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Electromagnetic Induction

1.1 Introduction

Electricity and magnetism were considered separate and nonrelated branches for a long time. In the early decades of the nineteenth century, experiments on electric current carried out by Oersted, Ampere and a few others established the fact that electricity and magnetism are interrelated to each other. They found that moving electric charges (i.e. electric current) produce magnetic field. As for Illustration, a current carrying wire deflects a magnetic needle placed in its vicinity. This phenomenon raises the questions like: Is the converse effect possible ? Can moving magnets (i.e. magnetic field) produce electric currents or not ? Does the nature permit such a relation between electricity and magnetism ?

Around 1830, experiments conducted by Michael Faraday in England, and Joseph Henry in USA, demonstrated conclusively that electric current was induced in closed coil under the influence of changing magnetic flux. The phenomenon in which electric current is induced in a conductor by varying magnetic flux is called electromagnetic induction.

Practically the phenomenon of electromagnetic induction is of great importance. The historical experiments of Michael Faraday and Henry have led directly to the development of electric generators and transformers. Today's civilization owes its progress to a great extent to the discovery of electromagnetic induction.

In the present chapter we shall study about Faraday's experiments, induced current and induced emf; and phenomena like self-induction, mutual induction, eddy currents based on it.

1.2 Faraday's Experiments

The discovery and understanding of electromagnetic induction are based on a series of experiments performed by Faraday. We shall study some of these experiments.

Experiment 1 : As shown in figure 1.1, Faraday took a ring of soft iron in his historic experiment. An insulated conducting coil was wound on one side of the ring and connected to a battery.

On the opposite side of this coil, another conducting coil was wound and connected with a sensitive galvanometer. The coil connected to a battery acts like a solenoid. When electric current is passed through the coil (i.e. solenoid), it produces a magnetic field.

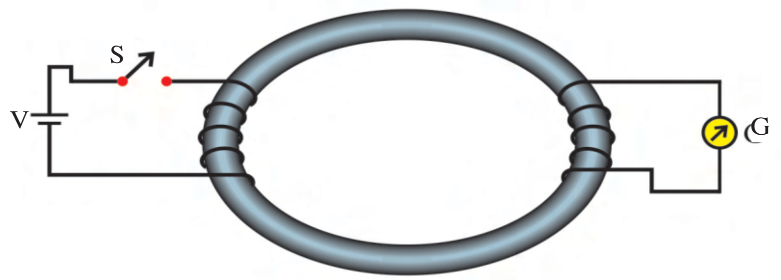


Figure 1.1 Faraday's Experiment

Galvanometer measures electric current passing through the other coil. As the ring is of the soft iron, the magnetic field lines produced remain confined in the ring itself. Almost all magnetic field lines pass through the ring and hence through the inner part of second coil and form closed loops. In other words, the ring connects two coils through the magnetic field lines.

When Faraday passed a steady electric current through the left coil, no effect was observed in the galvanometer. Faraday was slightly disappointed, but Faraday's intuition worked at that moment. During minute observations of every moment, he observed that the galvanometer shows a momentary deflection whenever the battery is switched on or off. In both the cases, the deflections of galvanometer were in opposite directions.

From his observation that galvanometer showed no deflection when a steady current is passed, Faraday concluded that the steady current is not important but change of current plays an important role in this experiment.

Experiment 2 : In his second experiment, Faraday arranged two bar magnets in V shape as in the figure 1.2.

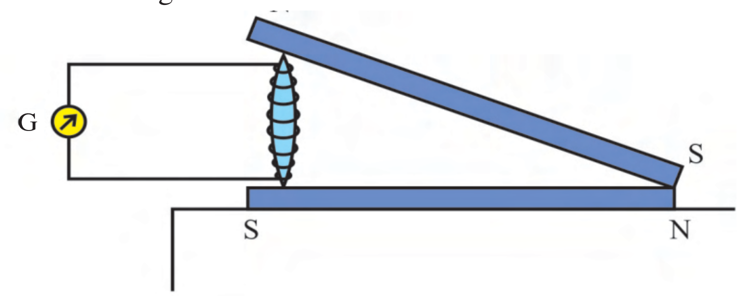


Figure 1.2 Faraday's Experiment of Two Barmagnets

At the other open end of V shape he kept one soft iron rod with an insulated copper wire wound around it. A galvanometer was connected with the conducting wire.

He observed that the galvanometer shows deflection on moving the end of upper magnet up and down. As the magnet moves nearer to the rod, magnetic flux linked with the coil increases. When the magnet touches the iron rod flux associated with the coil becomes maximum and as the magnet moves away from the rod, magnetic flux in the coil decreases.

From this experiment Faraday concluded that to induce electric current in a coil, change in magnetic flux is required and not the flux itself.

Experiment 3 : As shown in figure 1.3, an insulated conducting coil C_1 is connected to a galvanometer G.. When the North Pole (N) of a bar magnet is moved towards the coil, the pointer in the galvanometer shows deflection, indicating the presence of electric current in the coil.

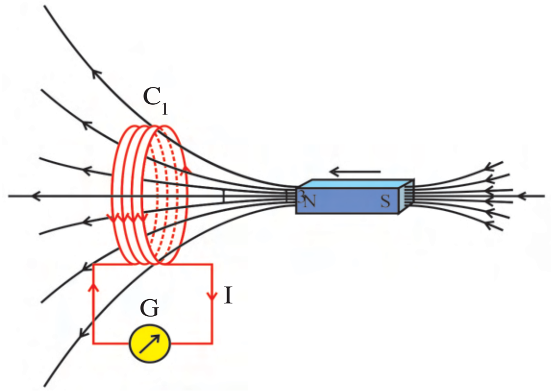


Figure 1.3 Faraday's Experiment of Bar Magnet and Coil

Galvanometer shows the deflection as long as the bar magnet is in motion. The galvanometer does not show any deflection when the magnet is held stationary.

When the magnet is pulled away from the coil, the galvanometer shows deflection in the opposite direction, which indicates reversal of the current's direction.

Moreover, when the South Pole (S) of the bar magnet instead of North Pole (N) is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed in the case of North Pole.

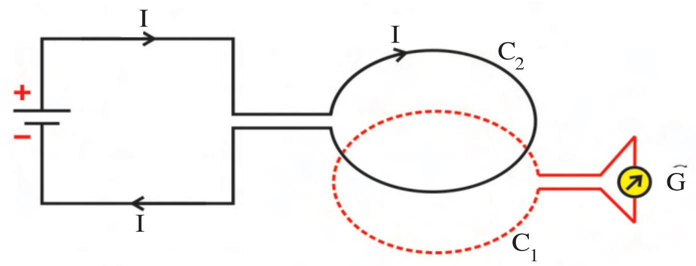


Figure 1.4 Faraday's Experiment on Two Coils

Further, the induced current is found to be larger when the magnet is pushed towards or away from the coil faster.

Instead, when the bar magnet is held fixed and the coil C_1 is moved towards or away from the magnet, the same results are observed.

When a bar magnet is replaced by a current carrying coil C_2 as shown in Figure 1.4, and relative motion is produced between the two coils C_1 and C_2 (nearest or farthest) then also the galvanometer connected with the coil C_1 shows deflection.

Further, if any one of the coils C_1 or C_2 is given a rotation with respect to each other, then also deflection is observed in the galvanometer.

The results of this experiment shows that :

(1) The relative motion between the magnet and the coil (or between the two coils) is responsible for generation (induction) of electric current in the coil.

(2) If the relative motion between the magnet and the coil is increased/decreased, more/less current is induced.

(3) The direction of induced current is reversed, if the direction of relative motion is reversed.

(4) If the magnet and the coil (or two coils) are moving with same speed in the same direction (if their relative velocity is zero), no current is induced in the coil.

Note : In the above experiment, electric current is induced due to the relative motion between the coil and the magnet and relative motion between the two coils respectively. However, Faraday showed that this relative motion is not an absolute requirement.

Faraday named the current produced in the other coil as the "induced current".

Here, the current generated in the other coil indicates that emf is produced in it which gives energy for the motion of charges. Faraday called this emf as "induced emf" and this phenomenon as electro magnetic induction.

Now, electric field is also produced in the second coil due to emf generated in it. Just as electric field is established in a wire by applying potential difference across its two ends, there is an electric field established in the second coil. Thus, we obtained the electric field due to the changing magnetic field with time. This fact is of basic importance in Farday's discovery.

Faraday's discovery fulfilled the dream of converting "mechanical energy into electrical energy".

1.3. Magnetic Flux

Magnetic flux is defined in the same way as electric flux is defined. The magnetic flux linked through any surface placed in a magnetic field is the number of magnetic field lines crossing this surface normally. Magnetic flux is a scalar quantity, denoted by ϕ.

Magnetic flux through a plane of surface area A placed in a uniform magnetic field $\vec{B}$ (Figure 1.5). can be written as,

$$\phi = \vec{B} \cdot \vec{A}$$

$$= BA\cos\theta \qquad (1.3.1)$$

where θ = Angle between $\vec{B}$ and $\vec{A}$.

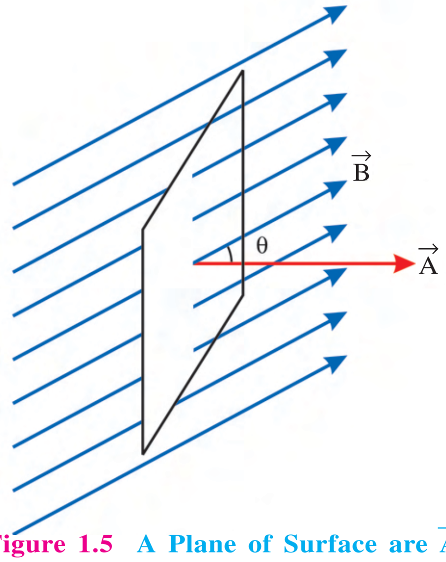


Figure 1.5 A Plane of Surface are $\vec{A}$ Placed in a Uniform Magnetic Field $\vec{B}$

Equation (1.3.1) can be extended to curved surfaces and non uniform fields too.

If the magnetic field has different magnitudes and directions at various parts of a surface as shown in figure 1.6, then the magnetic flux through the surface is given by,

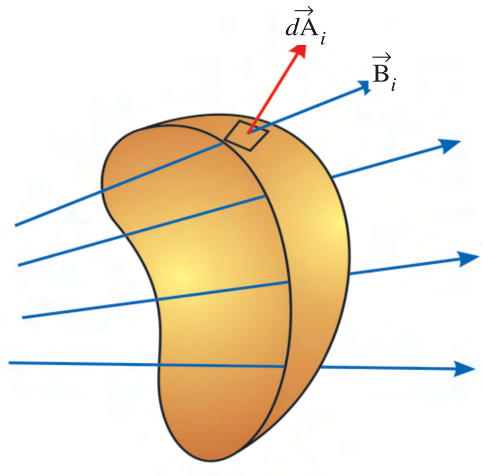


Figure 1.6 Magnetic field $\vec{B}_i$ at the i^{th} area Element $d\vec{A}_i$

$$\Phi = \vec{B}_1 \cdot d\vec{A}_1 + \vec{B}_2 \cdot d\vec{A}_2 + \vec{B}_3 \cdot d\vec{A}_3 + ...$$

$$\Phi = \sum_{\substack{\text{all} \\ \text{area elements}}} \vec{B}_i \cdot d\vec{A}_i \qquad (1.3.2)$$

where, $d\vec{A}_i$ is the area vector of i^{th} area element and $\vec{B}_i$ is the magnetic field at the area element $d\vec{A}_i$.

The SI unit of magnetic flux is weber (Wb) or Tm^2.

If the normal drawn to a plane points outward in the direction of the field ($\theta = 0$), then magnetic flux is positive. If the normal points in the opposite direction of the field ($\theta = \pi$), then flux is negative.

1.4. Lenz's Law

In article 1.2 we discussed about the induced emf but did not discuss about how much emf will be produced in which direction under the given conditions. In 1934, German physicist Lenz deduced a rule, known as Lenz's law which gives the direction (polarity) of the induced emf. We will first study this law and then Faradays law which gives the magnitude of induced emf.

As shown in figure 1.7, suppose a bar magnet is moved towards a conducting coil with its north pole (N) facing the coil. In this case the magnetic flux linked with the coil continuously changes and hence emf is induced in it. As a result the induced current flows through the coil which gives rise to the magnetic field and hence the coil acts like a magnet. Right now the direction of this current is not known to us.

In this situation, suppose the electric current flows in clockwise direction while looking at the coil normally from the same side as that of the bar magnet, then the side of the coil facing the magnet will act like a South Pole (S).

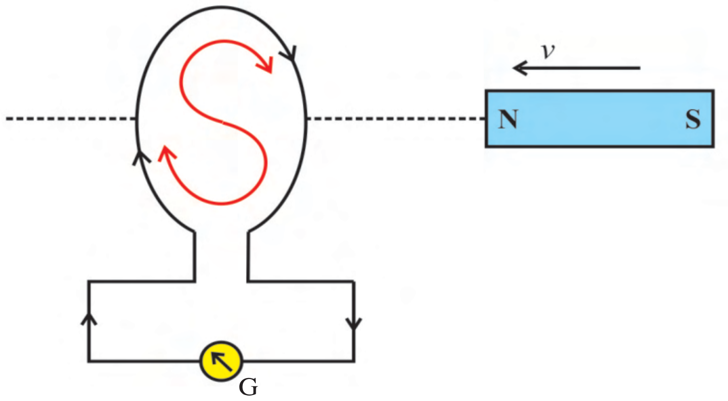


Figure 1.7

If this assumption is true, then by giving a gentle push to the magnet, the magnet will be attracted by the South Pole (S) of the coil and hence its speed will increase. As a result of this, the rate of change of flux linked with the coil increases and hence induced current will also increase. This makes the south pole of the coil more stronger which attracts the magnet towards itself with greater force. In this manner, the magnet will be accelerated more and more towards the coil (the velocity and kinetic energy of a magnet will continuously increase) and hence the induced current will also continuously increase. If an external resistance R is connected with the coil, the joule heat produced in it will continuously increase according to I^2Rt. In this case, no other mechanical work is done except giving a gentle push on the magnet even though heat energy I^2Rt is continuously being produced without expending any energy. According to the law of conservation of energy, we cannot produce energy without any cost. Hence our assumption that the "end of the coil facing the magnet acts like a South Pole (S) when the North Pole of the magnet is moved towards the coil" becomes wrong.

Now, according to the only other alternative left, this end of the coil, in the above case must behave like a North Pole (N), i.e. the current must flow in anticlockwise manner when viewed normally from the side of magnet, which opposes the change-here increase- in the flux.

If this is true, then there will be a repulsive force between the north pole of a magnet and induced north pole of the coil. Hence, mechanical work is required to be done by applying continuous external force in order to maintain the motion of the magnet towards the coil against this repulsive force. If this happens, the heat energy (I^2Rt) produced in the coil can be said to have produced at the cost of this mechanical work. This is consistent with the law of conservation of energy.

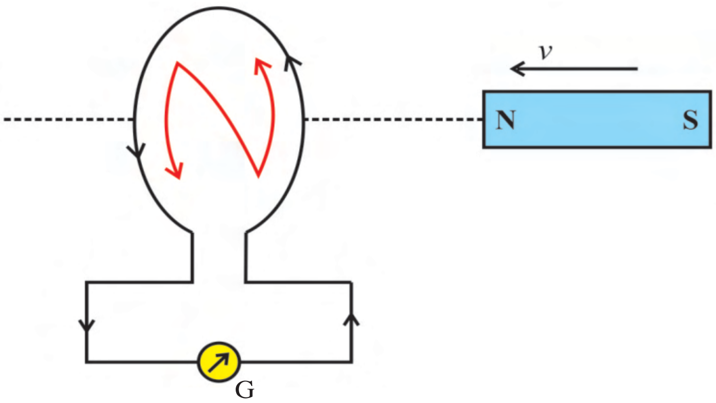


Figure 1.8 Direction of Induced Current

This discussion shows that, "induced emf (or induced current) is produced in such a direction that the magnetic field produced due to it opposes the very cause (here motion of the magnet) that produces it".

The above statement is known as Lenz's law which gives the direction of induced emf. Induced emf opposes the very cause which produces it.

1.5 Faraday's Law

From the experimental observations, Faraday arrived at a conclusion that an emf is induced in a coil when magnetic flux through the coil changes with time. Experimental observations discussed in article 1.2 shows the common fact that, the change of magnetic flux through a closed circuit (coil) induces emf in it. Faraday stated experimental observations in the form of a law called Faraday's law of electromagnetic induction which gives the magnitude of induced emf. The law is stated below.

"The magnitude of the induced emf produced in a closed circuit (or a coil) is equal to the negative of the time rate of change of magnetic flux linked with it".

Suppose the magnetic flux linked with each turn of the coil is ϕ at time t. The flux changes by $\Delta\phi$ in time interval Δt about this time is t. Then by Faraday's law,

Average induced emf = The negative of the time rate of change of magnetic flux during this time interval

$$\therefore \ \langle\varepsilon\rangle = -\frac{\Delta\phi}{\Delta t} \qquad (1.5.1)$$

Here, the negative sign indicates the presence of Lenz's Law.

$\therefore$ The instantaneous induced emf at time t,

$$\varepsilon = \lim_{\Delta t \to 0}\left(-\frac{\Delta\phi}{\Delta t}\right)$$

$$\varepsilon = -\frac{d\phi}{dt} \qquad (1.5.2)$$

Now, if the coil is made up of N turns and flux linked with each turn is ϕ then the total magnetic flux linked with the coil (flux linkage) $\Phi = N\phi$.

Moreover, if the rate of change of flux associated with each turn is the same, then the rate of change of flux for the coil of N turns $= -\frac{d}{dt}(N\phi) = -N\frac{d\phi}{dt}$.

The total induced emf in a coil of N turns,

$$\varepsilon = -N\frac{d\phi}{dt} \qquad (1.5.3)$$

1.6 Motional emf

The magnetic flux ($\phi = BA\cos\theta$) linked with a coil can be varied by many ways.

(1) The magnet can be moved with respect to the coil.

(2) The coil can be rotated in a magnetic field. (by changing angle θ between $\vec{A}$ and $\vec{B}$)

(3) The coil can be placed inside the magnetic field in a specific position and the magnitude of the magnetic induction ($\vec{B}$) can be changed with time.

(4) The magnet can be moved inside a non-uniform magnetic field.

(5) By changing the dimension of a coil (that is, by shrinking it or stretching it) in a magnetic field.

In all the cases mentioned above, the magnetic flux linked with the coil changes and hence emf is induced in the coil.

"The induced emf produced in a coil to due the change in magnetic flux linked with a coil due to some kind of motion is called motional emf"

One simple scheme of producing motional emf is shown in figure 1.9.

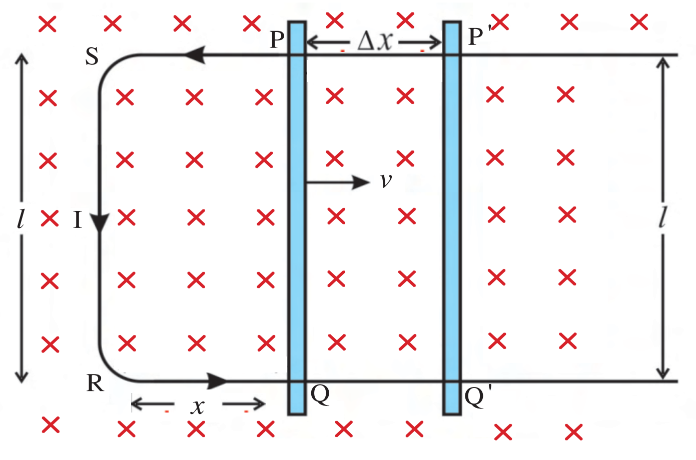


Figure 1.9 Motional emf

In figure 1.9, a U-shaped conducting wire is placed inside the uniform magnetic field $\vec{B}$ which is directed normally into the plane of paper in such a way that the plane formed by the conductor remains perpendicular to the magnetic field.

The conducting rod PQ is moved with a constant velocity v over the two arms of the U–shaped conductor. Assume that there is no loss of energy due to friction.

Here, the velocity of the rod is maintained constant by applying the force having same magnitude as that of the force which is acting opposite to the motion of the rod.

Suppose the normal distance between two arms of U-shaped conductor is RS $= l$ and RQ $=$ SP $= x$.

Note that the velocity of the rod is perpendicular to the magnetic field as well as its own length.

As the conducting rod PQ moves over the arms of U-shaped conductor, the area enclosed by a closed circuit PQRS changes and hence flux associated with a closed circuit also changes.

As a result emf is induced across two ends of a conducting rod PQ and induced current flows through the circuit.

Let PQ be the position of the rod at time t. In this situation the magnetic flux linked with the loop PQRS is,

$\phi = BA$

ϕ = (Magnetic induction) $\times$ (Area of PQRS)

$$\phi = Blx \tag{1.6.1}$$

As the rod goes on sliding, the value of x also changes with time. The rate of change of flux will give the induced emf in a rod.

Using Faraday's law, the induced emf,

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\varepsilon = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv \tag{1.6.2}$$

where, $\frac{dx}{dt} = v$ (velocity of rod)

This equation gives the value of induced emf produced in the circuit shown in figure 1.9. Here the motion of the rod is responsible for the generation of induced emf and hence this emf is called motional emf.

Thus, we are able to produce induced emf by moving a conductor instead of varying the magnetic field. (i.e. by changing the magnetic flux enclosed by the closed circuit)

In fact, if the rod is moves in a magnetic field without the U-shaped conductor, then also emf will be produced across the conductor which can be understood with the help of the following illustration.

The origin of the generation of induced emf : A conducting rod PQ moves in a magnetic field with its plane perpendicular to it as shown in figure 1.10. The positive ions and electrons in the rod will also move along with it (like a passenger in the train with a rod) in the direction of motion of the rod.

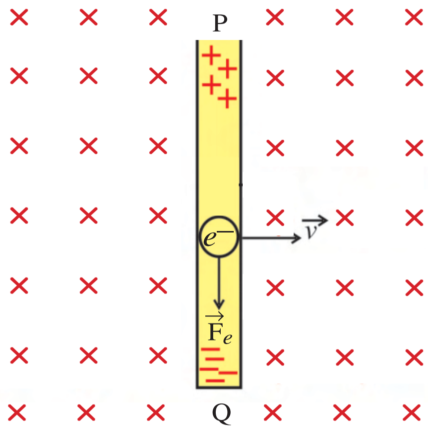


Figure 1.10 Induced emf in a Rod Moving in Magnetic Field

In the present case, they move with velocity $\vec{v}$ perpendicularly to the magnetic field $\vec{B}$. Hence. they experience Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$. Direction of this force can be determined by using right hand screw rule which is normal to the plane formed by $\vec{v}$ and $\vec{B}$.

Here, the positive ions will experience force from Q to P but as they remain fixed at their lattice points, they will not move under the influence of this force.

Now, according to the above equation, the force acting on electrons will be from P to Q. Since electrons are free to move, they deposit at Q end of the rod and make it negatively charged. Because of this positive charge of the ions opens up at P end and hence the resultant positive charge appears at P end.

Thus, end Q of the rod becomes negative and end P becomes positive and hence the rod behaves as a battery of emf $\varepsilon = Bvl$.

Conversion of Mechanical Energy into Electrical Energy : In the Illustration of U-shaped conductor, a rod is moving with velocity $\vec{v}$ perpendicular to the magnetic field $\vec{B}$ pointing into the plane of paper so that the lower end of the rod becomes negative and upper end becomes positive.

Here, the circuit gets completed and conventional current I flows in the direction PSRQ. Now the rod carrying electric current and moving through the magnetic field experiences a force according to $\vec{F} = I\vec{l} \times \vec{B}$.

If the resistance of the rod is R, the current flowing through a closed circuit is, $I = \frac{\varepsilon}{R} = \frac{Bvl}{R}$.

The force BIl acting on the rod, is opposite to the direction of velocity $\vec{v}$ of the rod. Thus, to continue the motion of the rod a force BIl must be applied towards right side. Such a force is called "Lenz force".

Here, mechanical power = Force × Velocity

$$P_m = Fv$$

$$P_m = BIlv$$

$$P_m = B\left(\frac{Bvl}{R}\right)lv = \frac{B^2v^2l^2}{R} \tag{1.6.3}$$

Electrical power generated in the circuits, $P_e = \varepsilon I$

$$P_e = (Bvl)I$$

$$P_e = (Bvl)\left(\frac{Bvl}{R}\right) = \frac{B^2v^2l^2}{R} \tag{1.6.4}$$

Equations (1.6.3) and (1.6.4) show that the electrical power generated is equal to the mechanical power spent i.e the mechanical work done in continuing the motion of the rod is obtained in the form of electrical energy. Here, we have ideally considered heat dissipation as zero.

From Faraday's law the magnitude of the induced emf,

$$|\varepsilon| = \frac{\Delta\Phi}{\Delta t}$$

However, $|\varepsilon| = IR = \frac{\Delta Q}{\Delta t}R$

$$\text{Thus, } \Delta Q = \frac{\Delta\Phi \text{ (Net change in Magnetic Flux)}}{\text{R (Resistance)}} \tag{1.6.5}$$

which gives the relation between the induced charge flow through the circuit and the change in magnetic flux. Note that induced charge does not depend on the rate of change of magnetic flux.

Illustration 1 : A conducting circular loop of surface area 2.5×10^{-3} m^2 is placed perpendicular to a magnetic field which varies as $B = (0.20\ T) \sin [(50\pi\ s^{-1})t]$. Find the charge flowing through any cross section during the time $t = 0$ to $t = 40$ ms. Resistance of the loop is 10 Ω.

Solution : Face area of the loop $A = 2.5 \times 10^{-3}\ m^2$

Resistance of the loop $R = 10\ \Omega$

Magnetic field changes as $B = B_0 \sin\omega t$

where $B_0 = 0.20$ T and $\omega = 50\pi\ s^{-1}$

The flux passing through the loop at time t is $\phi = AB_0 \sin\omega t$

The induced emf is $\varepsilon = -\dfrac{d\Phi}{dt} = -AB_0\omega\cos\omega t$

$$\text{Induced current } I = \frac{\varepsilon}{R} = \frac{-AB_0\omega}{R}\cos\omega t$$

$$= -I_0\cos\omega t$$

$$\text{where, } I_0 = \frac{AB_0\omega}{R}$$

The current changes sinusoidally with the time period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50\pi\,s^{-1}} = 40 \times 10^{-3}$ s

The charge flowing through any cross section during time $t = 0$ to $t = 0.04$ s is,

$$Q = \int_0^{0.04} I dt = -I_0 \int_0^{0.04} \cos\omega t\ dt$$

$$\therefore\ Q = -\frac{I_0}{\omega}[\sin\omega t]_0^{0.04}$$

$$\therefore\ Q = 0$$

Illustration 2 : A conducting circular loop is placed in a uniform magnetic field of 0.04 T with its plane perpendicular to the field. Somehow, the radius of the loop starts shrinking at a constant rate of $2\dfrac{\text{mm}}{\text{s}}$. Find the induced emf in the loop at an instant when the radius becomes 2 cm.

Solution : Let the radius of the loop be r at time t.

The magnetic flux linked with the loop at this instant is,

$$\phi = AB = \pi r^2 B$$

Here, $\dfrac{dr}{dt} = 2\ \text{mms}^{-1} = 2 \times 10^{-3}\ \text{ms}^{-1}$

When radius of the loop becomes $r = 2$ cm $= 2 \times 10^{-2}$ m, the induced emf in the loop,

$$\varepsilon = \frac{d\phi}{dt}$$

$$\varepsilon = \frac{d}{dt}(\pi r^2 B)$$

$$\varepsilon = 2\pi B r \frac{dr}{dt}$$

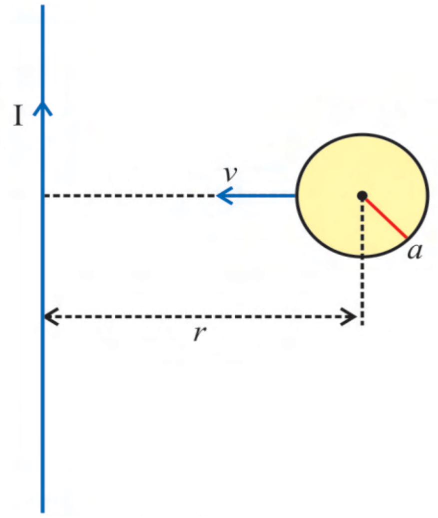
$$\varepsilon = 2\pi\ (0.04)\ (2 \times 10^{-2})\ (2 \times 10^{-3})$$

$$= 3.2\pi \times 10^{-6}\ \text{V}$$

$$= 3.2\pi\ \mu\text{V}$$

Illustration 3 : As shown in figure, a long wire kept vertically on the plane of paper carries electric current I. A conducting ring moves towards the wire with velocity v with its plane coinciding with the plane of paper. Find the induced emf produced in the ring when it is at a perpendicular distance r from the wire. Radius of the ring is a and $a << r$.

Solution : Magnetic field at a distance r from the wire carrying current is, $B = \frac{\mu_0 I}{2\pi r}$.



$\therefore$ Magnetic flux linked with the ring,

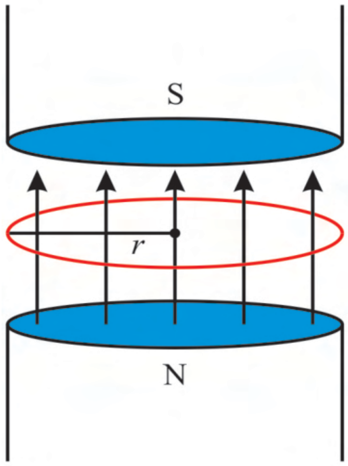
$$\phi = B(\pi a^2) = \frac{\mu_0 I}{2\pi r} \times \pi a^2 = \frac{\mu_0 I a^2}{2r}$$

$$\therefore \text{ Induced emf } \varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 I a^2}{2r}\right)$$

$$= \frac{\mu_0 I a^2}{2}\left(\frac{1}{r^2}\right)\frac{dr}{dt}$$

$$\therefore \varepsilon = \frac{\mu_0 I a^2}{2r^2} v \qquad \left(\because \frac{dr}{dt} = v\right)$$

Illustration 4 : A conducting ring of radius r is placed perpendicularly inside a time–varying magnetic field as shown in figure. The magnetic field changes with time according to $B = B_0 + \alpha t$ where B_0 and α are positive constants. Find the electric field on the circumference of the ring.



Solution : The magnetic field linked with the ring at time t,

$$B = B_0 + \alpha t$$

$$\therefore \phi = B(\pi r^2) = (B_0 + \alpha t)\pi r^2 \qquad .....(1)$$

From Faraday's law, emf produced in a ring,

$$\therefore \varepsilon = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}\left[(B_0 + \alpha t)\pi r^2\right]$$

$$\therefore \varepsilon = -\alpha\ \pi r^2 \qquad .....(2)$$

Now by definition, emf is the work done by the electric field for one complete revolution of a unit positive charge on the circumference of the ring. If $\vec{E}$ is the electric field intensity, the work done is,

$= \int \vec{E} \cdot \vec{dl}$ since $\vec{E}$ and $\vec{dl}$ are in the same direction,

$$\int \vec{E} \cdot \vec{dl} = E \int dl$$

$$= E(2\pi r) \qquad .....(3)$$

Comparing equations (2) and (3),

$E(2\pi r) = \alpha \pi r^2$ (neglecting negative sign)

$$\therefore \; E = \frac{\alpha r}{2}$$

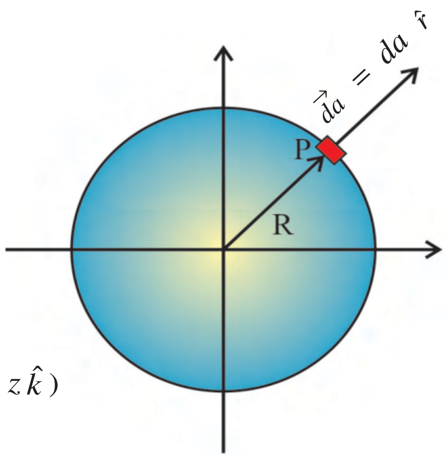
Note : See that the magnetic field goes on changing with time but the electric field in the ring remains constant. Though, this is not a general result. When the magnetic field changes non-linearly with time, the result will not be the same.

Illustration 5 : A field is given by $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$. Can this field be used to obtain induced emf ?

[**Hint :** A field must be magnetic one to obtain induced emf.]

Solution : If a given field is magnetic, its surface integration over any closed surface (flux passing through the closed surface) must be zero. For this we will consider the surface of a sphere of radius R whose center is at the origin of the coordinate system.

In figure $\vec{da} = da\ \hat{r}$ represents the vector of a surface element on the surface of this sphere. If coordinates of point P are (x, y, z)



$$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\therefore \; \hat{r} = \frac{\vec{R}}{|\vec{R}|} = \frac{x}{R}\hat{i} + \frac{y}{R}\hat{j} + \frac{z}{R}\hat{k}$$

Surface integration over given surface is,

$$\int_{\text{surface of sphere}} \vec{A} \cdot \vec{da} = \int_{\text{surface of sphere}} (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \frac{da}{R}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= \frac{1}{R}\int (x^2 + y^2 + z^2)\, da = R\int da$$

$$= R \times 4\pi R^2$$

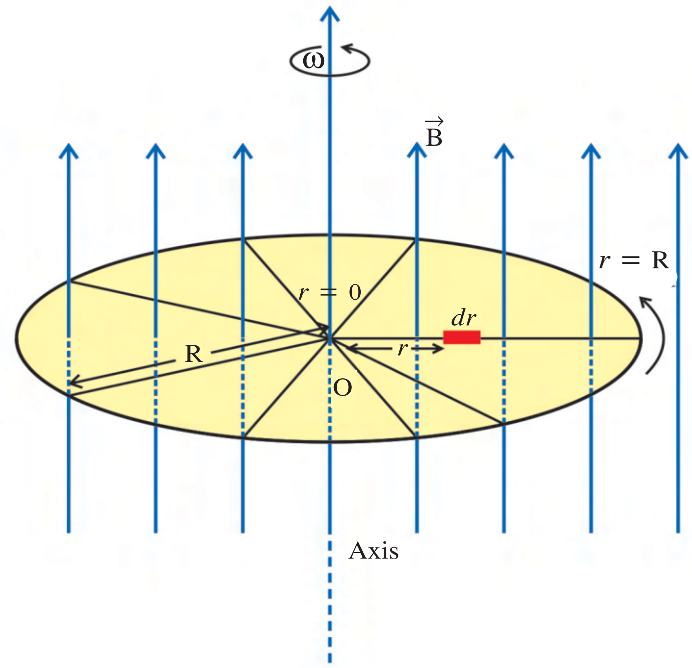
$$= 4\pi R^3 \neq 0$$

Thus the surface integral of given field is not zero over a closed surface and hence it cannot be magnetic field. Therefore, induced emf cannot be produced.

Illustration 6 : A wheel having n conducting concentric spokes is rotating about its geometrical axis with an angular velocity ω, in a uniform magnetic field B perpendicular to its plane. Prove that the induced emf generated between the rim of the wheel and the center is $\frac{\omega B R^2}{2}$, where R is the radius of the wheel. It is given that the rim of the wheel is conducting.

Solution : As shown in the figure, consider a small element dr on any spoke at a distance r from the center.

Linear velocity of this element $v = r\omega$



emf induced in a small element dr is,

$$d\varepsilon = Bvl$$

$$= B(r\omega)dr$$

Total emf induced along the entire length of any spoke is,

$$\varepsilon = \int_{r=0}^{r=R} d\varepsilon = \int_{r=0}^{r=R} B(r\omega)dr$$

$$\varepsilon = B\omega\int_0^R r dr = B\omega\left[\frac{r^2}{2}\right]_0^R$$

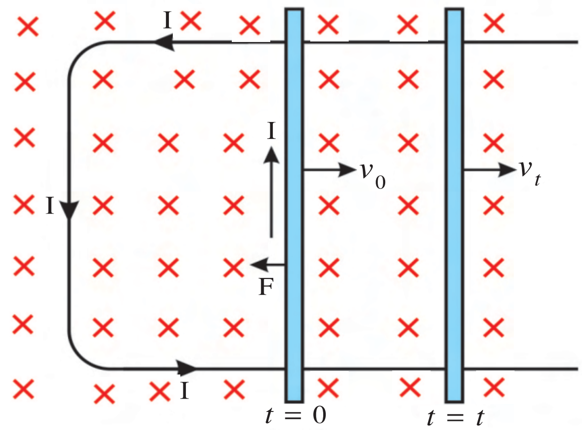
$$\therefore \ \varepsilon = \frac{1}{2}B\omega R^2$$

Application of the right hand screw rule with equation $\vec{F} = -e(\vec{v} \times \vec{B})$ shows that free electrons in a spoke will experience force towards the center of the wheel. Therefore, the free electrons accumulate at the center of the wheel leaving the rim positively charged. Thus, the end of the spoke at the center of the wheel behaves as a negative electrode and the end of the spoke on the rim behaves as a positive electrode.

If a rod of length L rotates with a uniform angular velocity ω about its perpendicular bisector and uniform magnetic field $\vec{B}$ exists parallel to the axis of rotation, what will be the potential difference between the two ends of the rod and between the center of the rod and the end ? Think !!

Illustration 7 : A U-shaped conducting frame is placed in a magnetic field B in such a way that the plane of the frame is perpendicular to the field lines. A conducting rod is supported on the parallel arms of the frame, perpendicular to them and is given a velocity v_0 at time $t = 0$. Prove that the velocity of the rod at time t will be given by $v_t = v_0 \exp\left(\frac{-B^2 l^2}{mR} t\right)$.

Solution : As shown in figure, when a conducting rod is given a motion in magnetic field, emf is induced in it and hence induced current flows through the rod i.e. the rod carries current. Here, the force acting on the rod due to magnetic field ($F = BIl$) is opposite to the motion of the rod. Therefore, the rod will decelerate and its velocity decreases with time.



Emf induced in a rod at time t is,

$$\varepsilon = -Bv_t l$$

$$IR = -Bv_t l$$

$\therefore$ Induced current at time t, $I = \frac{-Bv_t l}{R}$

Here, the rod is moving in a magnetic field. Therefore, the force acting on the rod at time t according to Lenz's law is,

$$F = BIl$$

$$= B\left(\frac{-Bv_t l}{R}\right)l$$

$$\therefore F = \frac{-B^2 l^2 v_t}{R} \qquad ......(1)$$

According to Lenz's law, this force is acting in the direction opposite to the motion of the rod, it produces deceleration $a = \frac{dv_t}{dt}$ in the rod.

From $ma = F$,

$$m\frac{dv_t}{dt} = \frac{-B^2 l^2 v_t}{R} \quad \text{(Using equation (1))}$$

$$\therefore \frac{dv_t}{v_t} = -\frac{B^2 l^2}{mR} dt$$

Integration on both the sides,

$$\int_{v_0}^{v_t} \frac{1}{v_t} dv_t = -\frac{B^2 l^2}{mR} \int_{t=0}^{t} dt$$

$$\left[ln\ v_t\right]_{v_0}^{v_t} = -\frac{B^2 l^2}{mR} [t]_0^t \qquad ......(2)$$

$$\therefore ln\ v_t - ln\ v_0 = -\frac{B^2 l^2}{mR} t$$

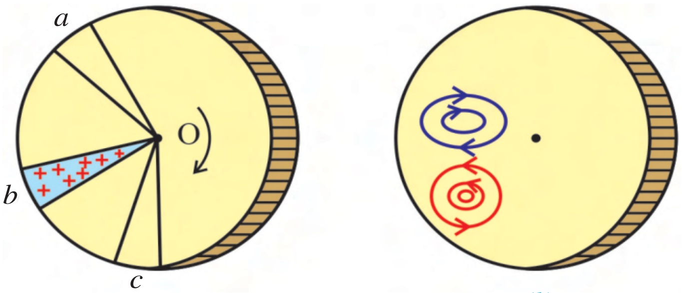
$$\therefore ln\left(\frac{v_t}{v_0}\right) = -\frac{B^2 l^2}{mR} t$$

$$\therefore \frac{v_t}{v_0} = \exp\left[\frac{-B^2 l^2}{mR} t\right]$$

$$\therefore v_t = v_0 \exp\left[\frac{-B^2 l^2}{mR} t\right]$$

1.7 Eddy Currents

Whenever a solid conductor is kept in a region of varying magnetic field, the magnetic flux linked with the conductor changes and induced emf is produced by induction. As a result, circulatory currents are induced in the plane normal to the direction of flux. These currents are distributed throughout the conductor. These are known as Eddy currents because of their circulatory nature. Eddy currents were first observed by physicist Foucault in 1895. The direction of flow of these currents is determined by Lenz's law. When a conductor rotates in a uniform magnetic field, then also eddy currents are produced in it.



(*a*) (*b*)

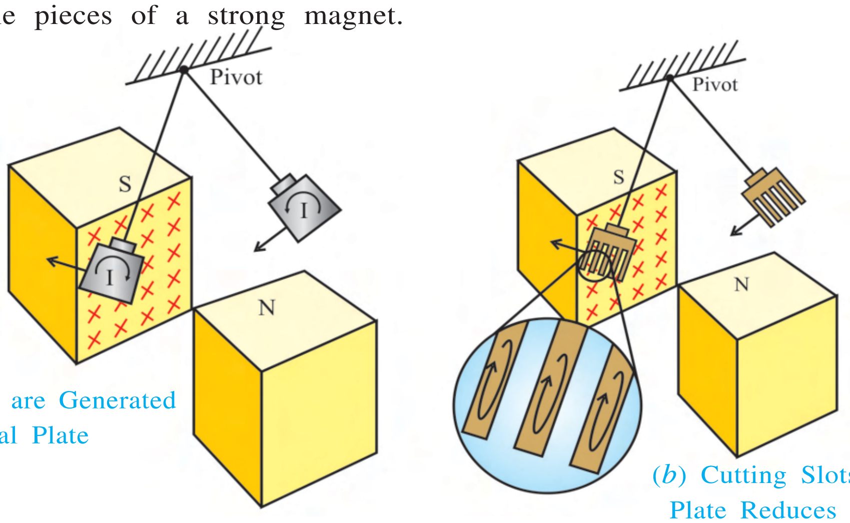
Figure 1.11 Eddy Currents in Disc

Suppose a magnetic field is applied to the portion of rotating disc in the direction perpendicular to the plane of disc. As shown in figure 1.11 (*a*), element Ob of the disc is moving across the field, the electrons of this element will start moving under the influence of the force $\vec{F} = -e(\vec{v} \times \vec{B})$. Element O*a* and O*c* are not in the field. So, they provide return conducting path to the charges displaced along O*b*. In this way eddy currents are set up in the disc.

Arago performed a simple experiment to find the direction of eddy currents. A metal disc was pivoted horizontally so that it can rotate about vertical axis. A magnetic needle was freely suspended just above the plane of a disc without touching it, when the disc was rotated rapidly, it will cut the flux of magnetic field. The needle was found rotating in the direction of rotation of the disc due to induced current. When the direction of rotation of the disc was reversed, the needle was found to rotate in the reverse direction.

Consider a metal plate falling downward into a uniform magnetic field applied normal to the plane of paper and pointing inward. The electrons inside the plate will experience a force $[\vec{F} = -e(\vec{v} \times \vec{B})]$ because of the motion of the plate. Under the influence of this force, electrons move on the paths which offer minimum resistance and constitute eddy currents. These currents, according to Lenz's law, flow in such a direction that the magnetic field produced due to them opposes the motion of the conductor. Hence, the plate appears to fall with acceleration less than g in the presence of magnetic field.

As shown in figure 1.12 (*a*), a metallic plate is allowed to oscillate like a simple pendulum between two pole pieces of a strong magnet.



(*a*) Eddy Currents are Generated in the Metal Plate

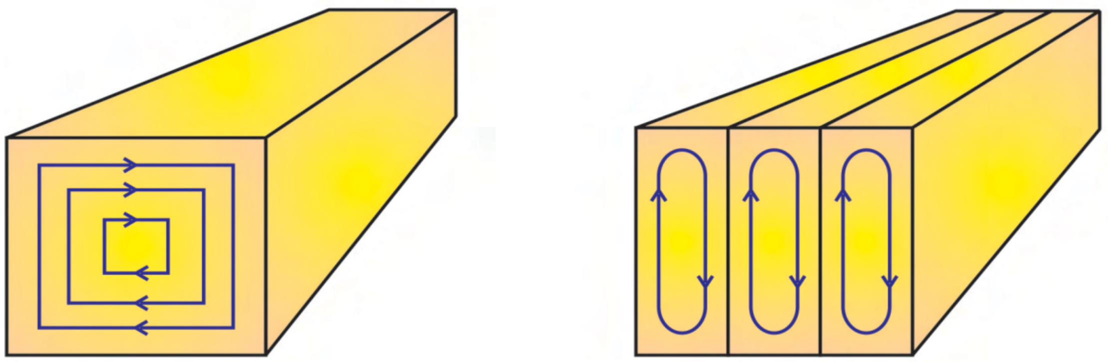
(*b*) Cutting Slots in the Metal Plate Reduces the Effect of Eddy Currents

Figure 1.12

It is observed that the oscillations of the plate is damped and in a short while the plate comes to rest in the magnetic field. Such a damping is called electromagnetic damping. Magnetic flux associated with the plate keeps on changing as the plate moves in and out of the region between magnetic poles. Eddy currents are produced in a plate due to change in magnetic flux. According to Lenz's law, these eddy currents oppose the motion of the plate in a magnetic field. The directions of eddy currents are opposite when the plate swings into the region between two poles and when it swings out of the region.

If rectangular slots are made in the metal plate as shown in figure 1.12 (*b*), area available to the flow of eddy currents become less. Thus, the length of the path of electrons is greatly increased in the plate. So magnitude of eddy current is decreased. As a result of this the effect of eddy current is reduced. Thus the pendulum plate with slots oscillates for a longer time because the effect of electromagnetic damping is reduced.

In the interior of the iron cores of the rotating armatures of motors and dynamos and also in the core of transformer, eddy currents are produced. Eddy currents are undesirable since they heat up the iron core and dissipate electrical energy in the form of heat energy. To reduce the effect of eddy currents, a laminated core instead of a single solid piece of iron is used (figure 1.13). The iron core is made up of several layers. These layers are separated by an insulating material (Varnish). In this way, eddy currents flow through the individual laminations instead of the whole core. Thus the length of the path of electron is greatly increased, as a result the strength of eddy currents can be minimized and energy loss is substantially reduced.



(*a*) Solid Piece of Metal

(*b*) Solid Piece Made Up of Insulated Layers

Figure 1.13

Applications of Eddy Currents

(1) Induction Furnace : When a metal specimen is placed in a rapidly changing magnetic field (produced by high frequency a.c.) eddy currents generated in the metal produce high temperatures sufficient to melt the metal. This process is used in the extraction of some metals from their ores.

Induction furnace can be used to produce high temperatures and can be utilized to prepare alloys.

(2) Speedometer : In a speedometer, a tiny magnet rotates according to the speed of the vehicle and produces the required changing magnetic field. The magnet rotates in an aluminium drum. Eddy currents are set up in the drum. The drum turns in the direction of the rotating magnet. A pointer attached to the drum indicates the speed of the vehicle on a calibrated scale.

(3) Electric Brakes : When a strong magnetic field is applied to the rotating drum attached to the wheel, eddy currents set up in the drum which exert a restoring torque on the drum so the motion of the drum stops. Using this fact, the eddy currents are used in braking system of trains so that the brakes can be applied smoothly.

(4) Electric Power Meters : The shiny metal disc in the electric power meter rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.

1.8 Self Induction

When an electric current is passed through an insulated conducting coil, it gives rise to a magnetic field in the coil so that the coil itself behaves like a magnet. The magnetic flux produced by the current in the coil is linked with the coil itself.

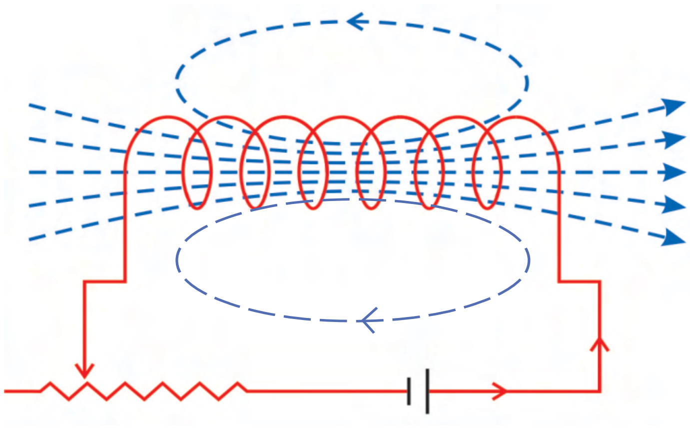


Figure 1.14 Self-Induction in a Coil

As the strength of the current in the coil is changed, the flux linked with the coil also changes. Under such circumstances an emf is induced in the coil too. Such emf is called a self-induced emf and this phenomenon is known as self-induction.

If the number of turn in a coil is N and the flux linked with each turn is ϕ, then the total flux linked through the coil = $N\phi$.

In this case, the total flux linked with the coil (which is called flux linkage) is directly proportional to the current I flowing through the coil.

$$N\phi \propto I$$

$$\therefore \quad N\phi = LI \qquad (1.8.1)$$

where the constant of proportionality L is called the self–inductance of a coil. From equation (1.8.1),

$$L = \frac{N\phi}{I} \qquad (1.8.2)$$

The self inductance L is a measure of the flux linked with coil per unit current.

The self-inductance L of a coil depends upon –

(1) The size and shape of the coil.

(2) The number of turns N.

(3) The magnetic property of the medium within the coil in which the flux exists.

If the coil is wound around a soft iron core, the flux linked with the coil increases because of very high permeability of soft iron. Therefore, the value of self-inductance L increases to a great extent.

Self-inductance L does not depend on current I.

Differentiating equation (1.8.1),

$N\phi = LI$ with respect to time t,

$$N\frac{d\phi}{dt} = L\frac{dI}{dt} \qquad (1.8.3)$$

In the case of self-induction, Faraday's law and Lenz's law holds good. Hence self-induced emf in the coil is,

$$\varepsilon = -N\frac{d\phi}{dt} \tag{1.8.4}$$

Self-induced emf is also called "back emf".

Equating equations (1.8.3) and (1.8.4), we get,

$$\varepsilon = -L\frac{dI}{dt} \tag{1.8.5}$$

If the rate of change of current $\left(\frac{dI}{dt}\right) = 1$ unit,

$\varepsilon = -L$

So the self-inductance of a circuit can be defined as under :

"The self-induced emf produced per unit rate of change of current $\left(\frac{dI}{dt} = 1\right)$ in the circuit is called self-inductance of the circuit."

Form equation $\varepsilon = -L\frac{dI}{dt}$,

Self inductance $L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$

Unit of $L = \frac{\text{Unit of emf (V)}}{\text{Unit of rate of change of current }(As^{-1})} = VsA^{-1}$

The SI unit of self-inductance L is Henry H and its dimensional formula is $M^1L^2T^{-2}A^{-2}$.

Henry : If the rate of change of current $\left(\frac{dI}{dt}\right) = 1\ As^{-1}$ and the induced emf is $\varepsilon = 1V$, then the self-inductance of the circuit is said to be 1 H.

The component of the circuit (e.g. coil) which posseses self-inductance is called an inductor. The symbol for an inductor in a circuit is shown in figure 1.15.

When an inductor is connected in a circuit, the end of the inductor through which the current enters and increases with time is considered positive and the end through which the current leaves is considered as negative. Thus, direction of induced emf can be determined.

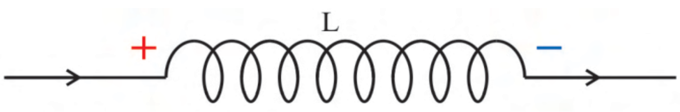


Figure 1.15 Symbol of Inductor

Magnetic Energy Stored in an Inductor : Suppose I is the current flowing through an inductor at time t and the rate of change of current in inductor is $\left(\frac{dI}{dt}\right)$.

Therefore, the induced emf between two ends of an inductor is, $\varepsilon = L\frac{dI}{dt}$. Here, the negative sign is ignored.

This self-induced emf is also called the back emf as it opposes any change in the current in a circuit. Physically, the self-inductance plays the role of inertia. It is the electro-magnetic analogue of mass (m) in mechanics. So, work needs to be done against the back emf (ε) in establishing the current. This work done is stored as magnetic potential energy.

For the current I at an instant in a circuit, the rate of work done is,

$$\frac{dW}{dt} = |\varepsilon| \, I \qquad (1.8.6)$$

Using equation (1.8.5),

$$\frac{dW}{dt} = LI\frac{dI}{dt} \qquad (1.8.7)$$

Total amount of work done in establishing the current I is,

$$W = \int_0^I dW$$

$$W = \int_0^1 LI \, dl$$

$$W = \frac{1}{2}LI^2 \qquad (1.8.8)$$

Thus, the electrical energy required to build up the current I is

$$W = \frac{1}{2}LI^2$$

This energy is stored in an inductor in the magnetic field linked with it.

This expression reminds us of $\frac{1}{2}mv^2$ for the (mechanical) kinetic energy of a particle of mass m. It shows that L is analogous to m. (i.e. L is the electrical inertia and opposes growth and decay of current in the circuit).

Illustration 8 : Find the value of the self-inductance of a very long solenoid of length l, having total number of turns equal to N, and cross-sectional area A.

Solution : The number of turns per unit length of the solenoid is $\frac{N}{l}$.

$\therefore$ The magnetic field at any point within the solenoid, on passing a current I will be

$$B = \frac{\mu_0 N I}{l}$$

The total flux linked with the entire solenoid will be,

$$\Phi = BAN$$

$$= \frac{\mu_0 NIA}{l} N$$

$$= \frac{\mu_0 N^2 IA}{l}$$

$\therefore$ Self-inductance, $L = \frac{\Phi}{I} = \frac{\mu_0 N^2 A}{l}$.

1.9 Mutual Induction

If two conducting coils are put close to each other, and a steady current is passed through one coil, magnetic flux links with the other coil. If the current flowing through the current carrying coil is changed, an emf is set up in the second coil according to Faraday's law. This phenomenon is called mutual induction.

Figure 1.16 shows two conducting coils near each other sharing a common central axis. Let the number of turns in coil 1 and 2 be N_1 and N_2 respectively.

Coil-1 is connected with battery, key and rheostat whereas coil-2 is connected to a sensitive galvanometer but contains no battery. When a steady current I_1 is passed through coil-1, it creates magnetic field (B_1) in coil-1. Some of the magnetic field lines of B_1 links with coil-2.

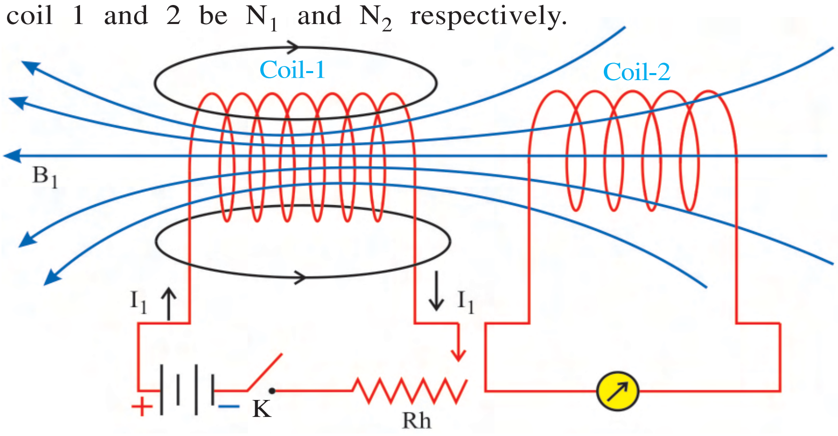


Figure 1.16 Mutual Inductance

For a given position of coil 1 and 2, it follows Bio-Savart law, that the flux Φ_2 linked with coil-2 will be proportional to the current I_1 in coil-1.

$$\Phi_2 \propto I_1$$

$$\therefore \; \Phi_2 = M_{21} I_1 \tag{1.9.1}$$

If the current I_1 in coil-1 is changed, there will be a corresponding change in the flux Φ_2 linked with coil-2. An emf ε_2 given by Faraday's law is induced in coil-2 which is given by,

$$\varepsilon_2 = -\frac{d\Phi_2}{dt}$$

$$\varepsilon_2 = -\frac{d}{dt}(M_{21} I_1)$$

$$\varepsilon_2 = -M_{21} \cdot \frac{dI_1}{dt} \tag{1.9.2}$$

The constant of proportionality M_{21} which appears in the equations (1.9.1) and (1.9.2) is termed as the mutual inductance of the system formed by two coils. It can be defined from equations (1.9.1) and (1.9.2).

Taking $I_1 = 1$ unit in the equation (1.9.1), we get $\Phi_2 = M_{21}$.

Thus, "The magnetic flux linked with one of the coils of a system of two coils per unit current flowing through the other coil is called mutual inductance of the system".

If the current is expressed in A and flux in Wb, then the unit of mutual inductance is WbA^{-1} = henry (H).

If, $\frac{dI_1}{dt} = 1$ unit in equation (1.9.2),

Then $\varepsilon_2 = -M_{21}$

Thus, "the emf generated in one of the two coils due to a unit rate of change of current in the other coil is called the mutual inductance of the system of two coils".

If we take $\frac{dI_1}{dt}$ in As^{-1} and ε_2 in V, then the unit of mutual inductance becomes $\frac{V}{As^{-1}} = VsA^{-1}$ = henry H. You can verify that the dimension of henry defined in either way are the same.

The mutual inductance M of a system of two coils depends upon their shape and size, their number of turns, distance between them, their relative orientation and the magnetic property of the medium on which they are wound.

Instead of coil-1, if we set up a current I_2 in coil-2 by means of a battery, this produces a magnetic flux Φ_1 that links with coil-1. If we change current I_2 flowing through coil-2, the emf induced in coil-1 by the argument given above is,

$$\varepsilon_1 = -M_{12}\frac{dI_2}{dt} \tag{1.9.3}$$

The mutual inductance will be same in both the cases discussed above. i.e. $M_{21} = M_{12} = M$. This result is called the reciprocity theorem.

Illustration 9 : Two long solenoids are of equal length l and the smaller solenoid having a cross-sectional area a is placed within the larger solenoid in such a way that their axes coincide. Find the mutual inductance of the system.

Solution : When a current I_1 is flowing through the smaller solenoid, the magnetic field strength within it is given by $\frac{\mu_0 N_1 I_1}{l}$.

Where N_1 = Number of turns for the smaller solenoid.

The flux linked with the larger solenoid due to this field is

$$\Phi_2 = \frac{\mu_0 N_1 N_2 I_1 a}{l}$$ (Where, N_2 = number of turns of the larger solenoid.)

$$\therefore\ M_{21} = \frac{\Phi_2}{I_1} = \frac{\mu_0 N_1 N_2 a}{l} \tag{1}$$

Now consider the situation in which a current I_2 is flowing through the outer solenoid, the field inside it is given by = $\frac{\mu_0 N_2 I_2}{l}$.

The flux due to this field linked with the inner solenoid is

$$\Phi_1 = \frac{\mu_0 N_1 N_2 I_2 a}{l}$$

$$\therefore\ M_{12} = \frac{\Phi_1}{I_2} = \frac{\mu_0 N_1 N_2 a}{l} \tag{2}$$

From the equation (1) and (2) we find $M_{21} = M_{12} = M$.

Illustration 10 : A small square loop of wire of side l is placed inside a large square loop of wire of side L. (L >> l). The loops are coplanar and their centres coincide. Find the mutual inductance of the system.

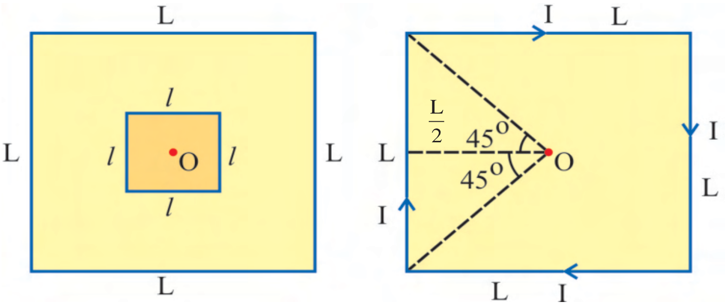
Solution : Let a current I pass through the large square loop of side L. Magnetic field at the centre O of the loop,

B = 4 × Magnetic field due to each side

$$B = 4 \times \frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)} (\sin 45^o + \sin 45^o)$$

$$B = \frac{2\mu_0 I}{\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$\therefore\ B = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$



Since l is very small compared to L, value of B can be considered uniform over the area A = πl^2 of the inner loop.

∴ Magnetic flux linked with the small square loop,

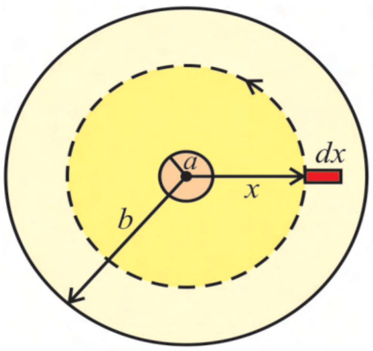
$$\Phi = BA = Bl^2 = \frac{2\sqrt{2}\mu_0 I l^2}{\pi L}$$

Mutual inductance of the system of two loops,

$$M = \frac{\Phi}{I} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}$$

Illustration 11 : Current I is passing through the central wire as well as the outer cylindrical layer of a co-axial cable in mutually opposite directions as shown in the figure. Find self inductance of this cable. The co-axial cable is normal to the plane of paper.

Solution : Magnetic field at a distance x from the central wire is,



$$B(x) = \frac{\mu_0 I}{2\pi x}$$

The magnetic flux passing through a strip of length l and width dx parallel to the axis is,

$$d\phi = B(x) l dx = \frac{\mu_0 I l}{2\pi x} dx$$

So the flux passing through the space of length l and breadth ($b - a$), between the two wires will be,

$$\phi = \int_a^b d\phi = \frac{\mu_0 I l}{2\pi x} \int_a^b \frac{1}{x} dx$$

$$= \frac{\mu_0 I l}{2\pi} [ln\ x]_a^b$$

$$= \frac{\mu_0 I l}{2\pi} ln \frac{b}{a}$$

Now, self-inductance, $L = \frac{\phi}{I} = \frac{\mu_0 l}{2\pi} ln \frac{b}{a}$

Illustration 12 : Prove that the energy density associated with the magnetic field of a very long solenoid is $\frac{B^2}{2\mu_0}$.

Solution : Magnetic induction linked with a solenoid is,

$$B = \frac{\mu_0 NI}{l} \quad (1)$$

Where, N = total number of turns of the solenoid,

l = length of the solenoid,

I = electric current.

Now, if self-inductance of the solenoid is L, the energy of magnetic field linked with it is,

$$U = \frac{1}{2}LI^2 \quad (2)$$

Keeping the value of I from equation (1) in equation (2),

$$U = \frac{1}{2}L\frac{B^2 l^2}{\mu_0{}^2 N^2} \quad (3)$$

But, self-inductance of a solenoid is,

$$L = \frac{\mu_0 N^2 A}{l} \quad (4)$$

where, A = Cross-sectional area of the solenoid.

Substituting the value of L from equation (4) in equation (3),

$$U = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\frac{B^2 l^2}{\mu_0{}^2 N^2}$$

$$\therefore \; U = \frac{1}{2\mu_0} A l B^2$$

$\therefore$ Now energy density is the energy per unit volume of the solenoid,

$$\rho_B = \frac{U}{Al} = \frac{1}{2\mu_0}B^2$$

Note : We have proved that energy of a charged capacitor is stored in the electric field between its two plates and energy density of electric field is $\rho_E = \frac{1}{2}\varepsilon_0 E^2$. Though the equations of ρ_B and ρ_E are obtained in cases of a solenoid and a capacitor, they are valid for more general cases also. If electric and magnetic field exist in some region of space (for Illustration electromagnetic waves), the energy density associated with the fields,

$$\rho = \rho_E + \rho_B$$

$$\therefore \; \rho = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

1.10 AC Generator

An important application of the phenomenon of electromagnetic induction is the generation of alternating currents (AC). Here, we shall discuss the principle of AC generator. One method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area. When a coil of surface area $\vec{A}$ rotates in a magnetic field $\vec{B}$, the effective area of the loop (the face perpendicular to the field) is $A\cos\theta$ (θ = angle between $\vec{A}$ and $\vec{B}$). This method of producing a flux change is the principle of operation of AC generator. An AC generator converts mechanical energy into electrical energy.

The basic components of an AC generator are shown in figure 1.17. It consists of a coil mounted on a rotor shaft. The axis of rotation of a coil (called armature) is perpendicular to the direction of magnetic field $\vec{B}$. When the coil (armature) is mechanically rotated in the uniform magnetic field ($\vec{B}$) by some external means, then the flux through the coil changes. So, an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings A_1, A_2 and brushes B_1, B_2.

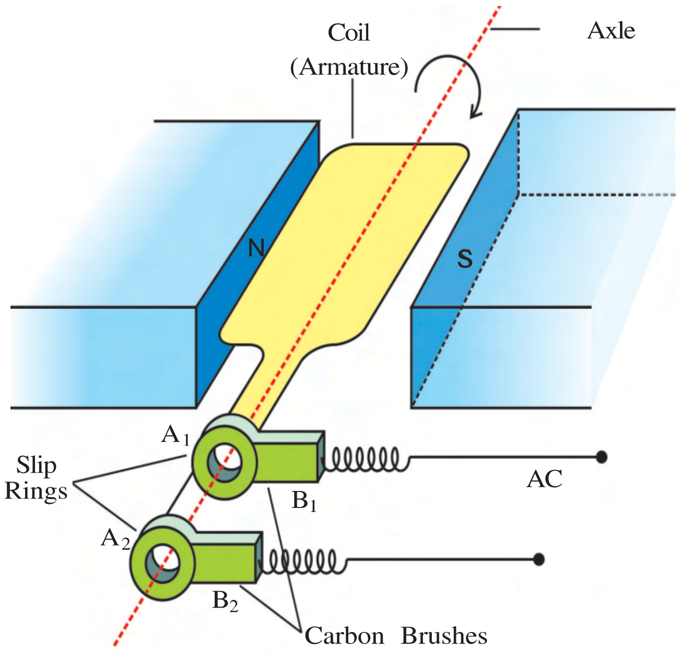


Figure 1.17 A.C. Generator

When the coil is rotated with a constant angular velocity ω in a uniform magnetic field $\vec{B}$, the angle θ between the magnetic field Vector $\vec{B}$ and the Area Vector $\vec{A}$ at any instant t is, $\theta = \omega t$ (assuming $\theta = 0$ at time $t = 0$).

As the coil having N turns is continuously rotating in a magnetic field, the magnetic flux $\Phi = NAB\cos\theta = NAB\cos\omega t$ associated with the coil keeps on changing with time.

The emf induced in the coil, according to Faraday's law is,

$$V = -\frac{d\Phi}{dt}$$

$$V = -\frac{d}{dt}(NBA\cos\omega t)$$

$$V = -NBA\frac{d}{dt}(\cos\omega t)$$

$$V = NBA\omega\sin\omega t \qquad (1.10.1)$$

where, $NBA\omega = V_m$ = Maximum induced emf in the coil.

$$\therefore\ V = V_m\sin\omega t \qquad (1.10.2)$$

Equation (1.10.2) suggests that the induced emf in the coil varies with time as per the function $\sin\omega t$. This emf is obtained between the brushes B_1 and B_2 which are in contact with the slip rings A_1 and A_2.

Since the value of the sine function varies between +1 and −1, the sign (polarity) of the induced emf changes with time. From figure (1.18), the emf has its extreme value when $\theta = \omega t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ as the rate of change of flux is maximum at these points. Emf will be zero for $\theta = \omega t = 0, \pi, 2\pi$.

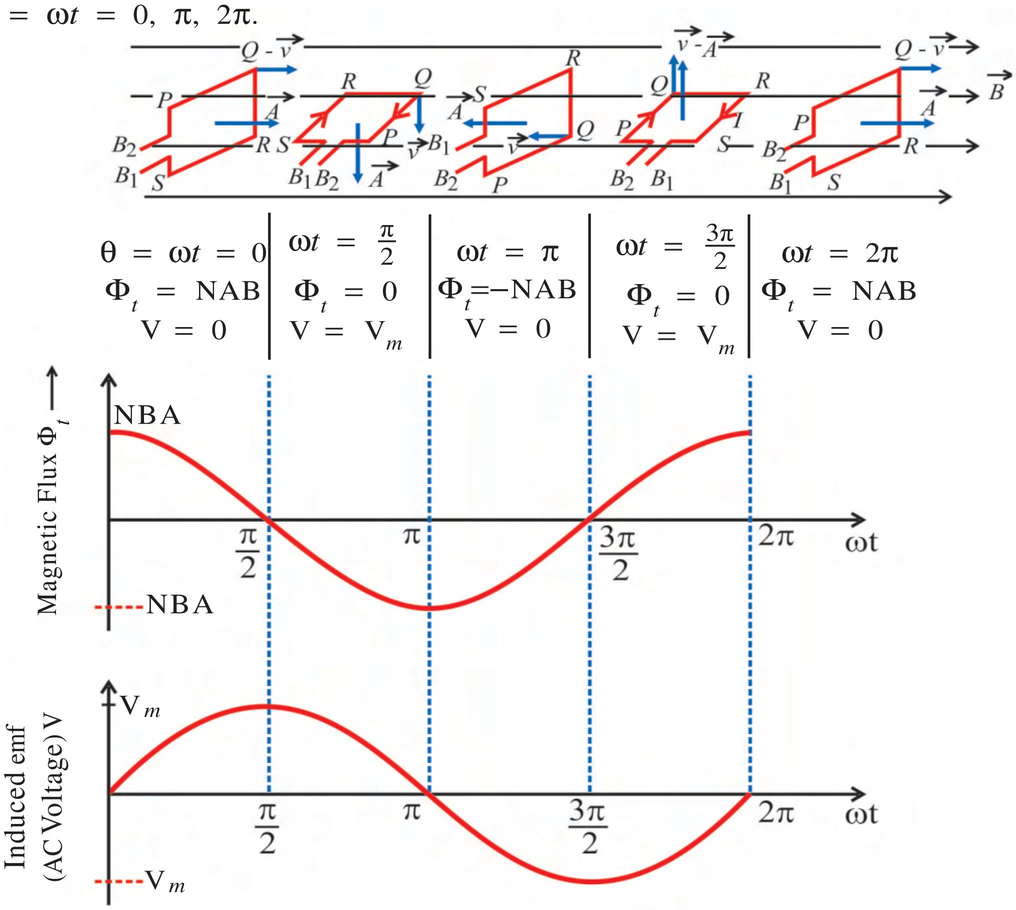


Figure 1.18 **Graph of Magnetic Flux (Φ_t) $\rightarrow$ ωt and AC Voltage (V) $\rightarrow$ ωt**

During the continuous rotation of the coil, the above mentioned situation is repeated. i.e. during a time interval of $\frac{\pi}{\omega} = \frac{T}{2}$, brushes B_1 and B_2 alternatively become positive and negative. Equation (1.10.2) gives the instantaneous value of the emf which varies between $+V_m$ and $-V_m$ periodically. Here, the voltage obtained between B_1 and B_2 is known as AC voltage (alternating voltage). B_1B_2 can be considered as AC voltage source.

Here, the direction of current changes periodically and therefore, such current is called alternating current (AC). Since $\omega = 2\pi f$, equation (1.10.2) can be written as,

$$V = V_m \sin 2\pi f t \tag{1.10.3}$$

where, f = Frequency of revolution of the generator's coil.

In commercial generators the mechanical energy required for rotation of the armature is provided by water falling from a height (e.g. from dams). These are called "Hydro-electric Generators". Alternatively, water is heated to produce steam using coal or other sources. The steam at high pressure produces the rotation of the armature. These are called "Thermal Generators". Instead of coal, if a nuclear fuel is used, it is called "Nuclear Power Generator". Modern day generators produce electric power as high as 500 MW, i.e. one can light up 5 million 100 W bulbs ! In most generators, the coils are held stationary and it is the electromagnets which are rotated. In India the frequency of AC is 50 Hz and in certain countries like USA, it is 60 Hz.

Illustration 13 : The number of turns in the coil of an AC generator are 50 and its cross sectional area is 2.5 m^2. This coil is revolving in a uniform magnetic field of strength 0.3 T with a uniform angular velocity of 60 rad s^{-1}. The resistance of the circuit comprising the coil is 500 Ω.

(1) Find the maximum induced emf and maximum current produced in the generator.

(2) Calculate the flux passing through the coil, when current is zero.

(3) Calculate the flux passing through the coil, when the current is maximum.

Solution : N = 50, A = 2.5 m^2, ω = 60 $rads^{-1}$, B = 0.3 T, R = 500 Ω

(1) The induce emf generated in an AC generator,

$$V = NBA\omega\sin\omega t = V_m\sin\omega t$$

$$\therefore \text{ Maximum emf } V_m = NBA\omega = 50 \times 0.3 \times 2.5 \times 60$$

$$= 2250V = 2.25 \text{ kV}$$

$$\text{Maximum current } I_m = \frac{V_m}{R} = \frac{2250}{500} = 4.5 \text{ A}$$

(2) The impedence is purely resistive. Therefore, when current is zero, voltage is also zero.

$$V = \frac{d\Phi}{dt} = 0$$

$$\therefore \Phi = \text{Maximum}$$

$$\therefore \Phi_m = NBA = 50 \times 0.3 \times 2.5 = 37.5 \text{ Wb}$$

(3) At the time of maximum current, voltage V will also be maximum.

$$V = NBA\omega\sin\omega t = \text{Maximum}$$

$$\therefore \sin\omega t = 1$$

$$\therefore \omega t = \frac{\pi}{2}$$

$$\therefore \text{ Flux } \Phi = NBA\cos\omega t = NBA\cos\frac{\pi}{2} = 0$$

Thus, when current is maximum flux will be zero.

SUMMARY

1. **Magnetic Flux :** The number of magnetic field lines crossing a surface normally is called magnetic flux linked with the surface.

   Magnetic flux through a plane of surface area A placed in a uniform magnetic field $\vec{B}$ is,

   $$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

   where, θ = angle between $\vec{B}$ and $\vec{A}$.

2. **Electromagnetic Induction :** The phenomenon in which electric current (and emf) is induced in a conductor or a closed circuit by varying magnetic field is called electromagnetic induction.

3. **Faraday's Law of Electromagnetic Induction :** Whenever the magnetic flux linked with a closed circuit (or coil) changes, an emf is induced in it.

"The magnitude of the induced emf produced in a closed circuit (or a coil) is equal to the negative of the time rate of change of magnetic flux linked with it".

$$\varepsilon = -\frac{d\phi}{dt} \qquad \text{(For 1 turn)}$$

$$\varepsilon = -N\frac{d\phi}{dt} \qquad \text{(For N turns)}$$

4. **Lenz's Law :** Induced emf (or induced current) is produced in such a direction that the magnetic field produced due to it opposes the very cause (e.g. the motion of the magnet) that produces it.

Lenz's law gives the direction of induced emf.

5. **Motional emf :** "The induced emf produced in a coil due to the change in magnetic flux linked with a coil due to some kind of motion is called motional emf".

If a conducting rod of length l moves with velocity v in a magnetic field B perpendicular to both its length and the direction of magnetic field, then the emf induced across its two ends is given by,

$$\varepsilon = -Blv$$

Force required to move the rod with a constant velocity v is,

$$F = BIl = \frac{B^2l^2v}{R}$$

Mechanical power $P = Fv = \dfrac{B^2l^2v^2}{R}$

6. **Relation between Induced Charge and Change in Magnetic Flux :**

Induced charge $\Delta Q = \dfrac{\Delta\Phi \text{ (Net Change in Magnetic Flux)}}{\text{R (Resistance)}}$

7. **Methods of Generating Induced emf :** The magnetic flux linked with a coil or a closed circuit can be changed and hence induced emf can be produced by following three methods.

(1) By changing the magnetic field $\vec{B}$.

(2) By changing the dimension (area A) of a coil.

(3) By changing the relative orientation (θ) of the coil in a magnetic field.

8. **Eddy Currents :** Whenever a solid conductor or a metallic plate is kept in a region of varying magnetic fields, the magnetic flux linked with it changes and circulatory currents are induced in it. These currents are called eddy currents. These currents are distributed throughout the conductor and their direction are determined by Lenz's law.

9. **Electromagnetic Damping :** When a pendulum made up of a metal plate is allowed to oscillate between two poles of a magnet, it performs damped oscillations due to eddy currents produced in it. Such a damping is called electromagnetic damping.

The effect of eddy currents can be minimized by making slots in the metal plate.

10. Self-Induction : When a current flowing through the coil is changed the magnetic flux linked with the coil itself changes. In such circumstances an emf is induced in the coil. Such emf is called self-induced emf and this phenomenon is called self-induction.

11. Self-Inductance : When a current I flows through a coil, flux linked with it,

$N\phi \propto I$

$N\phi = LI$

$L = \frac{N\phi}{I}$

The self-inductance L is a measure of the flux linked with the coil per unit current.

The self-inductance of a coil depends upon –

(1) The size and shape of the coil.

(2) The number of turns (N).

(3) The magnetic property of the medium within the coil in which the flux exists.

The self-induced emf in the coil is, $\varepsilon = -L\frac{dI}{dt}$

"The self-induced emf produced per unit rate of change of current $\left(\frac{dI}{dt} = 1\right)$ in the circuit is called the self-inductance of the circuit".

The SI unit of self-inductance L is henry H.

12. Henry : If the rate of change of current is $\left(\frac{dI}{dt} = 1As^{-1}\right)$ and the induced emf is $\varepsilon = 1V$, then the self-inductance of the circuit is 1H.

13. Self-Inductance of a Solenoid : $L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 l A$

where, μ_0 = permeability of free space

l = length of solenoid

N = total number of turns in a solenoid

A = area of cross-section of a solenoid

$n = \frac{N}{l}$ = number of turns per unit length of solenoid.

When the solenoid is wound over a soft iron core of relative permeability μ_r, then the self-inductance of a solenoid is, $L = \mu_r \mu_0 n^2 l A$.

14. Mutual Inductance : In the system of two coils, if the current flowing through one coil is changed, an induced emf is produced in the neighbouring coil. This phenomenon is called mutual induction.

15. Mutual Inductance : In the system of two coils, when a steady current I_1 is passed through coil-1, the magnetic flux linked with coil-2.

$\Phi_2 \propto I_1$ $\qquad$ $\Phi_2 = M_{21} I_1$

Mutual inductance : (Definition-1) : "The magnetic flux linked with one of the coils of a system of two coils per unit currents flowing through the other coil is called mutual inductance of the system".

If the current I_1 in coil-1 is changed, there will be a corresponding change in the flux Φ_2 linked with coil-2. Therefore, an emf is induced in coil-2 according to Faraday's law given by,

$$\varepsilon_2 = -M_{21}\frac{dI_1}{dt}$$

Mutual Inductance : (Definition-2) : The emf generated in one of the two coils due to a unit rate of change of currents in the other coil is called the mutual inductance of the system of two coils".

The mutual inductance (M) of a system of two coils depends upon their shape and size, their number of turns, distance between them, their relative orientation and the magnetic property of the medium on which they are wound.

16. Mutual Inductance of a System of Two Co-axial Long Solenoids :

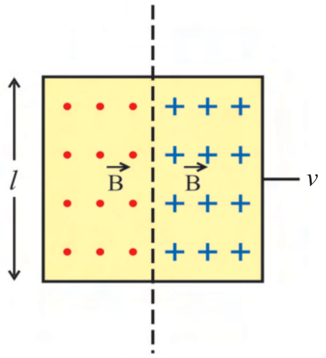
$$M = \frac{\mu_0 N_1 N_2 a}{l} = \mu_0 n_1 n_2 a l$$

where, n_1 and n_2 are the number of turns per unit length of two solenoids and a is the cross-sectional area of the inner solenoid.

EXERCISE

For the following statements choose the correct option from the given options :

1. A square conducting loop, whose plane is perpendicular to a uniform magnetic field, moves with velocity v normally to the magnetic field. If opposite sides of the loop, perpendicular to its velocity, remain in two mutually opposite uniform magnetic fields of strength B, the induced emf in this coil will be .... Length of each side is l.



(A) Bvl (B) $2Bvl$

(C) 0 (D) $\frac{Bvl}{2}$

2. The magnetic flux linked with a coil is changing with time t (second) according to $\phi = 6t^2 - 5t + 1$. Where ϕ is in Wb. At $t = 0.5$ s, the induced current in the coil is .... (the resistance of the circuit is 10 Ω)

(A) 1 A (B) 0.1 A (C) 0.1 mA (D) 10 A

3. A coil of surface area 100 cm^2 having 50 turns is held perpendicular to the magnetic field of intensity 0.02 Wbm^{-2}. The resistance of the coil is 2 Ω. If it is removed from the magnetic field in 1 s, the induced charge in the coil is ....

(A) 5 C (B) 0.5 C (C) 0.05 C (D) 0.005 C

4. When electric current in a coil steadily changes from +2 A to −2 A in 0.05 s, an induced emf of 8.0 V is generated in it. Then the self-inductance of the coil is .... H.

(A) 0.2 (B) 0.4 (C) 0.8 (D) 0.1

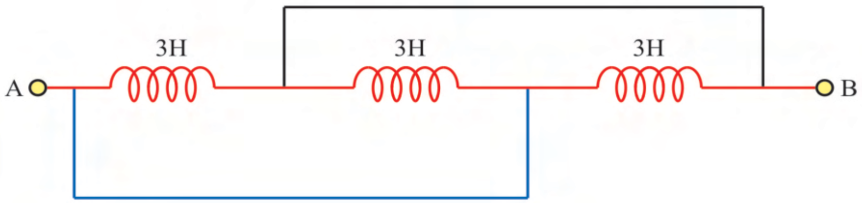
5. X and Y coils are joined in a circuit in such a way that when the change of current in X is 2 A , the change in the magnetic flux in Y is 0.4 Wb. The mutual induction of the system of two coils is .... H.

(A) 0.8 (B) 0.4 (C) 0.2 (D) 5

6. The mutual inductance of the system of two coils is 5 mH. The current in the first coil varies according to the equation $I = I_0 \sin\omega t$, where I_0 = 10A and $\omega = 100\pi$ rads^{-1}. The value of maximum induced emf in the second coil is ....

(A) 2π V (B) 5π V (C) π V (D) 4π V

7. Three pure inductances each of 3 H are connected as shown in figure. The equivalent inductance of this connection between points A and B is ....



(A) 1 H (B) 2 H (C) 3 H (D) 9 H

8. A square conducting coil of area 100 cm^2 is placed normally inside a uniform magnetic field of 10^3 Wbm^{-2}. The magnetic flux linked with the coil is .... Wb.

(A) 10 (B) 10^{-5} (C) 10^5 (D) 0

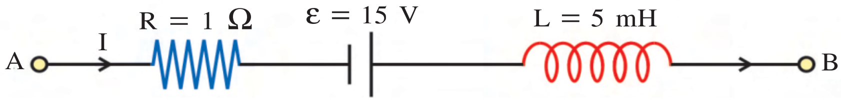
9. The distance between two extreme points of two wings of an aeroplane is 50 m. It is flying at a speed of 360 kmh^{-1} in horizontal direction. If the vertical component of earth's magnetic field at that place is 2×10^{-4} Wbm^{-2}, the induced emf between these two end points is .... V.

(A) 0.1 (B) 1.0 (C) 0.2 (D) 0.01

10. A wheel with 10 metallic spokes each 0.5 m long rotated with a speed of 120 rpm in a plane normal to the horizontal component of earth's magnetic field B_h at a place. If B_h = 0.4 G at the place, what is the induced emf between the axle and the rim of the wheel ? (1 G = 10^{-4} T)

(A) 0 V (B) 0.628 mV (C) 0.628 μV (D) 62.8 μV

11. The network shown in figure is a part of the circuit. (The battery has negligible resistance.)



At a certain instant the current I = 5 A and is decreasing at a rate of 10^3 As^{-1}. What is the potential difference between points B and A ?

(A) 5 V (B) 10 V (C) 15 V (D) 0V

12. A coil is placed in a time-varying magnetic field. Electrical power is dissipated in the form of Joule heat due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be ....

(A) halved (B) the same (C) doubled (D) quadrupled

13. The self-inductance of two solenoids A and B having equal length are same. If the number of turns in two solenoids A and B are 100 and 200 respectively, the ratio of the radii of their cross-section will be ....

(A) 2 : 1 (B) 1 : 2 (C) 1 : 4 (D) 4 : 1

14. A thin circular ring of area A is held perpendicular to a uniform field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is,

(A) $\frac{BR}{A}$ (B) $\frac{AB}{R}$ (C) ABR (D) $\frac{B^2A}{R}$

15. A magnet is moving towards a coil along its axis and the emf induced in the coil is ε. If the coil also starts moving towards the magnet with the same speed, the induced emf will be ....

(A) $\frac{\varepsilon}{2}$ (B) ε (C) 2ε (D) 4ε

16. A rod of 5 cm length is moving perpendicular to uniform magnetic field of intensity 2×10^{-4} Wbm^{-2}. If the acceleration of rod is 2 ms^{-2}, the rate of increase of the induced emf is ....

(A) 20×10^{-4} Vs^{-2} (B) 20×10^{-4} V (C) 20×10^{-4} Vs (D) 20×10^{-4} Vs^{-1}

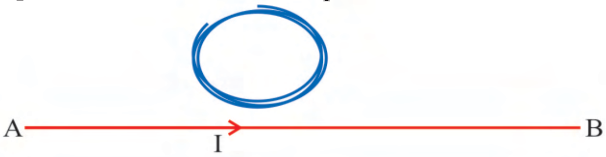
17. Current of 2A passing through a coil of 100 turns gives rise to a magnetic flux of 5×10^{-3} Wb per turn. The magnetic energy associated with coil is ....

(A) 5×10^{-3} J (B) 0.5×10^{-3} J (C) 5 J (D) 0.5 J

18. The flux linked per each turn of a coil of N turns changes from ϕ_1 to ϕ_2. If the total resistance of the circuit including the coil is R, the induced charge in the coil.

(A) $\frac{N(\phi_2 - \phi_1)}{t}$ (B) $\frac{N(\phi_2 - \phi_1)}{R}$ (C) $\frac{N(\phi_2 - \phi_1)}{Rt}$ (D) $N(\phi_2 - \phi_1)$

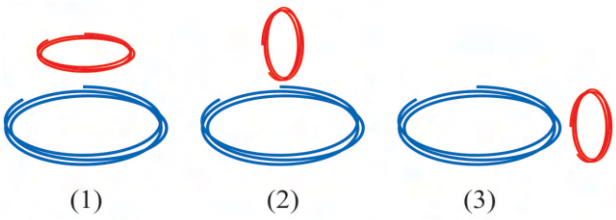
19. The current flows from A to B in a straight wire as shown in figure and it is decreasing with time. The induced current in the loop is placed near the wire in the same horizontal plane, when viewed from upward.



(A) In clockwise direction
(B) In anticlockwise direction
(C) Will not be produced
(D) Nothing can be said

20. Two circular coils can be arranged in any of the three situations as shown in figure. Their mutual inductance (M) will be.

(A) Maximum in situation (1)
(B) Maximum in situation (2)
(C) Maximum in situation (3)
(D) The same in all situations.



21. In AC generator, induced emf is zero at time $t = 0$. The induced emf at time $\frac{\pi}{2\omega}$ is ....

(A) $+2\,V_m$ (B) $+2\,V_m$ (C) zero (D) $+2\,V_m$

ANSWERS

1. (B)	**2.** (B)	**3.** (D)	**4.** (D)	**5.** (C)	**6.** (B)
7. (A)	**8.** (A)	**9.** (B)	**10.** (D)	**11.** (C)	**12.** (B)
13. (A)	**14.** (B)	**15.** (C)	**16.** (D)	**17.** (D)	**18.** (B)
19. (B)	**20.** (A)	**21.** (A)			

Answer the following questions in brief :

1. Give the statement of Lenz's law to determine the direction of induced emf.

2. State Faraday's law of electromagnetic induction.

3. What is the physical significance of negative sign appearing in the mathematical form of Faraday's law ?

4. Define motional emf.

5. What is Lenz's force ?

6. Will induced emf be produced in a wire kept in north-south direction allowed to fall freely ? Why ?

7. What are eddy currents ?

8. What is electromagnetic damping ?

9. How can the effects of eddy currents be reduced ?

10. Define self-inductance.

11. What will be the change in self-inductance if the current flowing in the coil is increased ?

12. Why self-inductance of a coil increases when the coil is wound on a soft iron core ?

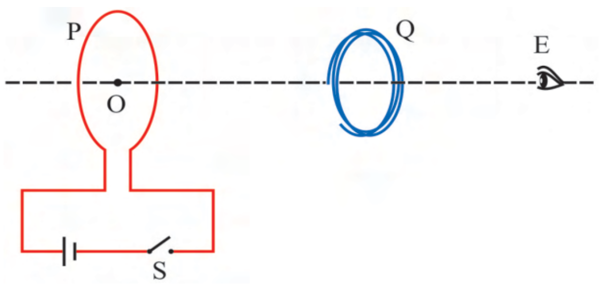
13. Why a metal plate falls downward with acceleration less than g in the presence of magnetic field ?

14. On what factors does the mutual inductance of a system of two coils depend ?

15. What is reciprocity theorem in the context of mutual inductance ?

16. A coil having N turns and resistance R Ω is connected to a galvanometer of resistance 4R Ω. This combination is moved in time t seconds from a magnetic flux Φ_1 Wb to Φ_1 Wb. What is the induced current in the circuit?

17. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the Switch S is closed, a clockwise current I_P flows in loop P (as seen by E) and an induced current I_Q flows in loop Q. In which direction this induced current I_Q will flow as seen by E ?



18. How can the magnetic flux associated with a coil or a closed circuit be changed ?

19. In an AC generator, the brushes in contact with the slip rings alternatively become positive and negative in the time interval of 5 ms. What is the frequency of the voltage generated ?

Answer the following questions :

1. Describe Faraday's historical experiment to induce current by winding insulated conducting coils on the ring of soft iron.

2. Discuss the results of Faraday's experiment performed with bar magnet and insulated conducting coil.

3. "Lenz's law is a special statement of law of conservation of energy." – Explain.

4. Obtain an equation for motional emf produced in a conducting rod which is moving on the two arms of U-shaped conductor perpendicular to magnetic field.

5. Using necessary figure (circuit), obtain an expression for self-induced emf produced in a coil.

6. Deduce an equation $U = \frac{1}{2}LI^2$ for an inductor.

7. Give two definitions of mutual inductance and write its unit.

8. Explain eddy currents.

9. Discuss the reason behind the production of induced emf in case of a conducting rod moving in the magnetic field with its velocity perpendicular to the magnetic field.

10. Explain conversion of mechanical energy into the electrical energy in case of conducting rod sliding over a U-shaped wire which is placed inside a magnetic field.

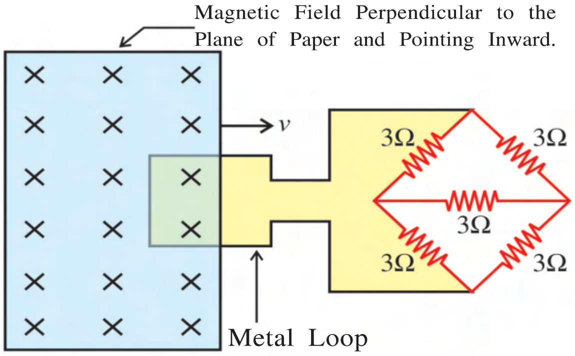
11. Give the applications of eddy currents.

12. With the help of neat diagram derive the expression for induced emf in an AC generator.

13. Give the characteristics of an induced emf in AC generator.

Solve the following examples :

1. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v in a uniform magnetic field of induction B = 2 Wbm^{-2} as shown in figure. The magnetic field is perpendicular to the plane of the loop and directed into the paper. The loop is connected to a network of resistors each of value 3 Ω. With what speed should the loop be moved so that a steady current of 1 mA flows in the loop.



[**Ans. :** 2×10^{-2} ms^{-1}]

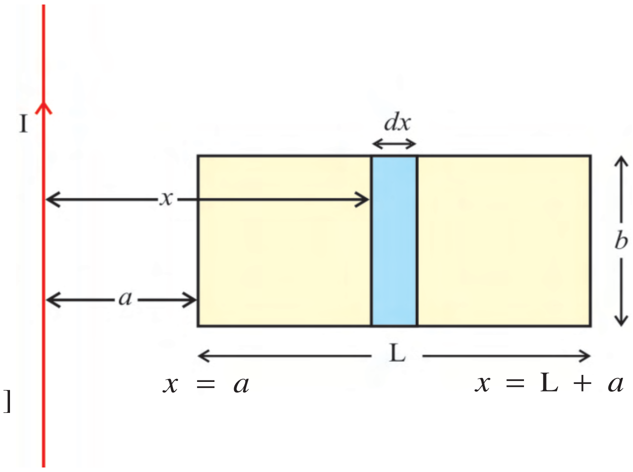
2. A coil having 200 turns has a surface area 0.12 m^2. A magnetic field of strength 0.10 Wbm^{-2} linked perpendicular to this area changes to 0.5 Wbm^{-2} in 0.2 s. Find the average emf induced in the coil. **[Ans. : 48 V]**

3. A coil of cross-sectional area A and having N turns lies in a uniform magnetic field B with its plane perpendicular to the field. In this position the normal to the coil makes an angle of 0^o with the field. The coil rotates at a uniform rate to complete one rotation in time T. Find the average induced emf in the coil during the interval when the coil rotates :

(i) From 0^o to 90^o position, (ii) From 90^o to 180^o position, (iii) From 180^o to 270^o position and (iv) From 270^o to 360^o position.

[**Ans. :** (i) $\frac{4NBA}{T}$ (ii) $\frac{4NBA}{T}$ (iii) $\frac{-4NBA}{T}$ (iv) $\frac{-4NBA}{T}$]

4. As shown in figure, a rectangular loop of length L and breadth b is placed near a very long wire carrying current I. The side of the loop nearer to the wire is at a distance a from the wire. Find magnetic flux linked with the loop.

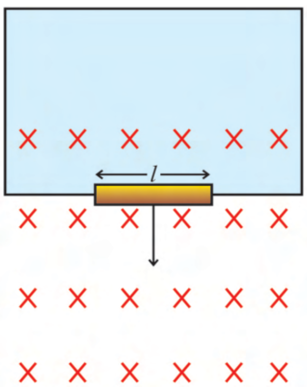


[**Hint :** $\int \frac{1}{x} dx = ln\ x$]

[**Ans. :** $\phi = \frac{\mu_0 Ib}{2\pi}\ ln\left(\frac{L+a}{a}\right)$]

5. A conducting bar of 2 m length is allowed to fall freely from a 50 m high tower, keeping it aligned along the east-west direction. Find the emf. induced in the rod when it is 20 m below the top of the tower g = 10 ms^{-2}. Earth's magnetic field is 0.7×10^{-4} T and angle of dip = 60^o. **[Ans. : 1.4 mV]**

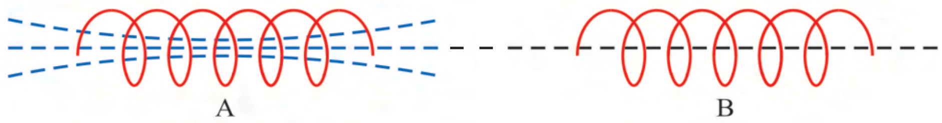
6. A conducting rod of length l, mass m and resistance R is falling freely through a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of paper as shown in figure. Find terminal velocity (v_t) of this rod.



[**Ans. :** $\frac{mgR}{B^2 l^2}$]

7. Find the equivalent inductance of two inductors having inductances L_1 and L_2 connected in parallel with the help of appropriate DC circuit. [**Ans. :** $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$]

8. Two coils placed near to each other have number of turns equal to 600 and 300 respectively. On passing a current of 3.0 A through coil A, the flux linked with each turn of coil A is 1.2×10^{-4} Wb and the total flux linked with coil B is 9.0×10^{-5} Wb. Find (1) self-inductance of A, (2) The mutual inductance of the system formed by A and B.



[**Ans. :** $L_A = 24$ mH; $M_B = 30\ \mu$H]

9. There are 1.5×10^4 turns in the winding of a toroidal ring. The radius of circular axis of the ring is 10 cm. The radius of cross-section of ring is 2.0 cm. Find inductance of the ring. [**Ans. :** 0.57 H]

10. A conducting loop of radius r is placed concentric with another loop of a much larger radius R so that both the loops are coplanar. Find the mutual inductance of the system of the two loops. Take R >> r. [**Ans. :** $\frac{\mu_0 \pi r^2}{2R}$]