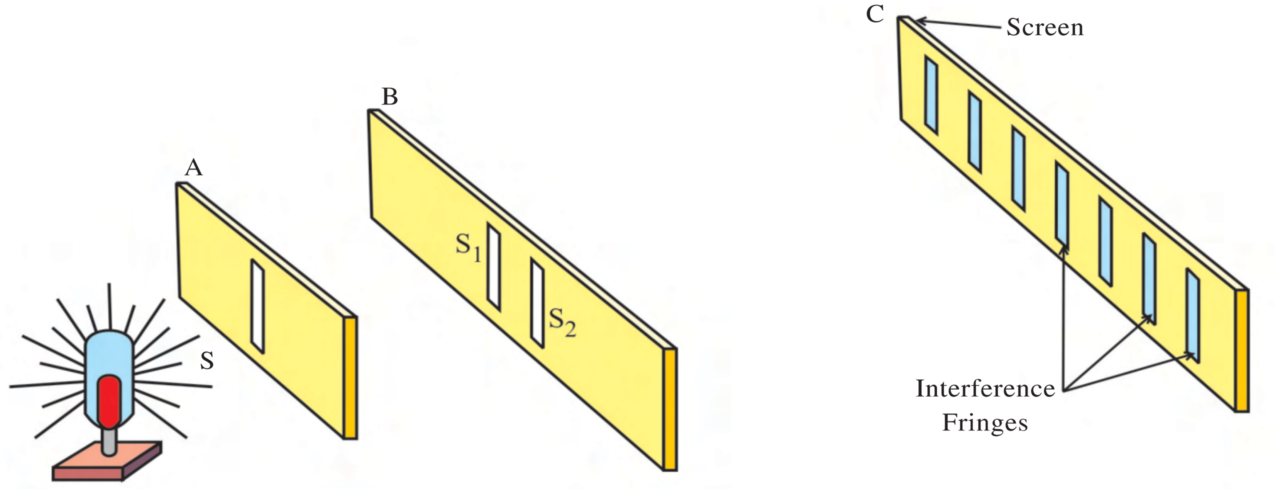


Figure 4.10 Young's Double Slit Experiment

A monochromatic light source emit cylindrical waves, which are collimated by slit S kept nearly on a screen A. Thus, slit now works as a secondary source of light and emit cylindrical waves towards the screen B. Two slits S_1 and S_2 on the screen B are kept such that $SS_1 = SS_2$. Also, distance between S_1 and S_2 is kept small, of the order of millimeter. Since S_1 and S_2 are equidistant from S, at a time only one wavefront is incident on them. According to Huygen's principle all the points on the same wave front vibrate in the same phase so that S_1 and S_2 act as coherent sources.

These cylindrical coherent waves emitted from S_1 and S_2 superpose on a screen C and produce stationary interference.

In the following figure 4.11, the cross section of the slit and the cylindrical wavefronts in a plane of the paper is shown.

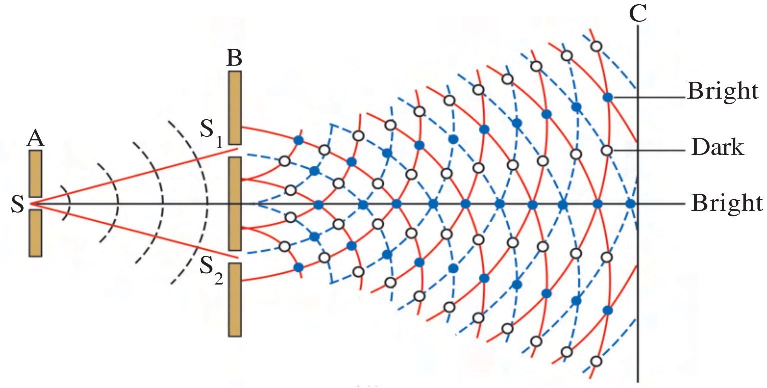


Figure 4.11 Interference Pattern Due to Cylindrical Wavefront. (Only for Information)

Here, points where constructive interference is produced are shown by solid circles, while those with destructive inference are shown by open circles.

Since in figure 4.10, secondary sources S_1 and S_2 are linear, on a screen C dark and bright fringes (bands) are seen.

It is to be noted that in his historical experiment Young had used pinholes in place of slits and white light instead of monochromatic light.

Illustration 2 : The ratio of intensities of rays emitted from two different coherent sources is α. For the interference pattern formed by them, prove that

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{1+\alpha}{2\sqrt{\alpha}}, \text{ where,}$$

I_{max} = Maximum of intensity in the interference fringes.

I_{min} = Minimum of intensity in the interference fringes.

Solution : For two waves, ratio of their intensities,

$$\frac{I_1}{I_2} = \alpha \text{ (given)}$$

But we know that $I \propto A^2$, where A is an amplitude.

$$\therefore \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} = \alpha$$

$$\therefore \frac{A_1}{A_2} = \frac{\sqrt{\alpha}}{1}$$

$$\therefore \frac{A_1+A_2}{A_1-A_2} = \frac{A_{max}}{A_{min}} = \frac{\sqrt{\alpha}+1}{\sqrt{\alpha}-1}$$

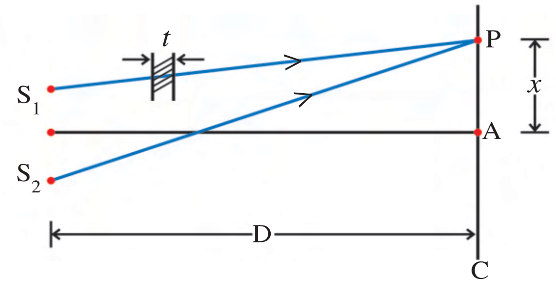
$$\therefore \frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2} = \frac{(1+\sqrt{\alpha})^2}{(\sqrt{\alpha}-1)^2} = \frac{(1+2\sqrt{\alpha}+\alpha)}{(1-2\sqrt{\alpha}+\alpha)}$$

$$\therefore \frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{(1+2\sqrt{\alpha}+\alpha)+(1-2\sqrt{\alpha}+\alpha)}{(1+2\sqrt{\alpha}+\alpha)-(1-2\sqrt{\alpha}+\alpha)}$$

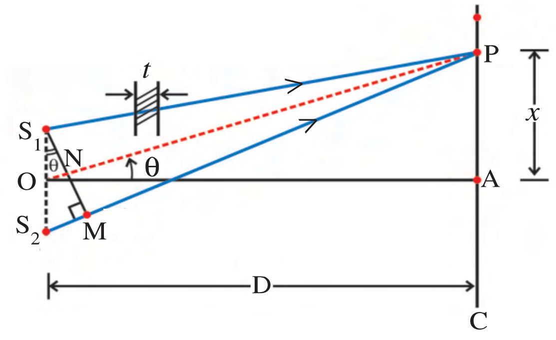
$$= \frac{\alpha+1}{2\sqrt{\alpha}}$$

Reciprocal of the above term, i.e. $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ is known as visibility of fringes.

Illustration 3 : Young's double slit experiment is used to determine the thickness of a thin transparent sheet. An experimental arrangement to find the thickness t of transparent material having refractive index n is shown in the figure. Let the central bright fringe, which was obtained at a point A on a screen in absence of the thin sheet shifts to point P. Derive the formula for thickness of the sheet.



Solution : In absence of the sheet, path difference between S_1A and S_2A is zero. Therefore, central bright fringe is located at point A. On introducing transparent sheet in the path of beam from source S_1, the fringes get displaced towards the beam in whose path a sheet is introduced. This is called the lateral shift (x) of fringes.



Now at point P the central bright fringe is obtained. That is, path difference $S_2P - S_1P = 0$

$\therefore\ \{(S_2P - t) + t_{medium}\} - S_1P = 0$

where t_{medium} = pathlength in a medium (optical path) = $t\ n$

$\therefore\ \{S_2P - t + t\ n\} - S_1P = 0$

$\therefore$ path difference,

$S_2P - S_1P = S_2M = (n - 1)t$ (1)

From ΔS_1S_2M, $S_2M = d\sin\theta$ (2)

Since two sources S_1 and S_2 are closely placed, θ (in rad) is very small.

$\therefore\ \sin\theta \approx \theta \approx \tan\theta$

From ΔOAP, $\tan\theta = \frac{x}{D}$ (3)

Using equation (3) into (2),

$$S_2M = \frac{xd}{D}$$

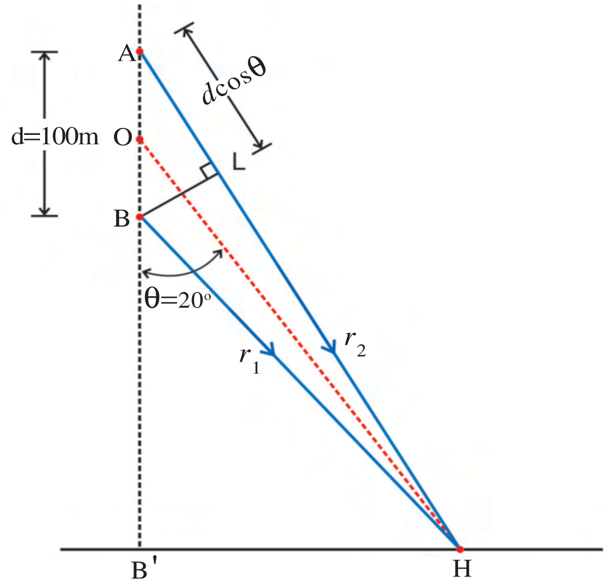
$\therefore$ from equation (4) and (1), $\frac{xd}{D} = (n - 1)t$

$\therefore$ thickness, $t = \frac{xd}{D(n - 1)}$

Illustration 4 : Two radio antennas A and B emit radio waves of frequency 1100 kHz. These waves get superposed at point H. If the distance between two antennas is 100 m and the line joining point H with the midpoint of these antennas makes an angle 20^o with the vertical, find resultant intensity in terms of maximum intensity (I_0) at H. Distance BH = 20 km. Take $\cos 20^o = 0.9397$, $\cos 62^o = 0.4695$.

Solution : Here, antennas A and B are two coherent sources of waves with frequency 1100×10^3 Hz.

$\therefore$ using an equation,



$$I = I_0 \cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

$$= \frac{2\pi \times 1100 \times 10^3}{3 \times 10^8}$$

and $(r_2 - r_1) = AL = d\cos\theta = 100 \times \cos 20^o$

$$= 100 \times 0.9397 \times \text{m}$$

$$= 93.97 \ \text{m}$$

$$\therefore \ I = I_0 \cos^2\left\{\frac{2\pi 1100 \times 10^3 \times 93.77}{2 \times 3 \times 10^8}\right\}$$

$$\therefore \ I_0 \cos^2\{\pi \times 0.3445\} = I_0\{\cos(62^o)\}^2$$

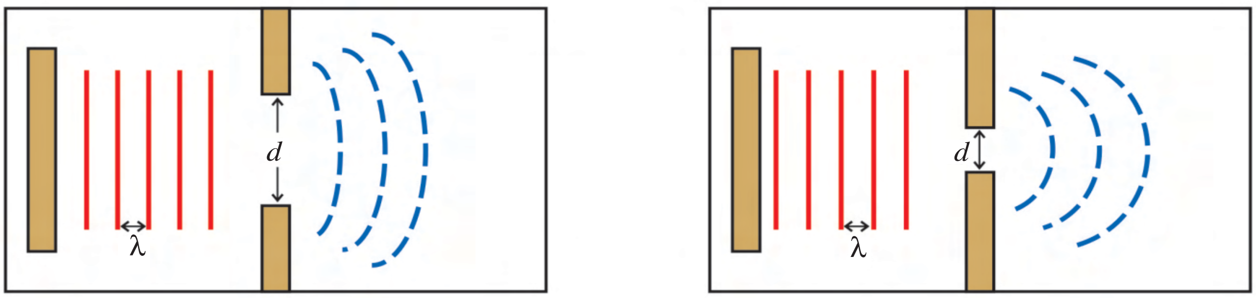
$$= I_0 \times (0.22)$$

$$\therefore \ \frac{I}{I_0} = 0.22$$

4.6 Diffraction

When waves encounter obstacles or openings like slits, they bend round the edges. This bending of waves is called diffraction. It was first discovered by Grimaldy. Since this is strictly against the idea of rectilinear propagation of light ray, we conclude that the ray optics cannot explain the phenomenon of diffraction.

To understand the phenomenon of diffraction consider the following day to day experience. We know that light and sound energy both travel in the form of waves. We have experienced that a person standing near an open door in one room may listen to a person standing on the other side of the wall but cannot see him. This implies that sound waves bend near the edge of the door showing the diffraction, but light waves do not! Then the question is why light waves do not diffract ? To explain this apparent paradox between sound waves and light waves, consider an experiment of ripple tank, as shown in the figure 4.12.



$d = 5\lambda$ $d = 2\lambda$

(a)   *(b)*

Figure 4.12 Ripple Tank Experiment for Diffraction

In this experiment, linear waves can be produced with the help of straight wooden strip by tapping periodically to the water surface. Near to this, a slit is formed by placing two blocks of wax. In this experiment the width of the slit and wave length of the waves produced can be taken as variable. Let the wave length of waves produced by the controlled oscillations of the stick be λ.

Suppose initially width (d) of the slit is kept as, $d = 5\lambda$. In this situation the waves emerging out of slit are found to be almost linear (see figure 4.12(a)). But when the width of the slit is reduced to $d = 2\lambda$, emerging waves are diffracted by considerable amount (figure 4.12(b)).

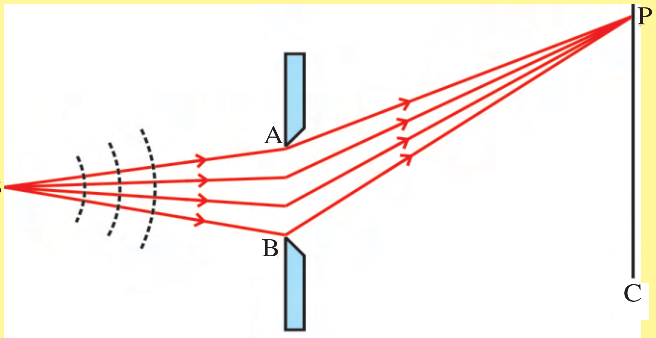
These observations show that smaller is the width of the slit, more will be the diffraction for a given wavelength. It is also found that if the wavelength and the width of the slit are so changed that the ratio $\frac{\lambda}{d}$ remains constant, amount of bending (= diffraction) does not change. Thus, we conclude that diffraction of a wave through a slit depends on the ratio $\frac{\lambda}{d}$. Also, more is the $\frac{\lambda}{d}$ ratio greater is the diffraction.

In the case of day to day life, wavelength of sound waves is typically of the order of 1m. The width of the door is also about 1 m, making the ratio, $\frac{\lambda}{d}$ nearly one. But considering average wavelength of visible portion of electromagnetic spectrum as 6000 $\overset{\circ}{A}$, i.e. 6×10^{-7} m, the ratio $\frac{\lambda}{d}$ will be of the order of 10^{-7}. This ratio is too small to produce any appreciable bending of light waves. Hence, in routine life light waves do not appear to diffract. However, if a very narrow slit is used, which increases the ratio $\frac{\lambda}{d}$, appreciable diffraction of light is also possible.

From the above discussion, we infer that in order to keep $\frac{\lambda}{d}$ ratio large for given wavelengths, width (opening) of the slit should be kept small. This requirement suggests that the complete wavefront does not pass through the slit. Slit allows only limited part of wavefront to pass through it. Thus, we say that "diffraction is the effect produced by the limited part of the wavefront."

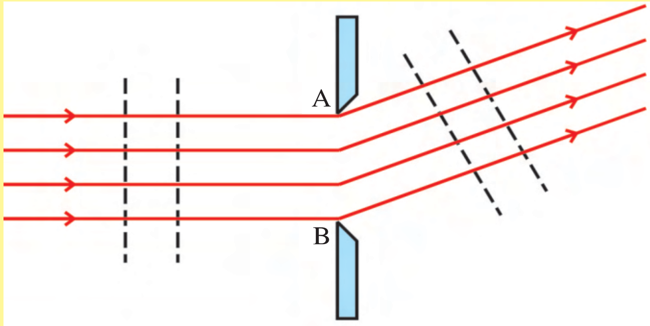
Types of Diffraction (Only for Information) : According to the type of the wavefronts hindered by the obstacle, diffraction is classified into two types. (1) Fresnel and (2) Fraunhoffer diffraction.

When the distances between the obstacle (slit) AB and the source of light S, as well as between the obstacle AB and screen C are finite the diffraction produced is known as Fresnel diffraction, (refer the figure (a)). In Fresnel diffraction waves are spherical or cylindrical.



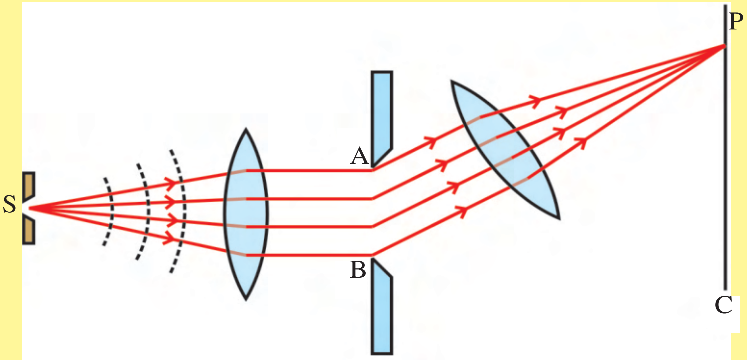
(a) Fresnel Diffraction

When the light incident on slit AB is coming from infinite distance (or the incident waves are plane) and the distance between the obstacle AB and screen C is also infinite, the diffraction is called Fraunhoffer diffraction, (refer figure (*b*)).



(*b*) Fraunhoffer Diffraction

Fraunhoffer diffraction can be obtained in the laboratory with an experimental arrangement as shown in figure (*c*).



(*c*) Laboratory Arrangement for Fraunhoffer Diffraction

Here, the source being at the focus of the convex lens, the rays incident on the slit AB are parallel. While on placing another lens in the passage of set of parallel rays diffracted in different direction, they can be focused at different points on the screen C. Thus in figure (*c*) the conditions of Fraunhoffer diffraction are fulfilled.

4.6 Diffraction Due to Single Slit

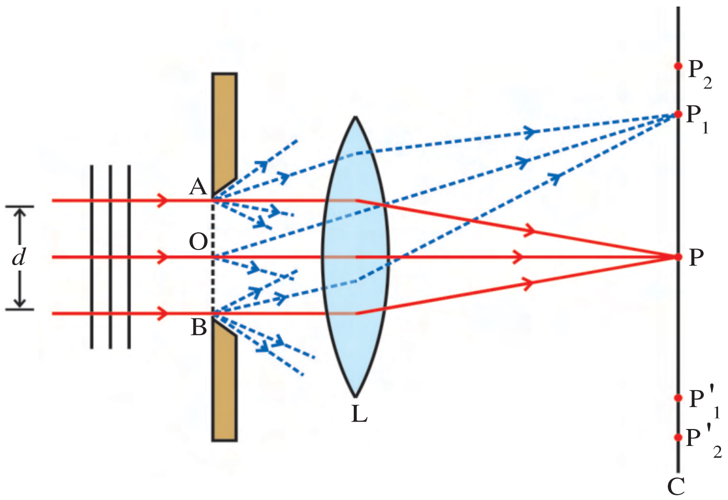


Figure 4.13 Diffraction Due to Single Slit

We now examine the diffraction pattern of plane waves (i.e. Fraunhoffer diffraction) of wavelength λ produced by a single slit of width d (See figure 4.13). When such plane wavefront arrive at a plane of slit, according to Huygen's principle, all points on the slit (like A, O, B) act as secondary sources having the same phase, and produce secondary waves. In order to produce a diffraction pattern of bright and dark fringes (i.e. interference maxima and minima) on the screen (C), converging lens (L) is often used.

Thus, now diffracted waves are converged on to the screen and produce interference pattern. Therefore, we can now use a procedure similar to the one we need to locate the fringes in Young's double slit experiment.

(1) Central Maximum : As shown in figure 4.14 (a), point P_0 of a screen C is lying on a perpendicular bisector of slit AB. Therefore, those waves originated from each points of a slit and diffracted normal to the plane of the slit (i.e, in the direction of incident waves, $\theta = 0$) will be all concentrated at point P_0 by a lens L. In figure 4.14 (a), out of many such waves only three representative rays are shown. Here, screen is at the focal plane of the lens. It is obvious from the figure that rays travelling less distance in air have to travel more distance through the lens. Since speed of waves in lens is less than their speed in air, their optical path will be equal. (Optical distance in a medium is equal to the product of refractive index of the medium to geometrical path length in air). Thus, all rays reaching to point P_0 having equal phase produce constructive interference, and point P_0 will be having maximum intensity. Point P_0 is known as Central Maximum.

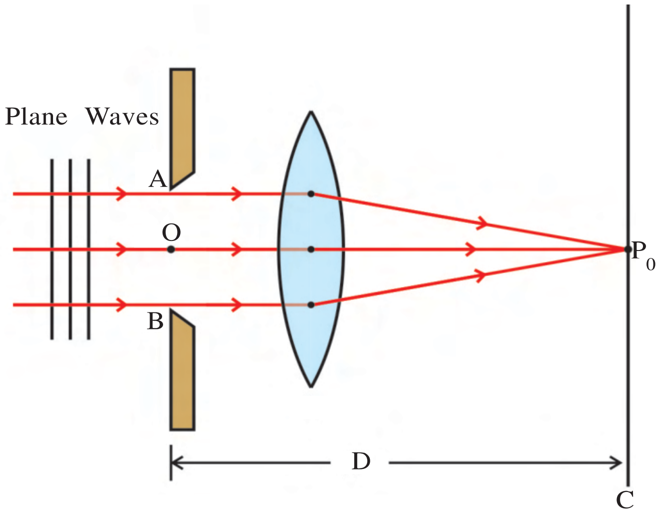


Figure 4.14 (a) Central Maximum

Only for Information : In laboratory experiment, lens (L) used to produce Fraunhoffer difference decides the width of the central maximum. But for lens-less diffraction by keeping screen at infinite distance (d << D), width of central maximum is roughly equal to the width of the slit (d).

For analysis of diffraction pattern (i.e. to know the intensity distribution and location of interference fringes) mathematical treatment is so complex (which is given at the end of the chapter as an appendix for information) that we will give only logical proof.

(2) First Minimum : As shown in figure 4.14 (b), consider waves which are diffracted at an angle θ with respect to perpendicular bisector XP_0 of the slit. Here, point X is the midpoint of slit AB. Therefore, AX = XB = $\frac{d}{2}$. Here, secondary waves originated from all points A, X, B of slit are thought to be divided in two parts : waves from A to X and waves

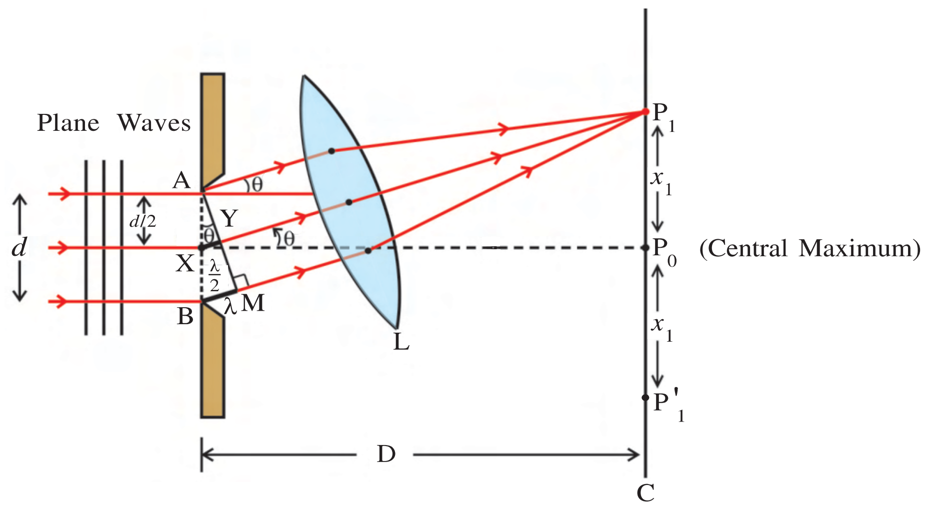


Figure 4.14 (b) First Minimum

from X to B. As per figure, all these waves diffracted at an angle θ are focused at point P_1 of a screen. To know whether constructive or distructive interference will take place at point P_1, we require to know phase difference between these waves. For that, draw AM $\perp$ BL. It is obvious that all the waves reaching from AM to P_1 have equal optical path.

But rays going from A and X, and reaching to P_1 have path difference of XY.

Let us assume that diffracted angle θ is such that $XY = \frac{\lambda}{2}$.

In this situation, waves from A and X will follow the condition of distructive interference at point P_1. And their resultant intensity will be zero.

Further, as corresponding to point A we have point X for which condition for distructive interference holds, like wise, corresponding to every point of part AX, we have successive points in section XB such that for every such pair, path difference at point P_1 is $\frac{\lambda}{2}$.

Thus, in totality, destructive interference will take place at point P_1 and it will be dark.

Point P_1 is known as First Minimum. From the symmetry of the figure it is obvious that at the same distance from P_0 on other side also we have first minimum (P_1').

(3) First Maximum : As shown in the figure 4.14 (*c*), suppose slit AB is assumed to be divided in three equal (odd number) parts AX_1, X_1X_2 and X_2B.

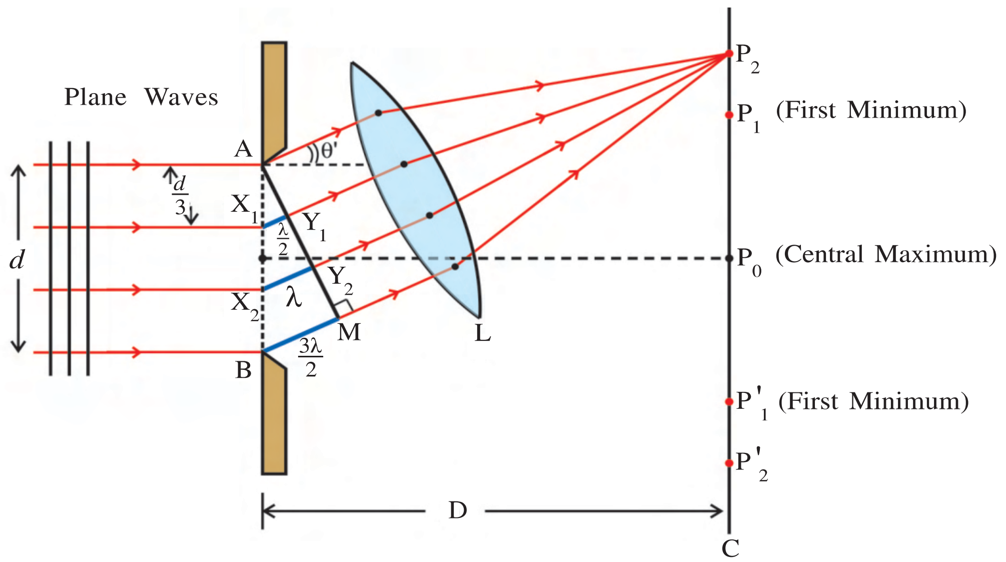


Figure 4.14 (*c*) First Maximum

Here, $AX_1 = X_1X_2 = X_2B = \frac{d}{3}$. As per figure, draw AM $\perp$ BL. Waves reaching from AM to P_2 will have equal optical path.

Waves starting from A and X_1 and imposing at point P_1 will have path difference X_1Y_1.

Let us assume that diffracted angle θ' is such that $X_1Y_1 = \frac{\lambda}{2}$, $X_2Y_2 = \lambda$, $BM = \frac{3\lambda}{2}$.

Since path difference between waves originated from A and X_1, and superimpose at point P_2 is $\frac{\lambda}{2}$, they interfere destructively. And intensity at point P_2 due to these waves will be zero.

In the same way, waves from every pair AX_1 and X_1X_2 will have path difference $\frac{\lambda}{2}$. And as explained above, resultant intensity at point P_2 due to them will be zero.

However, intensity of rays differacted at an angle θ' from section X_2B is not vanishing at point P_2. Therefore, due to this section of the slit, there remains some intensity at point P_2, and point P_2 will be bright.

Here, point P_2 is known as first maximum. It is obvious that the intensity at point P_2 is verymuch less as compared to P_0.

Of course, to know locations of higher order minima and maxima, and intensities of maxima relative to central maximum, above mentioned of logical method is not useful.

Intensity of diffracted light at any point on the screen (C) is given by the following formula (see information given in the appendix).

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \tag{4.6.1}$$

where I_0 is maximum intensity at point P_0 and

$$\alpha = \frac{\pi d \sin\theta}{\lambda} \tag{4.6.2}$$

Condition for Central Maximum : It is clear from the figure 4.13 that secondary waves from slit for which $\theta \approx 0$ (without undergoing diffraction) will meet at point P_0 on the screen, C. From equation (4.6.2), as $\theta \to 0$, $\alpha \to 0$.

Therefore, according to equation (4.6.2),

$$\text{Intensity} \quad I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0 \qquad \left(\therefore \lim_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1\right)$$

Thus, point P_0 will be bright, which we call the central maximum. On either side of it, at equal separation, we can observe successive minima and maxima.

Condition for Minima : If $\alpha = n\pi$; $n = 1, 2, 3, ...$, according to equation (4.6.1), we get successive minima for different values of n. From equation (4.6.2),

$$\frac{\pi d \sin\theta}{\lambda} = n\pi$$

$$\therefore \; d\sin\theta = n\lambda \tag{4.6.3}$$

Equation (4.6.3) gives the condition for minima. For $n = 1$ we get first minimum (point P_1), for $n = 2$ we get second order minimum (point P_3), etc. Due to symmetry on the other side of point P_0 corresponding minima (P_1', P_3',....) are also obtained.

Condition for Maxima : If $\alpha = (2n + 1)\frac{\pi}{2}$, $n = 1, 2, 3, ...$, according to equation (4.6.1), we get successive maxima for different values of n. From equation (4.6.2),

$$\frac{\pi d \sin\theta}{\lambda} = (2n + 1)\frac{\pi}{2}$$

$$\therefore \; d\sin\theta = (2n + 1)\frac{\lambda}{2} \tag{4.6.4}$$

Equation stated above gives the condition for maxima. For $n = 1$, we get first order maxima (points P_2 and P'_2), for $n = 2$ we get second maxima (points P_4 and P'_4), etc.

(1) For first order maximum (i.e. $n = 1$)

$$\alpha = (2 \times 1 + 1)\frac{\pi}{2} = \frac{3\pi}{2}$$

$$\therefore \; I = I_0 \left(\frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} \right)^2 = I_0 \left(\frac{-1}{\frac{3\pi}{2}} \right)^2 = \frac{4I_0}{9\pi^2} \approx \frac{I_0}{22}$$

(2) For second order maximum (i.e. $n = 2$)

$$\alpha = \frac{5\pi}{2} \Rightarrow I = I_0 \left(\frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}} \right)^2 = \frac{4I_0}{25\pi^2} \approx \frac{I_0}{62}$$

Thus, the intensity of maxima decreases rapidly with the order of maxima.

Further, from equation (4.6.2), $\frac{\pi d \sin\theta}{\lambda} = \alpha$, for a given order of maxima or minima (i.e. value of α is fixed) and fixed wavelength, $\sin\theta \propto \frac{1}{d}$. This suggests that the smaller the width of slit, the larger will be θ. From figures 4.14, then points P_1, P_2, ..., etc. will be found at larger angular separation. Thus, the diffraction pattern will spread/expand on the screen. However, intensity of diffraction maxima decreases in proportion to decrease in the width of a slit. To illustrate this point, graphs of intensity versus θ for two cases, $d = 5\lambda$ and $d = 10\lambda$, are shown in the figure 4.15.

Width of Central Maximum : "The distance between two first order minimum is known as width of central maximum." As per figure 4.14(b), width of the central maximum is $2x_1$.

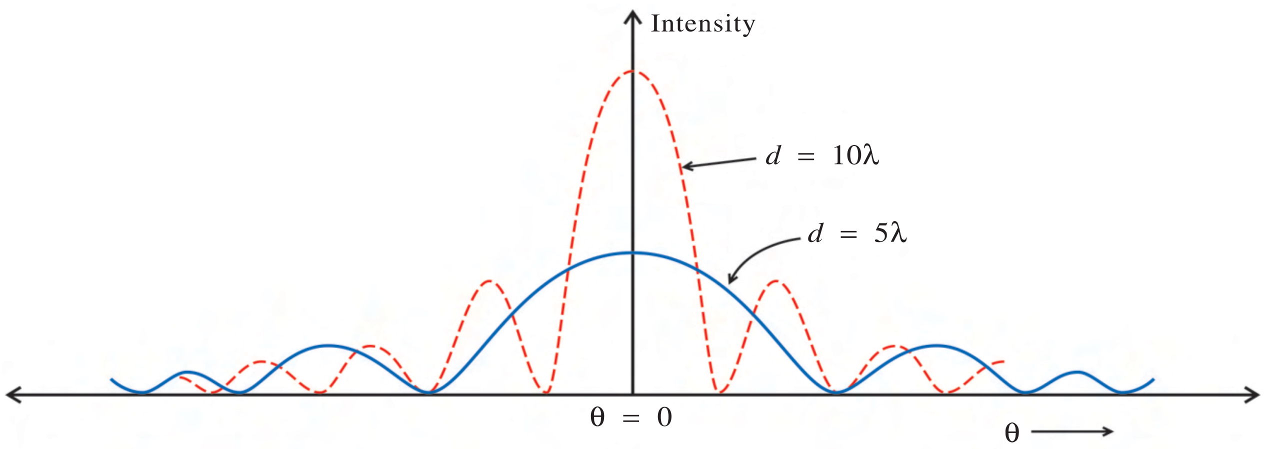


Figure 4.15 Intensity Distribution Due to Single Slit Diffraction

For first order minimum, $d\sin\theta = \lambda$ or $\sin\theta = \frac{\lambda}{d}$ (4.6.5)

Also, from the figure 4.14 (*b*), $\tan\theta = \frac{x_1}{D}$ (4.6.6)

But for small angle of diffraction θ (in rad) is small. Therefore, $\sin\theta \approx \tan\theta$. From equation (4.6.5) and (4.6.6)

$$\frac{x_1}{D} = \frac{\lambda}{d}$$

$\therefore$ Width of central maximum, $2x_1 = \frac{2\lambda D}{d}$

Angular width of central maximum is given by

$2\theta = \frac{2\lambda}{d}$ (See equation (4.6.5)).

In the case of optical instruments such as telescope or microscope, objective lens acts as a circular obstacle to the incoming wavefronts, and produces diffraction. In such diffraction pattern, due to circular aperture there is a central circular bright fringe, which is called the Airy's disc. It is surrounded by alternate dark and bright concentric rings called Airy's rings.

For Franuhoffer diffraction, the width of central maximum is the measure of the deviation. If the width of the beam is more than linear measure the obstacle (width in the case of slit and diameter of an objective for optical instruments), light will deviate more. If the width of the beam is either nearly equal to or smaller than the obstacle, it will be travelling straight. In this situation ray optics can be used. Thus, we can define a length, called Fresnel distance (Z_f) such that $Z_f = \frac{d^2}{\lambda}$, where d is the linear measure of the obstacle and λ is the wavelength of light. It defines the distance upto which bending is very less, and ray optics is applicable. However, one should not use this cirterion as a condition for using ray optics.

4.7 Comparison between Interference and Diffraction

In common, the patterns (fringes) obtained in both interference and diffraction are due to superposition of waves. Fundamentally, there are some differences between interference and diffraction, as given below

Interference	Diffraction
(1) It is obtained due to superposition of waves from different coherent sources. That is, it is the effect produced due to superposition of different wavefronts.	(1) It is obtained due to superposition of waves originated from the different parts of the same wavefront.
(2) Bright and dark all interference fringes are of equal width.	(2) Diffraction fringes are not of the same width. Central maximum is having the largest width, while width of maxima and minima decreases for higher order of diffraction.
(3) All bright fringes have equal intensities.	(3) Central maximum has highest intensity, and it decreases with higher order diffraction maxima.
(4) Interference dark bands are perfectly dark.	(4) Regions of minimum intensities may not be perfectly dark.

Illustration 5 : Angular width of a central maximum in a Franhoffer diffraction obtained by a single slit using light of wavelength 6000 $\mathring{A}$ is measured. If light of another wavelength is used, the angular width of the central maximum is found to be decreased by 30%. Find (i) the other wavelength (ii) If the experiment is repeated keeping the apparatus in a liquid, the angular width of central maximum decreases by the same amount (i.e. 30%), find its refractive index.

Solution : Angular width of central maximum is given by (1)

$$2\theta = \frac{2\lambda}{d} \Rightarrow \theta = \frac{\lambda}{d}$$

For first light, $\theta_1 = \frac{\lambda_1}{d}$ and for second light, $\theta_2 = \frac{\lambda_2}{d}$

$$\therefore \frac{\theta_2}{\theta_1} = \frac{\lambda_2}{\lambda_1} \quad (2)$$

But θ_2 is 30% less that of θ_1

That is, θ_2 = 70% of $\theta_2 = 0.7\ \theta_1$

Using in equation (2) $\frac{\lambda_2}{\lambda_1} = 0.7$

$$\therefore \lambda_2 = 0.7 \times 6000\ \mathring{A} = 4200\ \mathring{A}$$

That is, wavelength in a liquid is 4200 $\mathring{A}$.

$$n = \frac{\lambda_{air}}{\lambda_{liquid}} = \frac{6000}{4200} = 1.43.$$

Illustration 6 : Obtain the necessary condition to observe maxima in the case of Fraunhoffer diffraction in term of $\alpha\left(=\frac{\pi d \sin\theta}{\lambda}\right)$.

Solution : In the case of Fraunhoffer diffraction, intensity at a point is given by,

$$I = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right) \quad (1)$$

If at any point of maxima takes place, $\frac{dI}{d\alpha} = 0$

Using equation (1),

$$\frac{dI}{d\alpha} = I_0\left\{\frac{2\sin\alpha\cos\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^3}\right\} = 0$$

(Condition for maxima require $\frac{dI}{d\alpha} = 0$)

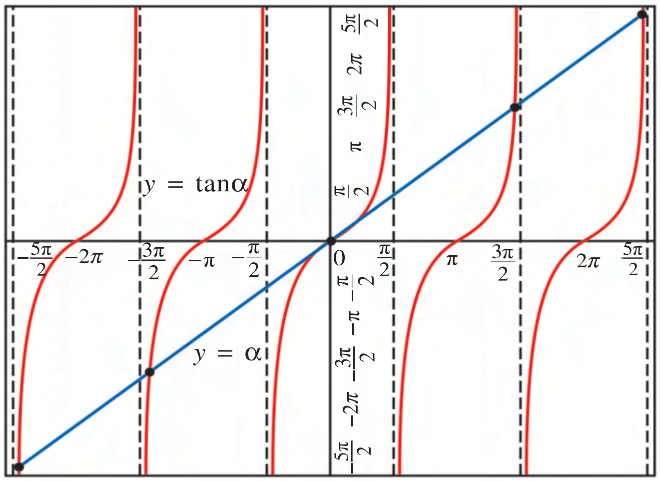
$$\therefore \frac{2\sin\alpha\cos\alpha}{\alpha^2} = \frac{2\sin^2}{\alpha^3}$$

$$\therefore \tan\alpha = \alpha \quad (2)$$

Equation (2) gives the necessary condition for maxima to take place.

Only for Information : To find the value of α from equation (2) for which diffraction maxima occur, graph of $y = \tan\alpha$ and $y = \alpha$ are plotted. Intersections of these graphs give value of α (in rad) for maxima.

It also explains that why we had not considered $\alpha = \frac{\pi}{2}$ value in condition for maxima.



4.8 Resolving Power of Optical Instruments

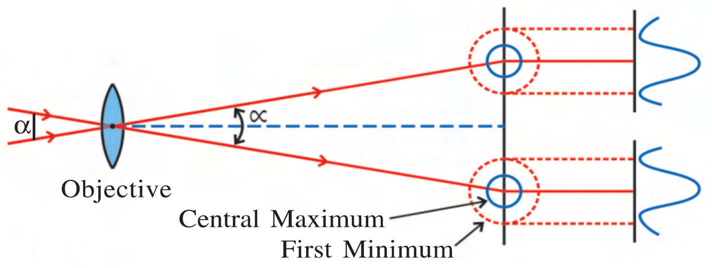
As studied in the previous semester, optical instruments are used to see an object clearly and comfortably. But when two objects or their images are very close to each other, they may appear as one. And it may not be possible for the eyes to see them as separate. Even optical instruments such as telescope or microscope used to see object have limitations in resolving two nearby objects on their images due to diffraction phenomenon. In this section, we will study resolving power of optical telescope and microscope.

Rayleigh's Criterion : When a beam of light (light waves) from a point object passes through the objective of an optical instruments, the lens acts like a circular aperture and produces a diffraction pattern (Airy's disc and Airy's rings) instead of sharp point image. If there are two point objects kept close to each other, their diffraction pattern may overlap. Then it may be difficult to distinguish them as separate. The criterion to get distinct and separate images of two closely placed point like objects was given by Rayleigh.

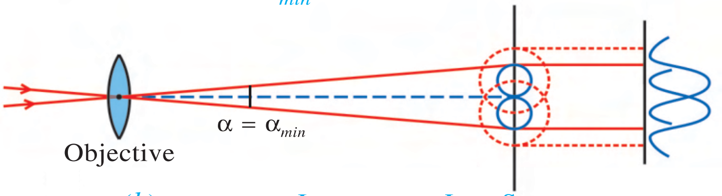
"The images of two point like objects can be seen as separate if the central maximum in the diffraction pattern of one falls either on the first minimum of the diffraction pattern of the other or it is at a grater separation."

For the case of circular aperture diffraction due to lens of diameter D, Rayleigh's criterion is given by, $\sin\theta \approx \theta = \frac{1.22\lambda}{D}$. Here, λ is wavelength of light.

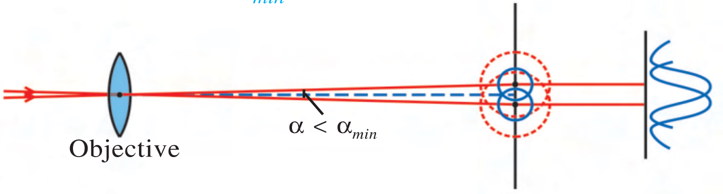
4.8 (*a*) Resolving Power of Telescope : Suppose we are observing two nearby stars with the help of a telescope. The ray coming from these stars make an angle α at the lens of the telescope as shown in the figure 4.16. Since only limited parts of incident wavefronts can pass through the lens, lens acts as an obstacle, and produces diffraction. An image of stars appear as two central bright spots surrounded by alternate dark and bright rings of decreasing intensity as we go away from the central bright spots. From the figure 4.16 (*a*), it is obvious that, if angle α is large, the diffraction pattern will be quite distinct. Hence, the images of the stars will be seen as separate.



(a) $\alpha > \alpha_{min}$ Images are Separate



(b) $\alpha = \alpha_{min}$ Images are Just Separate



(c) $\alpha < \alpha_{min}$ Images Overlap

Figure 4.16 Resolution of Images

But if two stars are close to each other (figure 4.16. (*b*) and (*c*)) angle α will be very small and the diffraction pattern of both stars may mingle with each other. In this situation it is difficult to see both the stars distinctly and clearly.

"The ability of an optical instrument to produce distinctly separate images of two closely placed objects is called its resolving power (R.P.)"

It is clear from the above discussion for optical instruments like telescope and microscope that R.P. depends on an angle α. If diameter of an objective of telescope is D and its focal length is f, then the width of central maximum obtained by it is given by $f\left(\frac{1.22\lambda}{D}\right)$. Here, λ is the wavelength of incident light. Width of central maximum on screen $= f\alpha$.

$\therefore$ The necessary minimum angle to see two images distinctly (α_{min}) is, $f\alpha_{min} = f\left(\frac{1.22\lambda}{D}\right)$

$$\therefore \quad \alpha_{min} = \frac{1.22\lambda}{D} \qquad (4.8.1)$$

Here, α_{min} is known as angular resolution of the telescope, while its inverse is known as resolving power or geometrical resolution.

$$\text{Thus, R.P. of telescope,} = \frac{1}{\alpha_{min}} = \frac{D}{1.22\lambda} \qquad (4.8.2)$$

Since R.P. of telescope is directly proportional to the diameter of its objective, telescopes with large objective lens are used to see very far closely placed celestial objects.

For example, angular resolution of Hubble telescope is 0.1'' (0.1 second), while angular resolution of human eye is approximately 1' – 2' (1 to 2 minutes).

4.8 (*b*) Resolving Power of Microscope : Image of a point like object by an objective of microscope is shown in figure 4.17. Let diameter of the lens be D and its focal length be *f*.

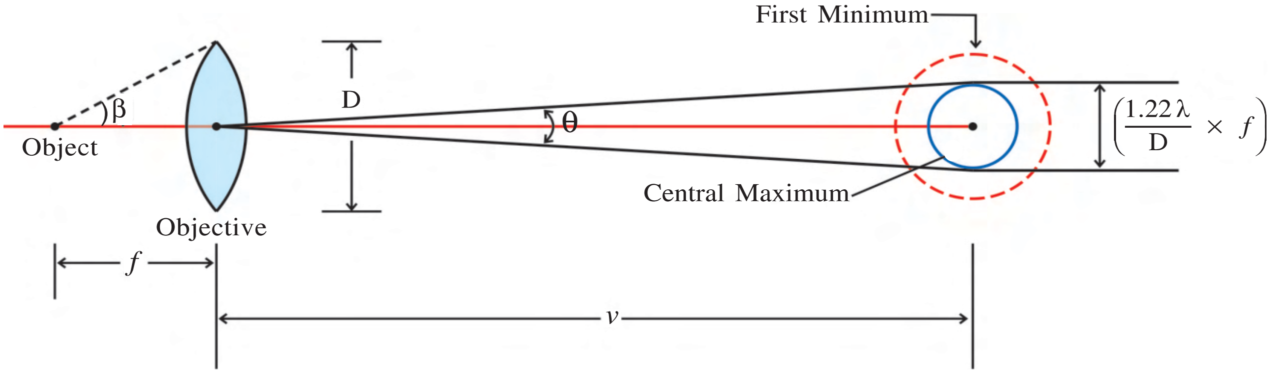


Figure 4.17 Image Formation by Microscope

As object distance is usually kept greater than that of f (remember theory of compound microscope of previous semester). Let an image distance be v. The angular width of central maximum due to the effect of diffraction is,

$$\theta = \frac{1.22\lambda}{D}$$

$$\therefore \text{ width of central maximum, } v\theta = \left(\frac{1.22\lambda}{D}\right)v \qquad (4.8.3)$$

If image of two point like objects are at a separation less than $v\theta$, then it will be seen as a mixed single object. It can be proved that a minimum distance (d_m) for which objects can be seen separately is given by,

$$d_m = \left(\frac{1.22\lambda}{D}\right)\frac{v}{m} \qquad (4.8.4)$$

When m $= \frac{v}{f}$ magnification. Substituting value of m in above equation,

$$d_m = \left(\frac{1.22\lambda}{D}\right)f \qquad (4.8.5)$$

From the figure 4.17, $\frac{\left(\frac{D}{2}\right)}{f} = \tan\beta$

$\therefore \frac{D}{f} = 2\tan\beta$. Using this in equation (4.8.5),

$$d_m = \left(\frac{1.22\lambda}{2\tan\beta}\right) \qquad (4.8.6)$$

For small angle β (in rad), $\tan\beta \approx \sin\beta$

$$\therefore d_m = \left(\frac{1.22\lambda}{2\sin\beta}\right) \qquad (4.8.7)$$

Reciprocal of d_m known as R.P. of microscope. That is,

$$\text{R.P. of microscope} = \frac{1}{d_m} = \left(\frac{2\sin\beta}{1.22\lambda}\right) \qquad (4.8.8)$$

Equation (4.8.8) is derived for air as medium between an object and objective lens. Instead, some medium with large refractive index (n) may be used between object and objective to increase the R.P. of microscope. In this situation, R.P. of microscope is given by $\left(\frac{2n\sin\beta}{1.22\lambda}\right)$.

Here, the term $n\sin\beta$, is known as Numerical Aperture. Normally, appropriate type of oil immersion is used to increase the resolution. It is also true that R.P. of microscope is inversely proportional to wavelength λ.

Illustration 7 : In the following two cases upto what minimum distance two point like objects can be seen distinctly by a human eye ? (1) Distance between eye and objects is 25 cm and (2) Distance between eye and object is 5 m. Diameter of pupil of eye is 2.5 mm. Consider wavelength of light 5500 $\overset{\circ}{\mathrm{A}}$.

Solution : Considering an eye as a simple microscope $d_{min} = \frac{1.22\lambda f}{D}$.

Here, f is the focal length of human eye. Remember that ciliary muscle of eye sets the focal length of the lens to the object distance.

$$(1)\ d_{min} = \frac{1.22 \times 5500 \times 10^{-10} \times 0.25}{2.5 \times 10^{-3}} = 6.71 \times 10^{-5} \mathrm{m}$$

$$(2)\ d_{min} = \frac{1.22 \times 5500 \times 10^{-10} \times 5}{2.5 \times 10^{-3}} = 1.34 \times 10^{-3} \mathrm{m}$$

Illustration 8 : Huble space telescope is at a distance 600 km from earth's surface. Diameter of its primary lens (objective) is 2.4 m. When a light of 550 nm is used by this telescope, at what minimum angular distance two objects can be seen separately ? Also obtain linear minimum distance between these objects. Consider these objects on the surface of earth and neglect effects of atmosphere.

Solution : $\alpha_{min} = \frac{1.22\lambda}{D} = \frac{1.22 \times 550 \times 10^{-9}}{2.4}$

$= 2.8 \times 10^{-7}$ rad

$= 0.058''$ $(\because\ 1'' = 4.85 \times 10^{-6}$ rad$)$

Linear distance between objects $= \alpha_{min}L$,

where L = distance between telescopes and objects.

$\therefore$ linear distance between objects $= 2.8 \times 10^{-7} \times 600 \times 10^{3}$

$= 0.17$ m

Illustration 9 : Calculate the useful magnifying power of a telescope of 11 cm objective. The limit of angular resolution of eye is $2'$ and wavelength of light used is 5500 $\overset{\circ}{\mathrm{A}}$.

Solution : The magnifying power of a telescope is given by,

$M = \frac{D}{d}$, where D = diameter of the objective

d = diameter of the eyepiece.

For normal (useful) magnification, diameter of eyepiece should be equal to the diameter of the pupil (d_e) of the eye. Therefore, useful magnification is

$$M = \frac{D}{d_e} \qquad (1)$$

From the equation of limit of resolution of telescope.

$$d\theta = \frac{1.22\,\lambda}{D}$$

$$= \frac{1.22 \times 5500 \times 10^{-10}}{11 \times 10^{-2}} = 6.1 \times 10^{-6} \text{ rad}$$

Limit of angular resolution of eye ($d\theta'$) is given as $2'$.

$$\therefore\ d\theta' = \frac{2 \times 3.14}{60 \times 180^{o}} = 5.815 \times 10^{-4} \text{ rad}$$

$$\therefore \text{ Useful magnification } M = \frac{d\theta'}{d\theta} = \frac{5.815 \times 10^{-4}}{6.1 \times 10^{-6}}$$

$$= 95.3$$

4.9 Polarization

Interference and diffraction phenomenon have manifested wave nature of light. In fact, these both effects, are observed for any kind of waves whether longitudinal or transverse. In the previous chapter we have studied that light (visible part of electromagnetic spectrum) is transverse waves. Its transverse character can be experimentally verified through the polarization phenomenon. In the case of longitudinal waves, particles of the medium oscillate in the direction of propagation only. On the other hand, in transverse waves vibration of particles or field vectors are possible in all directions perpendicular to the direction of propagation. In a sense, transverse waves enjoy preference in oscillations perpendicular to wave propagation. Due to this preferential character of particle or field oscillation, we may define the concept of polarization, which gives information about the state of oscillations of particles or field vectors.

4.9 (*a*) Unpolarized and Plane Polarized Light : To consider the polarization phenomenon see the following figure 4.18.

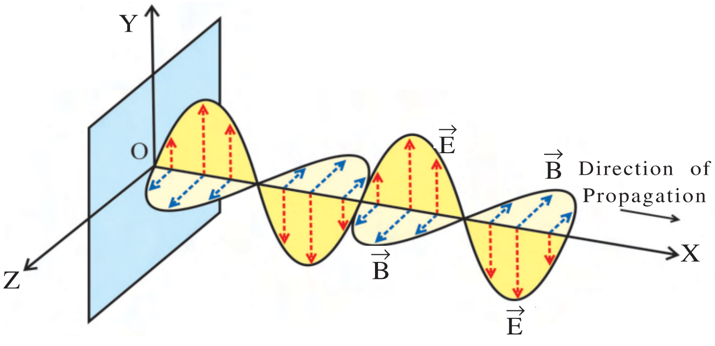


Figure 4.18 Propagation of Light

Suppose an atom or molecule is at point O and emitting electromagnetic wave as shown in the figure. It can be seen that the directions of $\vec{E}$, $\vec{B}$ and the propagation of waves are mutually perpendicular. In an ordinary light source like bulb, there are large number of such atomic emitters. They all emit electromagnetic waves with their $\vec{E}$ vectors (also called light vectors) vibrating randomly in all directions perpendicular to wave propagation. It means that $\vec{E}$ of one wave is not parallel to $\vec{E}$ of another wave. (Again we consider only $\vec{E}$ vectors for further discussion.) Also, the waves emitted by different atoms of a source and propagating in the same direction form a beam of light. If such beam of light is assumed to be coming out of the plane of the paper, light vectors of its waves will be found in all random direction in a plane of paper. Such light is called Unpolarized Light.

Such unpolarized light is schematically represented in figure 4.19 (*a*) and (*b*). For simplicity, we may resolve any light vector of unpolarized light into two perpendicular components (as shown in figure 4.19(*c*)) to the direction of propagation. However, we must remember that each of the wave in unpolarized beam of light is independently polarized.

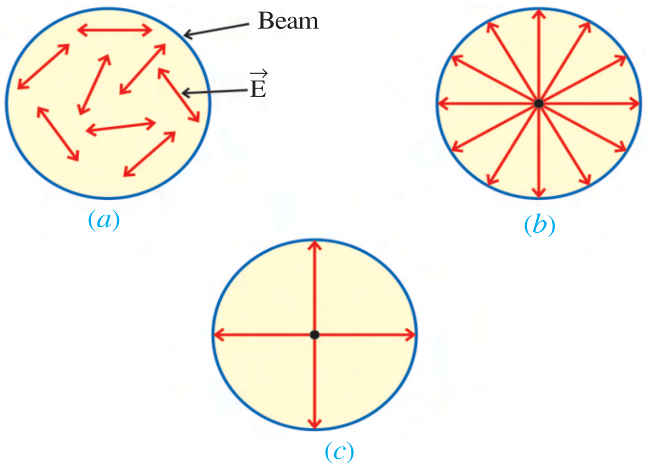


Figure 4.19 Unpolarized Light

"In a beam of light, if the oscillations of $\vec{E}$ vectors are in all directions in a plane perpendicular to the direction of propagation, then the light is called unpolarized light."

In 1815, Biot discovered that certain mineral crystals (like tourmaline) absorbs light selectively. This is called Selective Absorption or Dichroism. When light passes through tourmaline crystal freely transmit the light components which are polarized to a definite direction. While crystal absorbs light strongly whose polarization is perpendicular to this definite direction. This definite direction in a crystal is known as an optic axis. If the crystal is cut in proper size (1 to 2 mm thick) perpendicular components is totally absorbed (see figure 4.20). Hence, in the light emerging out of the tourmaline plate, which are parallel to the optic axis. Thus, emergent beam of light only coplanar and parallel $\vec{E}$ vectors are found. Such light is known as polarized light. Thus, tourmaline crystal is a natural polarizer or Polaroid.

"The beam of light in which light vectors are coplanar and parallel to each other is plane polarized or linearly polarized light."

The process by which getting the plane polarized light from unpolarized light is called polarization.

"The plane containing the direction of the beam and the direction of oscillation of $\vec{E}$ vectors is called the plane of oscillation (vibration)." In the figure 4.20, *abcd* is the plane of oscillation.

"A plane perpendicular to the plane of oscillation and passing through the beam of light is called the plane of polarization."

In above figure 4.20 *a'b'c'd'* is the plane of polarization.

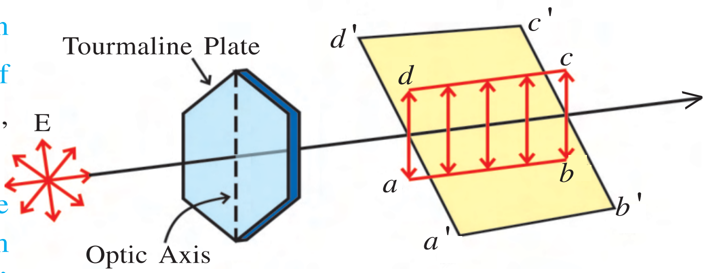


Figure 4.20 Polarization through Tourmaline Plate

4.9 (*b*) Malus' Law : The confirmation that the tourmaline acts as a polarizer can be checked as follows. Since tourmaline plate absorbs perpendicular components of $\vec{E}$ vectors, the intensity of emerging light is less than that of the incident unpolarized light. When tourmaline plate is rotated with an incident beam as an axis, intensity of emerging polarized light remains the same. This observation shows that in unpolarized light, in all directions in a plane perpendicular to the direction of propagation light vectors are uniformly distributed.

Now to analyze polarized light, another tourmaline plate B is arranged parallel to the plate A, as shown in the figure 4.21.

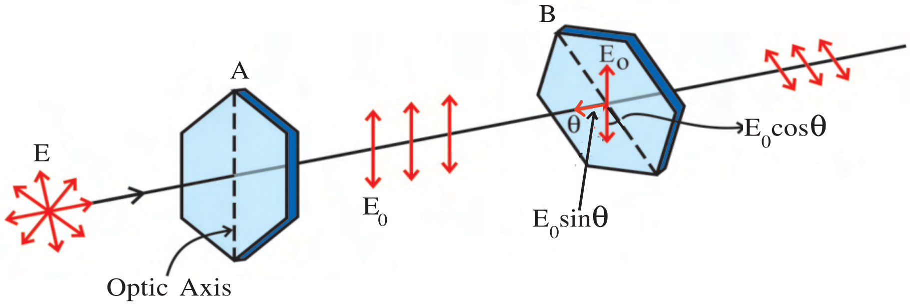


Figure 4.21 **Polarized and Analyzer**

An optic axis of plate B makes an angle θ with that of the plate A. In this situation, $\vec{E}$ vectors emerging from plate A (E_0) makes an angle θ with an optic axis of plate B. Therefore, we can resolve them into two components.

(1) $E_0\cos\theta$ parallel to the optic axis of plate B, and

(2) $E_0\sin\theta$ perpendicular to the optic axis of plate B.

Thus, only $E_0\cos\theta$ components will emerge out of plate B, while perpendicular components are absorbed. Since intensity is proportional to the square of amplitude, intensity of light incident on plate B is $I_0 \propto E^2_0\cos^2\theta$.

$$\therefore \quad \frac{I}{I_0} = \cos^2\theta$$

or

$$\therefore \quad I = I_0\cos^2\theta \tag{4.9.1}$$

Equation (4.9.1) is known as Malus Law. It is obvious from above equation that if plate B is completely rotated, twice the intensity of emerging light is zero (corresponding to $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$) and twice it becomes maximum (corresponding to $\theta = 0$ and π). This procedure will help us to verify whether the given light is polarized or not. Since tourmaline plate B is used to analyze a state of polarization of incident light, it is known as Analyzer.

4.9 (*c*) Nicol prism : In 1828 A.D. William Nicol made a Polaroid (polarizer and analyzer) from calcite crystal.

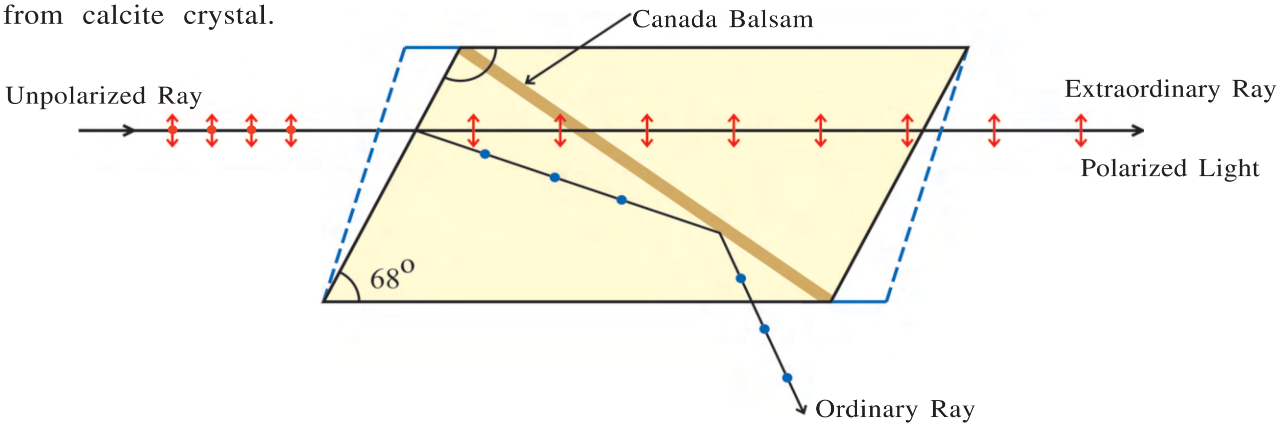


Figure 4.22 **Nicol Prism**

Nicol prism is made of two crystals of calcite. These crystals are cut at an angle of 68° with respect to its principal axis and then these pieces are joined with Canada balsam (a type of glue).

When unpolarized light is incident on such prism as shown in the figure, it divides into two rays, both rays are plane polarized. $\vec{E}$ vectors of one of the rays are perpendicular to plane as shown in the figure. This ray is called Ordinary Ray. The $\vec{E}$ vectors of another ray have oscillations parallel to the plane. This ray is called Extra Ordinary Ray. For these rays, refractive indaxes are $n_0 = 1.658$ and $n_e = 1.486$. The refractive index of Canada balsam is 1.55. As shown in the figure (4.22) the ordinary ray experiences total internal refraction at the surface of Canada balsam and comes out from one side of the prism while extraordinary ray comes out of the prism as plane polarized light.

4.9 (*d*) Polarization by Reflection and Brewster's Law : There are many methods of polarizing the light. We discussed one of them (with the help of tourmaline plate). Polarized light can also be obtained by reflection of light through transparent medium. In 1809, French scientist Malus found that when a ray of light is incident on surface of transparent medium, most of the $\vec{E}$ vectors in the reflected ray are perpendicular to the plane of incidence, that is reflected ray is partially polarized.

Here, the state of polarization of reflected ray depends on angle of incidence. Experimentally, it can be shown that when a ray of light is incident on a surface of transparent medium at some definite angle of incidence, reflected ray is found to be totally plane polarized. In this state all the $\vec{E}$ vectors in the reflected ray of light are parallel to each other and perpendicular to the plane of incidence. Such an angle of incidence is called Angle of Polarization of the given transparent medium. It depends on the type of the medium.

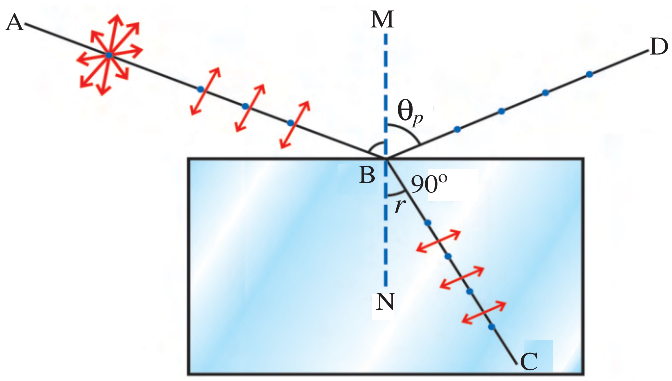


Figure 4.23 Polarization Through Reflection

A plane containing incident ray AB normal BM and a reflected ray BD is the plane of incidence in figure 4.23. The components of E, perpendicular to the plane of incidence are shown by (•). The components of $\vec{E}$ parallel to the plane of incidence are shown by (↔). The components perpendicular to the plane of incidence are known as σ components while components parallel to the plane of incidence are called π components.

When the angle of incidence is same as angle of polarization, only part of σ components are reflected. Hence, the reflected light is found to be totally plane polarized. In this situation π components are not found in reflected ray of light.

As in reflected ray of light a small part of σ components are present, it is very weak in comparision with the incident ray. At the surface of glass only 15% of σ components are reflected while 85% of σ components and all π components are refracted. Hence refracted ray is quite intense as compared to reflected ray.

Experimentally Brewster showed that when the reflected ray of light is totally plane polarized, the angle between reflected and refracted rays is 90^o. An important result obtained from this experiment is known as Brewster's Law.

Brewster's Law : "When a ray reflected from a surface of transparent object is totally plane polarized, the tangent of the angle of incidence (angle of polarization) is equal to the refractive index of the material of the object."

i.e., $n = \tan\theta_p$ (4.9.2)

where n = refractive index of the medium and θ_p is the angle of polarization.

Proof : As shown in figure 4.23, $\angle MBD + \angle DBC + \angle r = 180^o$

$\therefore\ \theta_p + 90^o + r = 180^o$

$\therefore\ r = 90^o - \theta_p$ (4.9.3)

According to Snell's law, refractive index

$$n = \frac{\sin\theta_p}{\sin r} = \frac{\sin\theta_p}{\sin(90^0 - \theta_p)} = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p \quad (4.9.4)$$

Equation 4.9.4 is known as Brewster's law.

4.9 (*e*) Uses of Polarization : Historically polarization was used to determine the type of the light (transverse) for longitudinal waves the oscillations of the particles of medium being parallel to direction of propagation, the polarization of longitudinal wave is never possible.

From the state of polarization of light emitted by an object or scattered by it, properties of the objects can be studied.

With the help of polarization it is found that in the rings of saturn there are ice cyrstals.

By studying state of polarization of ultraviolet light scattered by different viruses, their shape and size can be known.

The polarization of light is also useful in studying atoms and nuclei. The method known as photo-elasticity is used to study property of stress and strain of glass or bakelite.

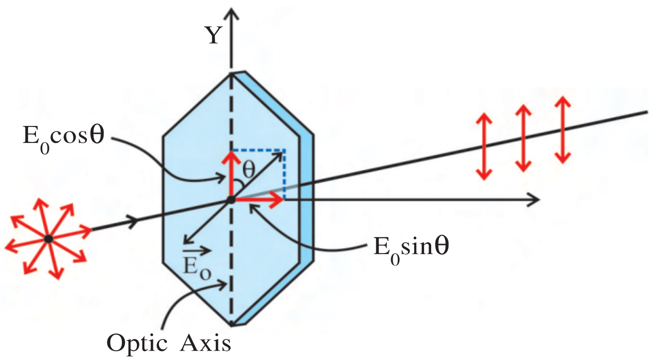
The type of sugar and concentration of its solution can be determined by passing plane polarized light through the solution of sugar. In LCD (Liquid Crystal Display) polarized light is used. They are used in calculators, watches and in the screens of laptops. To decrease the glare, sunglasses are also made from Polaroid.

Illustration 10 : Prove that when unpolarized light passes through a polarizer, the intensity of the transmitted light will be exactly half to the incident light.

Solution : As shown in the figure, let one such light vector make an angle θ w.r.t. optic axis. According to Malus' law, emergent intensity for this light vector will be,

$$I = I_0\cos^2\theta \quad (1)$$

where I_0 = intensity of the incident unpolarized light.



But we know that in unpolarized light, $\vec{E}$ vectors are distributed in all directions in a plane perpendicular to the direction of propagation. That is, all values of θ starting from 0 to 2π are equally possible.

Therefore, the average, emergent intensity is, given by

$$I_{ave} = \langle I \rangle = I_0 \left\langle \cos^2\theta \right\rangle$$

$$= \frac{I_0}{2\pi} \int_{\theta=0}^{\theta=2\pi} \cos^2\theta \, d\theta = \frac{I_0}{2\pi} \int_0^{2\pi} \left(\frac{1+\cos 2\theta}{2} \right) d\theta$$

$$= \frac{I_0}{4\pi} \left\{ [\theta]_0^{2\pi} + \left[\frac{\sin 2\theta}{2} \right]_0^{2\pi} \right\}$$

$$= \frac{I_0}{4\pi} \{(2\pi - 0) + 0\} = \frac{1}{2} I_0$$

That is, transmitted intensity is exactly half to the incident intensity.

Illustration 11 : A Plane polarized light is incident normally on the tourmaline plate. Its $\vec{E}$ vectors make an angle 60^o with the optic axis of the plate. Find the % difference between initial and final maximum values of $\vec{E}$ vectors.

Solution : According to Malus' law, $I = I_0 \cos^2\theta$

$$\therefore \frac{I}{I_0} = \cos^2(60^o) = (0.5)^2 = 0.25 = \frac{1}{4}$$

$$\therefore \frac{E^2}{E_0^2} = \frac{1}{4} \quad (\because \; I \propto E^2)$$

$$\therefore \frac{E}{E_0} = \frac{1}{2}$$

$$\therefore \frac{|E - E_0|}{E_0} = \frac{|1-2|}{2} = \frac{1}{2}$$

$$\%\Delta E = \frac{\Delta E}{E_0} \times 100 = \frac{1}{2} \times 100 = 50\%$$

Illustration 12 : A ray of light travelling in water is incident on a glass plate immersed in it. When the angle of incident is 51^o the reflected ray is totally plane polarized. Find the refractive index of glass. Refractive index of water is 1.33.

Solution : Angle of incidence, $\theta_p = 51^o$

Since at this incidence angle, reflected ray is totally plane polarized, using Brewster's law, refractive index of glass w.r.t. water is.

$$n' = \tan\theta_P = \tan 51^o = 1.235$$

$$\text{But, } n' = \frac{\text{refractive index of glass}(n_g)}{\text{refractive index of water}(n_w)}$$

$$\therefore n_g = n' n_w = 1.235 \times 1.33 = 1.64$$

Illustration 13 : A slit of width d is illuminated by white light. For what value of d will the first minimum for red light of wavelength $\lambda_R = 6500\ \mathring{A}$ appear at $\theta = 15^o$? What is the situation for violet colour having wavelength $\lambda_V = 4333\ \mathring{A}$ at the same point. $\sin 15^o = 0.2588$.

Solution : Since the diffraction occurs separately for each wavelength, we have to check condition for minima and maxima for each wavelength separately.

For the first minimum of red colour, $n = 1$, using equation,

$$d\sin\theta = n\lambda, \tag{1}$$

$$\text{slit width, } d = \frac{n \times \lambda_R}{\sin\theta} = \frac{1 \times 6500 \times 10^{-10}}{\sin 15^o}$$

$$= \frac{6.5 \times 10^{-7}}{0.2588} = 2.512 \times 10^{-6} \text{ m}$$

For violet colour, since wavelength is different we have to check whether the condition for minimum or maximum will satisfy.

$$\text{Using } d\sin\theta = n'\lambda_V \tag{2}$$

$$\therefore\ n' = \frac{d\sin\theta}{\lambda_V} = \frac{2.512 \times 10^{-6} \times 0.2588}{4333 \times 10^{-10}}$$

$$\therefore\ n' = 1.50$$

But to observe, minima, in equation (2), n' should be an integer. Thus, for violet colour condition for minimum does not satisfy.

$$\text{Using } d\sin\theta = (2n + 1)\frac{\lambda_V}{2},$$

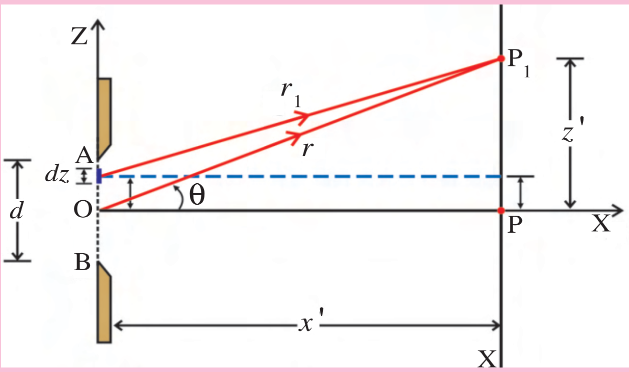
$$n' = \frac{d\sin\theta}{\lambda_V} - \frac{1}{2} = 1.5 - \frac{1}{2} = 1.0$$

This result suggests that for violet colour first maximum is observed.

Note : Irrespective of the width of the slit, at the position of first minimum of red colour, first maximum for violet colour takes place.

APPENDIX

For analysis of diffraction pattern in general (i.e. to know the intensity distribution and location of interference fringes) however, we ignore the converging lens and assume that the screen (C) is at very large distance. So the diffracted waves are considered to be effectively plane. However, it is tobe noted that even, while using the lens, situation remains the same. Since different secondary waves from the slit are passing through different thickness of the lens and therefore, they cover equal optical path length. (Optical path length in medium is equal to the product of refractive index of the medium and geometrical path length.)



Maxima and Minima Diffraction Due to Single Slit

As shown in the figure, the centre (O) of the slit is considered as the origin for Cartesian coordinate system. We further assume that slit AB is divided into large number of small segments (slit-segments) each of width dz. One such element is depicted at a distance z in the figure. We are now interested to find an equation for resultant intensity at different points on the screen due to superposition of waves from all such slit-segments.

The displacement at point P_1 due only to one such slit-element of width dz is given by,

$$de = E'\sin(\omega t - kr_1) \tag{1}$$

Where E' is the amplitude at point P_1. It is known that the larger the width dz, the larger is the amplitude E' (and therefore the intensity). That is, $E' \propto dz$ or $E' = A'dz$, where A' is the proportionality constant.

$$\therefore\ de = A'\sin(\omega t - kr_1)dz \tag{2}$$

Now, resultant displacement at point P_1 due to all slit-segments from B to A is,

$$e = A'\int_B^A \sin(\omega t - kr_1)dz = A'\int_{\frac{-d}{2}}^{\frac{+d}{2}} \sin(\omega t - kr_1)dz \tag{3}$$

from the figure, $r^2 = (x')^2 + (z')^2$

$\therefore\ (x')^2 = r^2 - (z')^2$

and $r_1^2 = x'^2 + (z' - z)^2$

$= (r^2 - z'^2) + (z' - z)^2$

$= r^2 - 2zz + z^2$

$$r_1^2 = r^2\left(1 - \frac{2z'z}{r^2} + \frac{z^2}{r^2}\right)$$

Since $r >> z$, $\frac{z^2}{r^2}$ is very small and it can be neglected, and the term $\frac{2z'z}{r^2}$ is very small compared to unity.

$$\therefore\ r_1^2 = r^2\left(1 - \frac{2z'z}{r^2}\right)$$

$$r_1 = \left(1 - \frac{2z'z}{r^2}\right)^{\frac{1}{2}}$$

Using Binomial theorem

$[(1 + x)^n \approx 1 + nx,\ x << 1]$, $r_1 \approx r\left(1 - \frac{1}{2}\frac{2z'z}{r^2}\right)$

$$\therefore\ r_1 = r - \frac{z'z}{r}$$

Also, from ΔOPP_1, $\sin\theta = \frac{z'}{r}$

$$\therefore\ r_1 = r - z\sin\theta \tag{4}$$

Using equation (4) into (3),

$$e = A' \int_{\frac{-d}{2}}^{\frac{+d}{2}} \sin(\omega t - kr + kz\sin\theta)\, dz$$

$$= \frac{-A'}{k\sin\theta}\left[\cos(\omega t - kr + kz\sin\theta)\right]^{\frac{+d}{2}}$$

$$= \frac{-A'}{\left(\frac{2\pi}{\lambda}\right)\sin\theta}\left[\cos\left\{(\omega t - kr) + \left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)\right\}^{\frac{-d}{2}} - \cos\left\{(\omega t - kr) - \left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)\right\}\right] \qquad \left(\text{writing } k = \frac{2\pi}{\lambda}\right)$$

Using an identity, $\cos(\theta_1 + \theta_2) - \cos(\theta_1 - \theta_2) = 2\sin\theta_1\sin\theta_2$

$$e = \frac{A'\lambda}{2\pi\sin\theta}\left[-2\sin(\omega t - kr)\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)\right]$$

$$= \left\{\left(\frac{A'\lambda}{\pi\sin\theta}\right)\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)\right\}\sin(\omega t - kr) \tag{5}$$

Thus, the resultant amplitude (E) at point P_1 is,

$$E = \left(\frac{A'\lambda}{\pi\sin\theta}\right)\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)$$

or

$$E = A'd\left(\frac{\sin\alpha}{\alpha}\right),$$

where we have assumed $\frac{\pi d\sin\theta}{\lambda} = \alpha$ (6)

Since intensity is directly proportional to the square of amplitude, resultant intensity at point P_1 is,

$$I = A^2d^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \tag{7}$$

with $I_0 = A'^2d^2$ = maximum intensity (8)

SUMMARY

1. An imaginary surface passing through particles of the medium or points in the space oscillating in phase is known as wavefront. It is used to describe the wave propagation.
2. Huygen's principle suggests that every point of a wavefront behaves as an independent secondary source, and emits by itself secondary spherical waves.
3. For the isotropic medium new wavefront maintains its original shape.
4. The physical effect produced by superposition of two or more waves is called interference. Using the principle of superposition resultant displacement at a point where interference takes place can be found.
5. Light sources emitting light waves with equal frequencies and either with constant or zero initial phase difference are known as coherent sources, otherwise sources are known as incoherent sources of light.
6. Coherent sources can only produce stationary interference.
7. In general, two methods are used to obtain coherent cources. (1) Division of wave front and (2) division of amplitude.
8. **For Superposing Waves :**

   (1) Phase difference of $2n\pi$, $n = 0, 1, 2, ...$ on path difference of $n\lambda$, $n = 0, 1, 2, ...$ produce constructive interference.

   (2) Phase difference of $(2n - 1)\pi$, $n = 1, 2, ...$ or path difference of $(2n - 1)\frac{\lambda}{2}$, $n = 1, 2, ...$ produce destructive interference.
9. Distance between two consecutive dark or bright interference fringes is same, $\bar{x} = \frac{\lambda D}{d}$. All bright fringes are equally bright.
10. Diffraction is the effect produced due to the limited part of the wavefront.
11. For Fraunhoffer diffraction, condition for minima can be given as,

    path difference $= n\lambda$; $n = 0, 1, 2, 3, ...$

    Corresponding to different values of n,

    $n = 1 \Rightarrow$ First order minimum,

    $n = 2 \Rightarrow$ Second order mimimum etc., we get different order minima.
12. For maxima is Fraunhoffer diffraction, path difference $= (2n + 1)\frac{\lambda}{2}$, $n = 1, 2, 3,...$
13. Corresponding to different values of n, we get different order maxima.
14. From central or zeroth order maximum, towards the higher order maxima, intensity rapidly decreases. It also decreases in proportion with the width of the slit.
15. The ability of an optical instrument to produce two nearby objects clearly and separate is defined as resolving power of an instruments.
16. Only transverse waves show polarization effect.
17. Ordinary light sources produce unpolarized light.
18. Different techniques are available to convert unpolarized light into the polarized light.

EXERCISE

For the following statements choose the correct option from the given options :

1. The distance between two slits in Young's experiment is 0.2 mm. If the wavelength of light used is 5000 Å, the angular position of 3^{rd} bright fringe from the central bright fringe is .... rad.

(A) 0.075 (B) 0.75 (C) 0.0075 (D) 0.057

2. In Young's experiment the distance between two slits is 0.4 cm and the distance of the screen from the slits is 100 cm. If the wavelength of the light used is 5000 Å, the distance of 4^{th} dark fringe from the central bright fringe is ....

(A) 4.37×10^{-2} cm (B) 4.37 mm (C) 8.74×10^{-2} cm (D) 8.74 mm

3. The distance between two slits in Young's experiment is 0.1 *m*m and the distance of the screen from the slits is 100 cm. If the wavelength of light is 5000 Å the width of the fringe is ....

(A) 5 mm (B) 2.5 mm (C) 2.5 cm (D) 5 cm

4. In Young's experiment the distance between two slits is halved and the distance between the screen and slits is doubled. The width of the fringe ....

(A) remains the same (B) becomes half (C) becomes double (D) becomes 4 times

5. A diffraction is formed with red light. If red light is repalced by blue light, ...

(A) the pattern does not change.

(B) the maxima and minima are narrow and more crowded.

(C) the maxima and minima are broadened and become distinct.

(D) diffraction pattern disappears.

6. In Young's experiment if transparent thin sheets are palced in front of two thin slits such that the central bright fringe remain at the same position. Thickness and refractive index of both sheets are t_1 and t_2, and n_1 and n_2, respectively. In this case, ...

(A) $\frac{t_1}{t_2} = \frac{n_1}{n_2}$ (B) $\frac{t_2}{t_1} = \frac{n_2}{n_1}$ (C) $\frac{t_1}{t_2} = \frac{(n_2-1)}{(n_1-1)}$ (D) $\frac{t_2}{t_1} = \frac{(n_2-1)}{(n_1-1)}$

7. A plate of refractive index 1.5 is placed in the passage of one ray in Young's experiment. If the central fringe is bright, the minimum thickness of the plate is ....

(A) 2λ (B) λ (C) $\frac{\lambda}{3}$ (D) $\frac{2\lambda}{3}$

8. To determine the position of a point like object precisely, ... light should be used.

(A) polarized (B) long wavelength (C) short wavelength (D) intense

9. The angular spread of central maximum, in diffraction pattern, does not depend on ....

(A) the distance between the slit and sources (B) wavelength of light

(C) width of slit (D) frequency of light

10. In Fraunhoffer diffraction by a single slit, the width of the slit is 0.01 cm. If the wavelength of light incident normally on the slit is 6000 Å the angular distance of second maximum from the mid line of central maximum is .... rad.

(A) 0.015 (B) 0.15 (C) 0.075 (D) 0.030

11. Detailed information can be obtained by the oil immersion objective of a microscope, because the objective has ....

(A) large value of magnification (B) greater value of resolution

(C) large diameter (D) none of the above

12. A person finds that the sun rays reflected by the still surface of water in a lake are polarized. If the refractive index of water is 1.327, the sun will be seen at the angle of .... with the horizon.

(A) 57° (B) 75° (C) 37° (D) 53°

13. Ordinary light incident on a glass slab, at the polarizing angle, suffers a deviation of 22°. in the medium. The value of angle of refraction is ....

(A) 74° (B) 22° (C) 90° (D) 34°

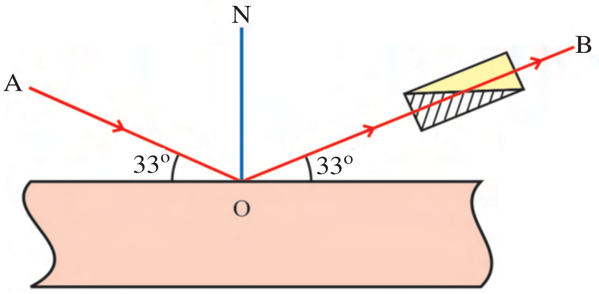
14. The ratio of resolving power of telescope, when lights of wavelengths 4000 Å and 5000 Å are used, is ....

(A) 16 : 25 (B) 5 : 4 (C) 4 : 5 (D) 9 : 1

15. The diameter of the lens of a telescope is 1.22 m. The wavelength of light is 5000 Å. The resolving power of the telescope is .... m^{-1}.

(A) 2×10^5 (B) 2×10^6 (C) 2×10^2 (D) 2×10^4

16. AO is a ray incident on a glass having refractive index 1.54, as shown in the figure. A Nicol prism is appropriately kept in the path of the reflected ray OB. Now, the Nicol prism is rotated. The intensity of light emferging from the Nicol Prism ....



(A) becomes zero and remains zero.

(B) slightly increases and decreases.

(C) does not change.

(D) decreases gradually and becomes zero and then again increases.

17. Unpolarized light falls on two polarizers placed one on top the other. What must be the angle between the characteristic directions (optic axis) of the polarizer if the intensity of the transmitted light is one third of the incident beam from source.

(A) 54.7^{0} (B) 35.3^{0} (C) 0^{0} (D) 60^{0}

ANSWERS

1. (C)	**2.** (A)	**3.** (B)	**4.** (D)	**5.** (B)	**6.** (C)
7. (A)	**8.** (C)	**9.** (A)	**10.** (A)	**11.** (B)	**12.** (C)
13. (D)	**14.** (B)	**15.** (B)	**16.** (D)	**17.** (B)	

Answer the following questions in brief :

1. State Huygen's principle.
2. What is interference ?
3. State the principle of superposition.
4. What are coherent source ?
5. Give relation between optical path length and geometrical path length.
6. What is Airy's disc ?

7. Define resolving power of an optical instrument.
8. State Rayleigh's criterion.
9. Define plane of polarization.
10. Define linearly polarized light.

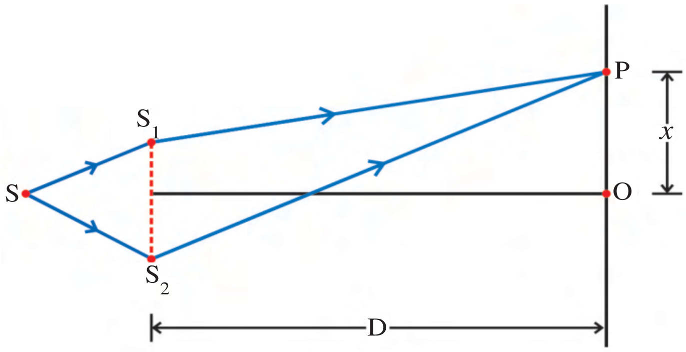
Answer the following questions :

1. Explain the use of wavefront to understand wave propagation.
2. Prove that the distance between consecutive dark and bright fringes in interference pattern is given by $\frac{\lambda D}{2d}$.
3. Explain central maximum obtained due to single slit Frahunhoffer diffraction.
4. Determine the width of central maximum in Franhoffer diffraction.
5. Explain the importance of Fresnel distance.
6. Give two points of comparison for interference and diffraction pattern.
7. Define unpolarized light and polarized light.
8. With diagram, give construction of Nicol prism.
9. State and prove Brewster's law.
10. Give uses of polarization.

Solve the following questions :

1. Two coherent line sources are 0.7 mm apart. If the centre of the fourth dark fringe of the interference pattern formed by the light emitted from them, on a screen placed at a distance of 1 m, is at 3 mm from the centre of the central bright fringe. Find the wave length of the monochromatic light used. [**Ans. :** 6000 $\overset{o}{\mathrm{A}}$]
2. In an Young's experiment, the distances between two slits and that between slits and the screen are 0.05 cm and 1 m, respectively. Find the distance between 3rd bright and 5th dark fringes. Take the wavelength of light equal to 5000 $\overset{o}{\mathrm{A}}$. [**Ans. :** 1.5 mm]
3. In Young's experiment fifth bright fringe produced by light of 4000 $\overset{o}{\mathrm{A}}$ superposes on the fourth bright fringe of an unknown wavelength. Find the unknown wavelength.

   [**Ans. :** 5000 $\overset{o}{\mathrm{A}}$]
4. In Young's experiment, the distance between two slits is 1 mm and the distance between two consecutive bright fringes is 0.03 cm. Now, on displacing the screen away from the slits by 50 cm, the distance between two consecutive dark fringes is doubled. Find the wavelength of light used. [**Ans. :** 6000 $\overset{o}{\mathrm{A}}$]
5. If the difference of time taken by two waves emitted from coherent sources to reach a point is an integral multiple of the period of the wave show that the constructive interference will occur at that point.

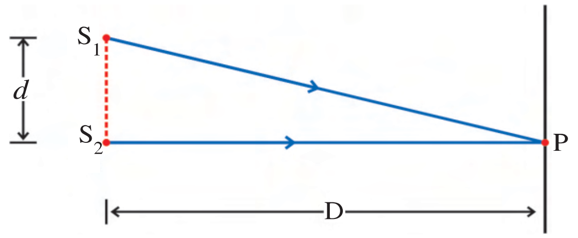
6. 

As shown in the figure, for interference by two rays is such that $SS_2 - SS_1 = 0.25\ \lambda$, where λ is the wavelength of the light used, obtain the conditions for constructive and destructive interference at point P.

7. In Young's double slit experiment, if the distance between two slits is double, than the wavelength of light used. Prove that a maximum 5 bright fringes will be obtained on the screen.

8. In Young's experiment a beam of light of wavelength 6500 Å and 5200 Å is used. Find the minimum distance from the central bright fringe where bright fringes produced by both the wavelength get superposed. The distance between two slits is 0.5 mm and the distance between the slits and the screen is 100 cm. [**Ans. :** 0.52 cm]

9. White light is used in Young's double slit experiment, as shown in the figure. At a point on the screen directly in front of slit S_2, certain wavelengths are producing destructive interference (i.e. they are missing in the diffraction pattern). Find these wavelengths, corresponding to first and second order diffraction. Here, $d << D$.



[**Ans :** (i) $\frac{d^2}{D}$, $n = 1$, (ii) $\frac{d^2}{3D}$, $n = 2$]

10. Three light waves are superposed at a certain point, where their electric field components are given as $E_1 = E_0 \sin\omega t$, $E_2 = E_0 \sin(\omega t + 60^\circ)$, $E_3 = E_0 \sin(\omega t - 30^\circ)$. Find their resultant E(t) at that point. (1) Find resultant amplitude E_R by resolving $\vec{E}$ into sine and cosine components in the phasor diagram. (2) Through resultant vector in the phase diagram, phase can be found. [**Ans. :** $E(t) = E_R \sin(\omega t + \beta)$ with $E_R = 2.4E_0$, $\beta = 8.8^\circ$]

11. In Fraunhoffer diffraction, the wavelength of light incident normally on the slit is $\frac{d}{2}$ where d is the width of the slit. What will be the number of bright fringes formed on an infinitely extended screen placed at any distance from the slit.

[**Ans. :** 3 maxima are formed]

12. Light of wavelength 5000 Å is incident on a slit of width 2 mm in Fraunhoffer diffraction. Find the width of second maximum on the screen placed at the focal plane of the convex lens of a focal length 100 cm. The lens is placed close to the slit. [**Ans. :** 0.025 cm]

13. An apparatus for Young's experiment is immersed in a liquid of refractive index 1.33. The distance between two slits is 1 mm and that between slits and screen is 1.33 m. The wavelength of light used is 6300 Å in air.

(1) Find the distance between two consecutive bright fringes. (2) Keeping the apparatus in the liquid, one of the slits is covered with a glass plate of refractive index 1.53. If in this condition the first order dark fringe is displaced in the position of zeroth order bright fringes. Find the thickness of the plate. [**Ans. :** (i) 0.63×10^{-3} m (ii) 1.57×10^{-6} m]