5

MAGNETISM AND MATTER

5.1 Introduction

The word magnet is derived from the name of an island in Greece called Magnesia, where magnetic ore deposits were found as early as 800 BC. Shepherds on this island complained that the nails of their shoes were getting stuck to the ground. The tip of their staff were also getting stuck to chunks of magnetite while they pastured their flocks. Greeks observed that the stone of magnetite (Fe_3O_4) attracts the pieces of iron.

The chinese were the first to use magnetic needles for navigation on ships. Caravans used the magnetic needles to navigate across the Gobi desert. Magnetism is much older than the genesis of life and the subsequent evolution of human beings on earth. It exists everywhere in the entire universe. The earth's magnetism predates human evolution.

In 1269 a Frenchman named Pierre-de Maricourt mapped out the directions of magnetic lines on the surface of a spherical natural magnet by using magnetic needle. He observed that the directions of magnetic lines formed on the sphere were passing through two points diametrically opposite to each other, which he called the poles of the magnet. Afterwards other experiments also showed that every magnet, regardless of its shape and size, has two poles called north and south poles. Some commonly known facts regarding magnetism are as follows :

(1) The Earth behaves as a magnet with the magnetic field pointing approximately from geographic south to north direction.

(2) When a bar magnet is suspended from its mid–point such that it can rotate freely in horizontal plane, then it continues to rotate (oscillate) until it aligns in the north–south direction. The end of the magnet pointing towards the north is called the magnetic **North pole** of the magnet, and the end pointing towards the south pole is called the magnetic **South pole** of the magnet.

(3) Like magnetic poles repel each other, and the unlike poles attract each other.

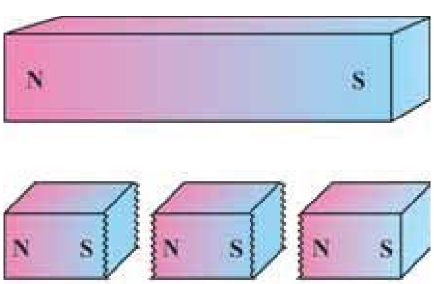


Figure 5.1 Magnet and its Fragments Behaving as Independent Magnets

(4) The positive and negative charges in **electric dipole** may be separated and can exist independently, called **electric monopoles.** The magnet with two poles may be regarded as a **magnetic dipole.** But the magnetic poles are always found in pairs. The north and south magnetic poles cannot be separated by splitting the magnet into two parts. Even if the bar magnet is broken into two or more parts, then also each fragment of the magnet behaves as an independent magnet with north and south magnetic poles with somewhat weaker magnetic field (See figure 5.1). Thus an independent magnetic monopole does not exist. The search for magnetic monopoles is going on.

(5) Magnets can be prepared from iron and its alloys.

In this chapter you will learn the equivalence between magnetic field of a bar magnet and a solenoid, the magnetic dipole moment of a current carrying loop and the dipole moment of orbiting electron in an atom.

The magnetic field strength produced by a magnetic dipole at a point on its equator and at a point along its axis is calculated. The magnetic field of the earth, geomagnetic elements, as well as, para, dia and ferro–magnetic materials are also discussed with suitable examples in this chapter. At the end of this chapter, the applications of permanent magnets and electromagnets are explained in brief.

5.2 The Bar Magnet

The great scientist Albert Einstein got a magnet as a gift when he was a child. He was much fascinated by it and used to play with it. When the magnet attracted iron nails, pins etc., he wondered how the magnet could attract the things without touching them.

Figure 5.2 shows the arrangement of iron filings sprinkled on a plane paper, which is kept on a bar magnet. When the paper is tapped twice or thrice, the iron filings rearrange in a systematic pattern representing the magnetic field lines. Similar picture of magnetic field lines can be formed if the bar magnet is replaced by a short solenoid, through which a DC current passes.

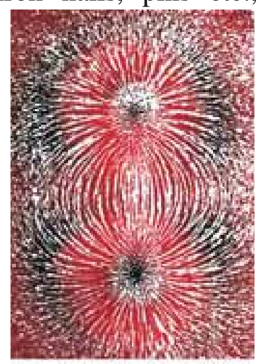
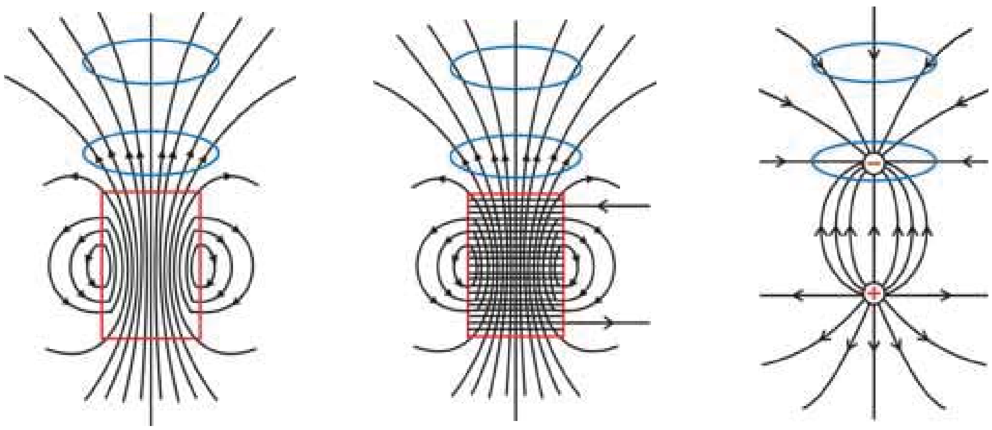


Figure 5.2 Systematic Arrangement of Iron Filings Representing Magnetic Field Lines of a Bar Magnet



(a) Bar Magnet (b) Solenoid (c) Electric-Dipole

Figure 5.3 Magnetic and Electric Field Lines (Only for Information)

Figure 5.3 shows the magnetic field lines due to a bar magnet and a short solenoid. Electric field lines due to an electric dipole are also shown for comparison.

Following conclusions can be made from the study of figure 5.3 :

(1) **The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. The magnetic field lines emerge out from the magnetic north pole, reach the magnetic south pole and then passing through the magnet, reach the north pole to complete the loop.** In the electric dipole, these field lines begin from a positive charge and end on the negative charge or escape to infinity.

It is impossible to have a static arrangement of electric charges, whose electric field lines form closed loops. This is a typical property of the static electric field.

(2) The tangent to a magnetic field line at a point through which it passes, indicates the direction of magnetic field $\vec{B}$ at that point.

For example, a compass needle may be used to trace out the magnetic field lines of a bar magnet by putting it at different positions surrounding the bar magnet.

(3) The magnitude of magnetic field in the region surrounding a magnet can be represented by the number of magnetic field lines passing normally through a unit area in that region. In figures 5.3 (a) and 5.3 (b) the magnitude of magnetic field B is larger around region (i) than in region (ii).

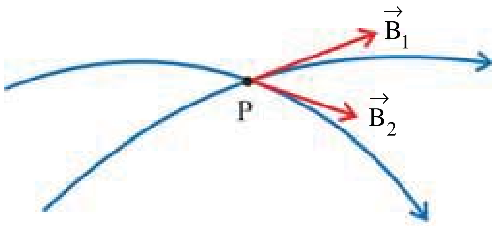


Figure 5.3 (d)

(4) **The magnetic field lines do not intersect with each other.** If they intersect at a point, then the tangents to the lines at the point of intersection would represent two different directions of the magnetic field at that point, which is impossible. (See figure 5.3(d))

If the magnetic field lines intersect at point P, the magnetic fields $\vec{B}_1$ and $\vec{B}_2$ point in different directions.

5.3 Current Loop as a Magnet and its Magnetic Moment

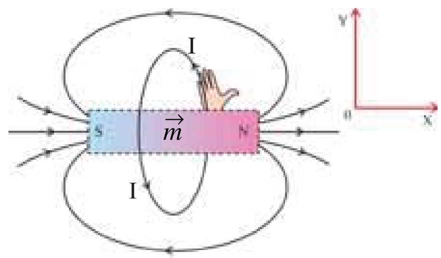


Figure 5.4 Magnetic Field Produced by a Current Loop Like that of a Bar Magnet of Magnetic Dipole Moment $\vec{m}$

In Chapter-4 you studied that, a loop of area A and carrying current I behaves as a magnet, with magnetic depole moment

$$m = IA \tag{5.3.1}$$

The direction of magnetic moment $\vec{m}$ of the loop can be found using right hand rule as shown in Figure (5.4)

$$\text{Thus, } \vec{m} = I\vec{A} \tag{5.3.2}$$

If there are N turns in the loop, then

$$\vec{m} = NI\vec{A} \tag{5.3.3}$$

For the points on the axis of the loop of radius a, far from its centre ($x >> a$), the magnetic field (Chapter-4) is given by

$$B(x) = \frac{\mu_0 I a^2}{2x^3} \tag{5.3.4}$$

$$= \frac{\mu_0}{2\pi}\frac{I\pi a^2}{x^3} = \frac{\mu_0}{2\pi}\frac{IA}{x^3} \qquad (A = \pi a^2 = \text{area of the loop})$$

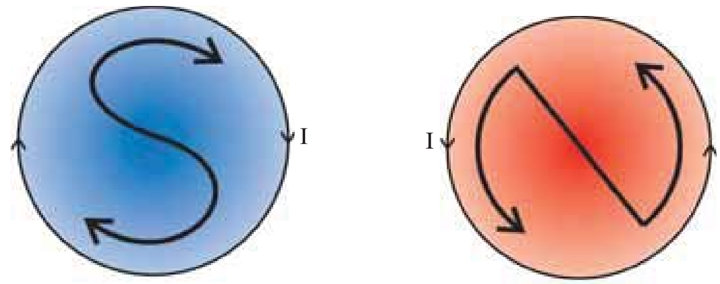
$$\therefore \; B(x) = \frac{\mu_0}{2\pi}\frac{m}{x^3} \tag{5.3.5}$$

Since $B(x)$ and m have same direction,

$$\vec{B}(x) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3} \tag{5.3.6}$$

which is the axial magnetic field in terms of magnetic dipole moment $\vec{m}$ of the loop at $x >> a$. Equation (5.3.6) is equally applicable for a (short) bar magnet of magnetic dipole moment $\vec{m}$.

5.3.1 Direction of Magnetic Pole in a Current Carrying Loop :



(a) Magnetic South Pole (b) Magnetic North Pole

Figure 5.5

Figure 5.5(a) shows the current I flowing in clockwise direction in a circular loop lying in the plane of the page. According to right hand rule, the side of the loop towards us behaves as a magnetic south pole whereas the opposite side of the loop behave as a magnetic north pole. The symbolic notation S indicates magnetic south pole pointing towards us.

Simliarly, if the current flows in anticlockwise direction in the loop, the side of the loop towards us behaves as a magnetic north pole and opposite side as a magnetic south pole (See Figure 5.5(b)). The symbolic notation N indicates the magnetic north pole pointing outwards.

5.4 Magnetic moment of an electron rotating around the nucleus of an atom :

Dear students, now you know that a magnetic field is produced by the motion of charged particles or by an electric current. Any material is made up of atoms, and in these atoms definite number of electrons (depending on the nature of the element), move in various possible orbits. Such motion of electrons in orbits can be considered as an electric current around a closed path, with magnetic moment IA (I = electric current, and A = area enclosed by the orbit). The magnetic dipole moment of an atom of any given element, depends upon the distribution of electrons in various orbits and on their spins.

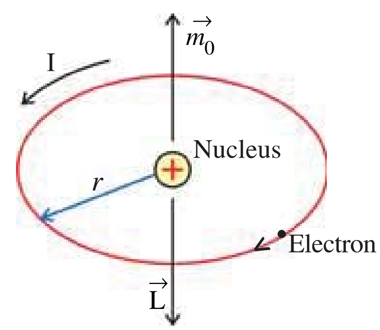


Figure 5.6 Non-zero Magnetic Moment of Atom

As shown in figure 5.6, consider an electron moving with constant speed v in a circular orbit of radius r about the nucleus. If the electron travels a distance $2\pi r$ (circumference of the circle) in time T, then its orbital speed is $v = \frac{2\pi r}{T}$. Thus the current I associated with this orbiting electron of charge e is, $I = \frac{e}{T}$.

Here, $T = \frac{2\pi}{\omega}$, and $\omega = \frac{v}{r}$

$\therefore \ I = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$

The orbital magnetic moment associated with this orbital current loop is

$$m_0 = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{1}{2} evr \quad (5.4.1)$$

where $A = \pi r^2$ = area enclosed by the circular orbit.

For this electron, the orbital angular momentum is $L = m_e vr$. Hence, the orbital magnetic moment of the electron can be represented as

$$m_0 = \left(\frac{e}{2m_e}\right)(m_e vr) = \left(\frac{e}{2m_e}\right)L \qquad (5.4.2)$$

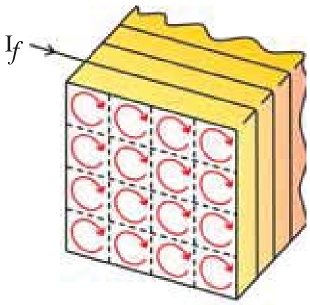
Equation (5.4.2) shows that the magnetic moment of the electron is proportional to its orbital angular momentum L. But since the charge of electron is negative, the vectors $\vec{m_0}$ and $\vec{L}$ point in opposite directions, perpendicular to the plane of the orbit.

$$\therefore \ \vec{m_0} = -\left(\frac{e}{2m_e}\right) \vec{L} \tag{5.4.3}$$

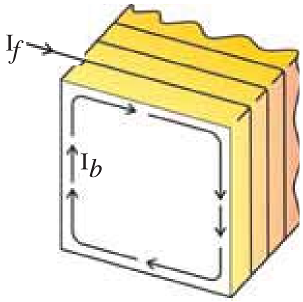
The ratio $\frac{e}{2m_e}$ is a constant called the gyro–magnetic ratio, and its value is 8.8×10^{10} C kg^{-1}.

5.5 Magnetism in Matter

In general, the magnets are prepared from iron (Fe). The atoms of iron normally possess magnetic dipole moment, but an ordinary piece of iron does not behave as a magnet .



(a) Atomic Current Loops Due to Current I_f (Magnetized Iron)



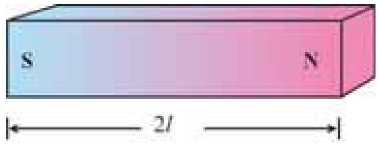
(b) Overall Bound Current Loop Equivalent to Loops in Part (a)

Figure 5.7

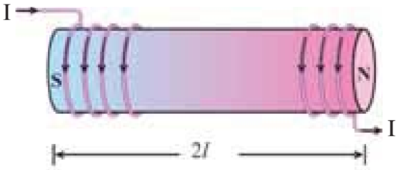
The same iron piece can be converted into a magnet, if it is kept in a strong magnetic field for some time and then the applied magnetic field is removed. As shown in figure 5.7 a wire is wound on a piece of iron. If $I_f = 0$, then the magnetic dipole moments of current loops of atoms are randomly oriented. Thus the resultant magnetic moment of the iron piece becomes zero and the iron piece does not behave as a magnet.

When sufficient current I_f passes through the wire, a strong magnetic field is generated in the iron piece, due to which the elemental atomic currents redistribute in the iron piece. Thus a resultant bound current I_b is generated in the iron piece (See Figure 5.7(b)). When the current I_f is slowly reduced to zero, all of the elemental atomic currents do not return to original state even though the external magnetic field becomes zero. This way the iron piece sustains magnetic field.

5.6 Equivalence between a Bar Magnet and a Solenoid



(a)



(b)

Figure 5.8 A Bar Magnet and a Solenoid

Figure 5.8 shows a bar magnet and a solenoid. If the pole strength of bar magnet is p_b (even though such individual poles do not exist), and the distance between two poles is $2l$ then according to definition, the magnetic dipole moment of bar magent is

$$m_b = 2lp_b \tag{5.6.1}$$

$$\therefore\ p_b = \frac{m_b}{2l} \tag{5.6.2}$$

The suffix b here indicates that the magnetic moment is due to bar magnet.

Note : Only for infomation : The poles (p_b) of the bar magnet are not on the end faces of the bar magnet, but are situated inside, in such a way that the distance between the two poles (magnetic length) is $2l_m$, which is slightly less than the geometric length $2l$ of the bar magnet. For practical purposes the magnetic length $2l_m = \frac{5}{6} \times 2l$, is taken as geometric length $2l$, in this book.

In a solenoid of cross sectional area A, carrying current I, each turn can be treated as a closed current loop, and hence a magnetic dipole moment IA can be associated with each turn. As the magnetic dipole moment of every turn is in the same direction, the magnetic dipole moment of the solenoid is a vector sum of dipole moments of all turns. If there are total N turns in length $2l$ of the solenoid, then its magnetic moment is

$$m_s = \text{NIA} \tag{5.6.3}$$

From equations (5.6.1) and (5.6.3), we can define equivalent pole strength of solenoid as

$$p_s = \frac{m_s}{2l} = \frac{\text{NIA}}{2l} = n\text{IA} \tag{5.6.4}$$

where $n = \frac{\text{N}}{2l}$ = number of turns per unit length of solenoid.

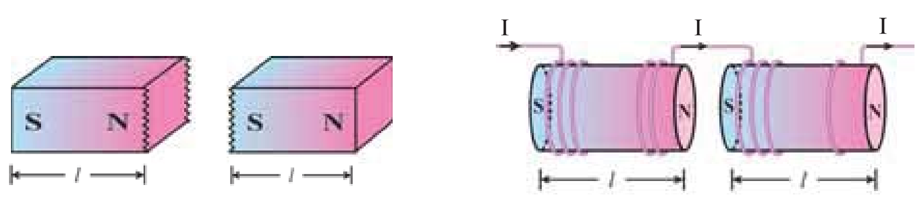
From equation (5.6.4), the unit of pole strength is A m.

As mentioned in the article (5.3) the magnetic field along the axis of dipole moment $\vec{m}$ is

$$\vec{\text{B}}(x) = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{x^3} \tag{5.6.5}$$

Hence, the magnetic field produced by a bar magnet or a solenoid can be calculated by replacing $\vec{m}$ by $\vec{m}_b$ or $\vec{m}_s$, respectively, in equation (5.6.5).

What happens if bar magnet is broken ?



(a) (b)

Figure 5.9 Broken Bar Magnet and a Solenoid

If the solenoid of figure 5.8 is broken into two equal pieces as shown in figure 5.9.(b), then the pole strength of each piece of solenoid remains same as nIA, since the number of turns per unit length (n) remains same. By analogy we can say that the pole strength of each piece of bar magnet also remains same.

In both cases, the magnetic length becomes half of the original length. Hence the magnetic dipole moment also becomes half.

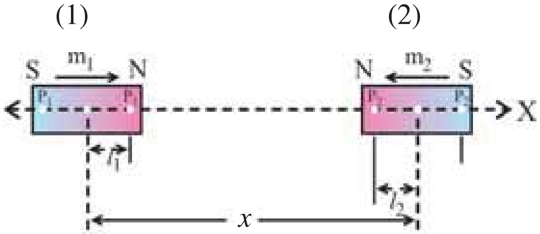
5.6.1 The Electrostatic Analogue : Comparing equatians (5.6.1) and (5.6.5) with corresponding equations for electric charge (chapter 1), it can be observed that the magnetic field at large distances due to a bar magnet or current loop of magnetic moment $\vec{m}$ can be obtained directly from the equations of electric field due to an electric dipole of dipole moment $p = 2aq$, by making following replacements.

$$\vec{E} \rightarrow \vec{B}, \; \vec{p} \rightarrow \vec{m}, \; \frac{1}{4\pi\varepsilon_0} \rightarrow \frac{\mu_0}{4\pi}$$

Table 5.1 Analogy between Electric and Magnetic Dipoles

Quantity	Electrostatics	Magnetics
Constant	$\frac{1}{4\pi\varepsilon_0}$ q (charge)	$\frac{\mu_0}{4\pi}$ p (pole strength)
Dipole moment	$\vec{p} = q(2\vec{a})$	$\vec{m} = p(2\vec{l})$
Equatorial Field	$\vec{E}(y) = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(y^2+a^2)^{\frac{3}{2}}}$	$\vec{B}(y) = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{(y^2+l^2)^{\frac{3}{2}}}$
$y >> a$ $y >> l$	$= -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{y^3}$	$= -\frac{\mu_0}{4\pi}\frac{\vec{m}}{y^3}$
Axial Field	$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}z}{(z^2-a^2)^2}$	$\vec{B}(z) = \frac{\mu_0}{4\pi}\frac{2\vec{m}z}{(z^2-l^2)^2}$
$z >> a$ $z >> l$	$= \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{z^3}$	$= \frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3}$
Force	$\vec{F} = q\vec{E}$	$\vec{F} = p\vec{B}$
Torque (in External Field)	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{m} \times \vec{B}$
Energy (in External Field)	$U = -\vec{p}\cdot\vec{E}$	$U = -\vec{m}\cdot\vec{B}$

Illustration 1 : Find the force between two small bar magnets of magnetic moments $\vec{m}_1$ and $\vec{m}_2$ lying on the same axis, as shown in the Figure. (p_1 and p_2 are the pole strength of magnets (1) and (2) respectively)



$x >> l_1, \quad x >> l_2$

Solution : To find the force on magnet (2) due to magnet (1), calculate the magnetic field due to magnet (1) at the poles of magnet (2). The axial magnetic field at the north pole of magnet (2) due to magnetic moment m_1 is (from the geometry of Figure)

$$B_N = \frac{\mu_0}{4\pi}\ \frac{2m_1}{(x-l_2)^3} \quad (1)$$

Similarly, the axial magnetic field at the south pole of magnet (2) is

$$B_S = \frac{\mu_0}{4\pi} \cdot \frac{2m_1}{(x+l_2)^3} \quad (2)$$

The repulsive force F_N acting on the north pole of magnet (2) having pole strength p_2 is (like $F = qE$ in electrostatics)

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi}\frac{2p_2 m_1}{(x-l_2)^3} \quad (3)$$

which is acting away from magnet (1)

Similarly, the attractive force F_S acting on the south pole of magnet (2) is

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi}\frac{2p_2 m_1}{(x+l_2)^3} \quad (4)$$

which is acting towards magnet (1)

Hence the resultant force on magnet (2) is

$$F = F_N - F_S$$

$$= \frac{\mu_0}{4\pi} \cdot 2p_2 m_1 \left[\frac{1}{(x-l_2)^3} - \frac{1}{(x+l_2)^3}\right] = \frac{\mu_0}{2\pi}\ p_2 m_1 \left[\frac{(x+l_2)^3 - (x-l_2)^3}{\{(x-l_2)(x+l_2)\}^3}\right]$$

$$= \frac{\mu_0}{2\pi}\ p_2 m_1 \left[\frac{6x^2 l_2}{(x^2 - l_2{}^2)^3}\right]$$

[Because $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$ and $l_2^3 << x^2 l_2$ in numerator]

$$\therefore\ F = \frac{\mu_0 m_1}{2\pi} \cdot \frac{2l_2 P_2 \cdot 3x^2}{x^6} \quad \text{(Since } l_2^2 << x^2\text{, and hence } l_2^2 \text{ can be neglected)}$$

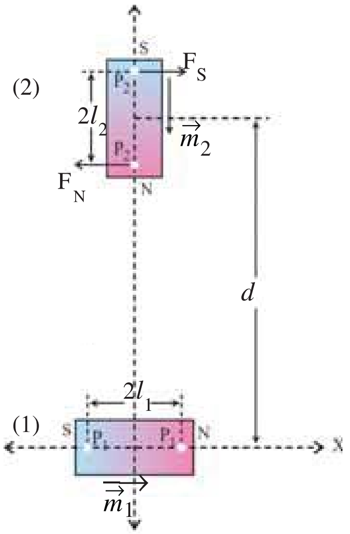
$$\therefore\ F = \frac{3\mu_0 m_1 m_2}{2\pi x^4} \quad (5)$$

Where $m_2 = 2l_2 p_2$ = magnetic moment of magnet (2)

This resultant force is repulsive for the magnet positions shown in Figure, and acts on magnet (2) in a direction away from magnet (1).

[What will be the resultant force between the two bar magnets, if the direction of one of the magnets is reversed ? Think !]

Illustration 2 : Find the torque on small bar magnet (2) due to small bar magnet (1), when they are placed perpendicular to each other as shown in Figure. ($l_1 << d$, $l_2 << d$)



Solution : From the geometry of Figure, it is seen that both the poles of magnet (2) are lying on the equatorial line of magnet (1).

The magnetic field B_N produced by the small bar magnet (1) at distance $(d - l_2)$ on its equatorial plane is

$$B_N = \frac{\mu_0}{4\pi}\frac{m_1}{(d-l_2)^3} \quad (1)$$

Similarly the magnetic field B_S produced by the magnet (1) at south pole of magnet (2), lying at a distance $(d + l_2)$ on its equatorial plane is

$$B_S = \frac{\mu_0}{4\pi}\frac{m_1}{(d+l_2)^3} \quad (2)$$

Thus as shown in figure the forces F_N and F_S acting on the north and south poles of magnet (2) having pole strength p_2 are

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi}\frac{m_1 p_2}{(d-l_2)^3} \quad (3)$$

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi}\frac{m_1 p_2}{(d+l_2)^3} \quad (4)$$

As $l_1 << d$ and $l_2 << d$, l_1 and l_2 can be neglected in comparison with d in equations (3) and (4).

$$\therefore\ F_S = F_N = \frac{\mu_0}{4\pi}\frac{m_1 p_2}{d^3} \quad (5)$$

As the non-colinear forces F_S and F_N are acting on magnet (2) in opposite direction, they form a couple. Hence the torque due to these forces is

$$\vec{\tau} = 2\vec{l_2} \times \vec{F}_N = 2\vec{l_2} \times \vec{F}_S \quad (\because \vec{\tau} = \vec{r} \times \vec{F})$$

Since $\vec{F}_N \perp \vec{l_2}$ and $\vec{F}_S \perp \vec{l_2}$, the magnitude of the torque with respect to centre of magnet (2)

$$\tau = 2F_N l_2 = \frac{\mu_0}{4\pi}\frac{m_1 2l_2 p_2}{d^3} = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{d^3} \quad (6)$$

where $2l_2 p_2 = m_2$ = magnetic moment of magnet (2).

5.7 Torque Acting on a Magnetic Dipole (Bar Magnet) in a Uniform Magnetic Field

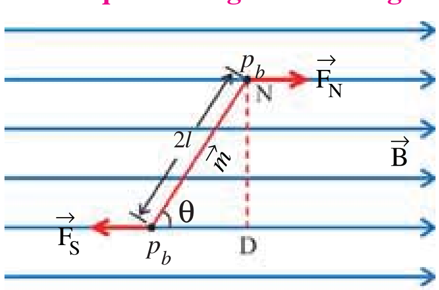


Figure 5.10 Torque Acting on a Magnetic Dipole of Magnetic Moment $\vec{m}$ in Uniform Magnetic Field $\vec{B}$

In Chapter-4 we have studied that the torque acting on a rectangular coil of magnetic moment $\vec{m}$, placed in a uniform magnetic field $\vec{B}$ is

$$\left.\begin{aligned} \vec{\tau} &= \vec{m} \times \vec{B} \\ \therefore\ \tau &= mB\sin\theta \end{aligned}\right\} \quad (5.7.1)$$

Where θ is the angle between $\vec{m}$ and $\vec{B}$ (sometimes magnetic moment is also represented by symbol $\vec{\mu}$).

This fact can be observed by placing a bar magnet or magnetic needle of magnetic dipole moment $\vec{m}$ in a uniform magnetic field $\vec{B}$ (See figure 5.10). In terms of pole strength, the magnetic field $\vec{B}$ can be considered equivalent to the force acting on unit pole strength. The magnetic field exerts equal and opposite forces $\vec{F}_N$ and $\vec{F}_S$ on the north and south poles. But since these forces do not lie on a straight line, they form a couple. Perpendicular distance between these two forces is ND. Under the influence of this couple, the magnetic dipole rotates to a new position making angle θ with the direction of magnetic field $\vec{B}$.

If the angle θ (in radian) in equation (5.7.1) is small, then $\sin\theta \approx \theta$.

$$\therefore \ \tau = m\mathrm{B}\theta \tag{5.7.2}$$

This torque, in the figure, is trying to rotate the dipole in a clockwise direction. Now if we try to rotate the dipole in anticlockwise direction further by a small angle θ with respect to this equilibrium position, then the torque represented by equation (5.7.1) will act in opposite direction. Thus we may write this restoring torque with negative sign as

$$\tau = -m\mathrm{B}\theta \tag{5.7.3}$$

According to Newton's second law of motion (for rotational motion)

$$\mathrm{I_m}\frac{d^2\theta}{dt^2} = -m\mathrm{B}\theta \tag{5.7.4}$$

Where $\mathrm{I_m}$ is the moment of inertia of the magnetic dipole with respect to an axis perpendicular to the plane of figure and passing through the centre of the dipole.

$$\therefore \ \frac{d^2\theta}{dt^2} = -\frac{m\mathrm{B}}{\mathrm{I_m}}\theta = -\omega^2\theta \tag{5.7.5}$$

Equation (5.7.5) is similar to the differential equation for angular simple harmonic motion. Hence the angular frequency

$$\omega = \sqrt{\frac{m\mathrm{B}}{\mathrm{I_m}}} \tag{5.7.6}$$

$$\therefore \ \mathrm{T} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\mathrm{I_m}}{\mathrm{mB}}} \tag{5.7.7}$$

$$\text{which gives } \mathrm{B} = \frac{4\pi^2\mathrm{I_m}}{\mathrm{mT}^2} \tag{5.7.8}$$

The potential energy of the magnetic dipole in the external field $\vec{B}$ is given by

$$\mathrm{U_B} = \int \tau d\theta = \int m\mathrm{B}\sin\theta d\theta = m\mathrm{B}\int \sin\theta d\theta$$

$$\therefore \ \mathrm{U_B} = -m\mathrm{B}\cos\theta = -\vec{m}\cdot\vec{\mathrm{B}} \tag{5.7.9}$$

In equation (5.7.9) we have taken the constant of integration to be zero by considering the potential energy to be zero at $\theta = 90°$, i.e. when the magnetic dipole is perpendicular to the field.

At $\theta = 0°$, $\mathrm{U_B} = -m\mathrm{B}\cos 0° = -m\mathrm{B}$,

which is the minimum value of potential energy representing most stable position of the magnetic dipole.

At $\theta = 180°$, $\mathrm{U_B} = -m\mathrm{B}\cos 180° = m\mathrm{B}$,

which is the maximum value of potential energy representing most unstable position of the magnetic dipole.

Illustration 3 : A magnetic needle placed in uniform magnetic field has magnetic moment 6.7×10^{-2} A m^2, and moment of inertia of 15×10^{-6} kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field ?

Solution : The periodic time of oscillation is, $T = \frac{6.70}{10} = 0.67$ s, and

$$B = \frac{4\pi^2 I_m}{mT^2} = \frac{4 \times (3.14)^2 \times 15 \times 10^{-6}}{6.7 \times 10^{-2} \times (0.67)^2} = 0.02 \text{ T}$$

Illustration 4 : A short bar magnet is placed in an external magnetic field of 600 G. When its axis makes an angle of 30° with the external field, it experiences a torque of 0.012 N *m*.

(a) What is the magnetic moment of the magnet ?

(b) What is the work done in moving it from its most stable to most unstable position ?

(c) The bar magnet is replaced by a solenoid of cross-sectional area 2×10^{-4} m^2 and 1000 turns, but having the same magnetic moment. Determine the current flowing through the solenoid.

Solution : $B = 600$ G $= 600 \times 10^{-4}$T, $\theta = 30°$, $\tau = 0.012$ N m, N = 1000,

$A = 2 \times 10^{-4}$ m^2

(a) From equation (5.7.1)

$\tau = mB\sin\theta$

$\therefore 0.012 = m \times 600 \times 10^{-4} \times \sin 30°$

$\therefore m = 0.40$ A m^2 (since $\sin 30° = \frac{1}{2}$)

(b) From equation (5.7.9), the most stable position is at $\theta = 0°$ and the most unstable position is at $\theta = 180°$. Hence the work done,

$W = U_B(\theta = 180°) - U_B(\theta = 0°) = mB - (-mB) = 2mB$

$= 2 \times 0.40 \times 600 \times 10^{-4} = 0.048$ J

(c) From equation (5.6.3)

$m_S = NIA$

But $m_S = m = 0.40$ A m^2, from part (a).

$\therefore 0.40 = 1000 \times I \times 2 \times 10^{-4}$

$\therefore I = 2$ A

5.8. Gauss's Law for Magnetic Field

From Figure (5.3-*a*) and (5.3-*b*) we can see that, for any closed surface like (i) or (ii), the number of magnetic field lines entering the closed surface is equal to the number of field lines leaving the surface. Since the magnetic field lines always form closed loops, the magnetic flux, associated with any closed surface is always zero.

$$\therefore \oint_{\substack{\text{closed} \\ \text{surface}}} \vec{B} \cdot d\vec{a} = 0 \qquad (5.8.1)$$

where $\vec{B}$ is the magnetic field and $d\vec{a}$ is an infinitesimal area vector of the closed surface. **"The net magnetic flux passing through any closed surface is zero."** This statement is called Gauss's law for magnetic field.

According to the Gauss's law for electric field

$$\oint \vec{E} \cdot \vec{da} = 0 = \frac{\Sigma q}{\varepsilon_0} \qquad (5.8.2)$$

In equation (5.8.2) if $\sum q = 0$, then

$$\oint \vec{E} \cdot \vec{da} = 0 \qquad (5.8.3)$$

Comparing this equation with equation (5.8.1), we can write that the Gauss law for magnetic fields indicate that there does not exist any net magnetic monopole (magnetic charge ?) that is enclosed by the closed surface. The unit of magnetic flux is Weber (Wb).

1 Wb = 1 T m^2 = 1 NmA^{-1}

5.9. The Magnetism of Earth and Magnetic Elements

We all are aware of the fact that the Earth has its own magnetic field. The magnetic field on the surface of Earth is of the order of 10^{-5} T (T = tesla).

The magnetic field on the Earth resembles that of a (hypothetical) magnetic dipole as shown in figure 5.11.

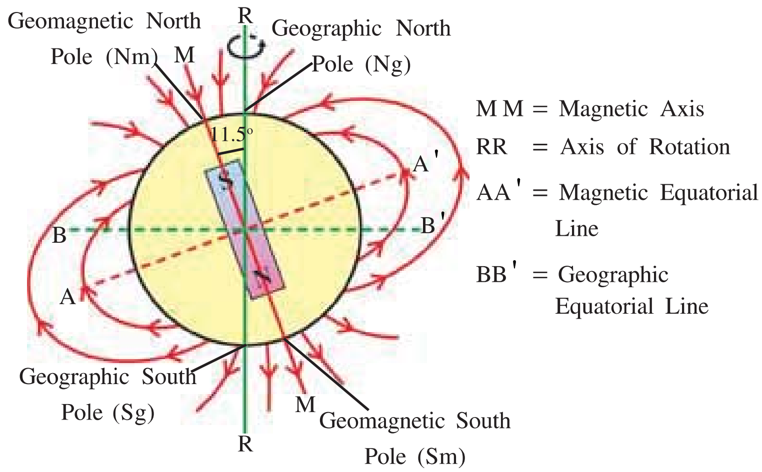


Figure 5.11 Magnetic Field of Earth

The magnitude of magnetic moment $\vec{m}$ of this (hypothetical) dipole is of the order of 8.0×10^{22} J T^{-1}. The axis MM of the dipole moment $\vec{m}$ does not coincide with the axis of rotation RR of the Earth, but is tilted by about 11.5°. The dipole axis MM intersects the Earth's geomagnetic north pole somewhere in north Canada, and the geomagnetic south pole in Antarctica. The magnetic field lines emerge out in the southern hemisphere and enter in the northern hemisphere. The actual south pole of earth's magnetic dipole is lying in the direction in which the north pole of magnetic needle, capable of rotating freely in the horizontal plane, remains stationary. Generally, we call this direction on earth as "Earth's magnetic north." The geomagnetic poles of Earth are located approximately 2000 km away from the geographic poles.

The geographic and geomagnetic equators intersect each other at longitude 6° west and 174° east. In India, Thumba near Trivandrum is on the magnetic equator, and hence it has been selected as the rocket launching station.

Each place on earth has a particular latitude and longitude which can be obtained from a good book of horoscope or map. The longitude circle passing through any place determines its geographic North-South direction. An imaginary vertical plane at a place on the Earth containing the longitude circle and the geographic axis of the Earth is called the **geographic meridian** (See figure 5.12).

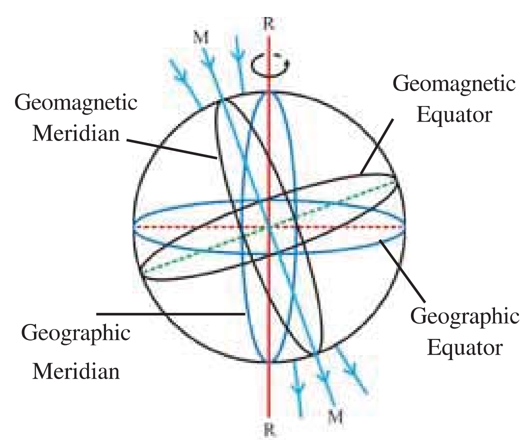


Figure 5.12 Geographic and Geomagnetic, Equator and Meridian of the Earth

Further, the magnetic field lines of geomagnetic dipole are also passing through every place on Earth. **Hence an imaginary vertical plane at a place on the Earth, passing through the magnetic axis and containing magnetic field lines is called magnetic meridian at that place.**

Illustration 5 : The Earth's magnetic field at some place on magnetic equator of Earth is 0.4 G. Estimate the magnetic dipole moment of the Earth. Consider the radius of Earth at that place to be 6.4×10^6 *m*. ($\frac{\mu_0}{4\pi} = 10^{-7}$T m A^{-1}, and 1 G = 10^{-4} T)

Solution : The magnitude of equatorial magnetic field, according to equation (5.6.6) is

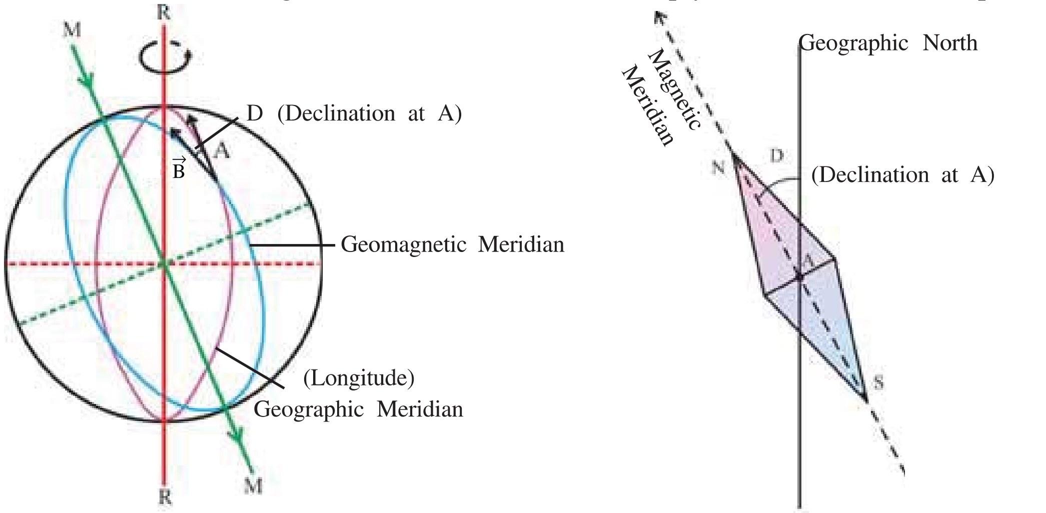
$$B_E = \frac{\mu_0 m}{4\pi y^3}$$

But $B_E = 0.4$ G $= 4 \times 10^{-5}$ T

$$\therefore \; m = \frac{4\pi y^3 B_E}{\mu_0} = \frac{B_E y^3}{\left(\mu_0/4\pi\right)} = \frac{4\times10^{-5}\times(6.4\times10^6)^3}{10^{-7}} = 1.05 \times 10^{23} \text{ A m}^2$$

5.9.1. Geomagnetic Elements : In order to describe the magnetic field of Earth scientifically, certain magnetic parameters are defined, called geo-magnetic elements.

Magnetic Declination : The angle between the magnetic meridian and the geographic meridian at a place on surface of Earth is called magnetic declination at that place. Thus, the angle between the true geographic north and the magnetic north at any place on the surface of Earth is the magnetic declination (D) or simply declination at that place.



(a) Declination at Point A on the Surface of Earth

(b) Magnetic Needle Placed in Horizontal Plane at Point A

Figure 5.13

As shown in figure (5.13-*a*) consider point A on the surface of Earth. At this point, the direction of true geographic north is determined from tangent to A the longitude circle of geographic meridian. A magnetic needle free to rotate in horizontal plane aligns along the magnetic meridian at point A. The north pole of the needle points towards the geomagnetic north pole (tangent to the magnetic meridian at A). The angle between the geographic meridian and magnetic meridian at point A indicates the declination at the point A.

The declination is larger at higher latitudes and smaller near the equator. The declination is small in India, it being 0°58′ west at Bombay, and 0°41′ east at Delhi. Thus, at both these places the magnetic needle shows true north quite accurately.

Magnetic dip angle or inclination : Magnetic dip angle or inclination is the angle ϕ (up or down) that the magnetic field of Earth makes with the horizontal at a place in magnetic meridian.

Magnetic field lines are not locally horizontal at all places on Earth. At a place near north Canada, magnetic field lines point vertically downwards, whereas at a place on the magnetic equator, these field lines are horizontal. At the magnetic equator, dip angle is zero. As we move towards magnetic pole, the dip angle increases and becomes 90° at magnetic poles.

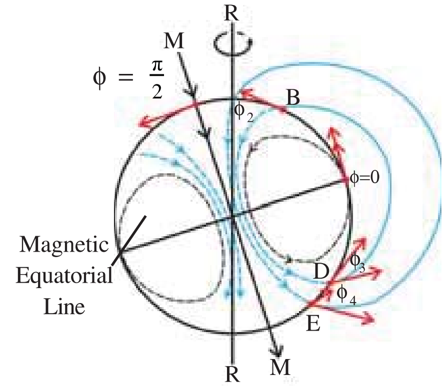


Figure 5.14 Magnetic Dip Angle at Different Places on the Earth's Surface in Magnetic Meridian [Only for Information]

Horizontal Component and Veritical Component of Earth's Magnetic field

Figure 5.15 shows the Earth's magnetic field ($\vec{B}$), angle of declination (D) and the angle of dip (ϕ) at a place (P).

The magnetic field $\vec{B}$ at point P is resolved into horizontal component $\vec{B}_H$ pointing towards geomagnetic north pole, and vertical component $\vec{B}_V$ pointing towards the centre of Earth. The angle made by $\vec{B}_H$ with geographic meridian is the angle of declination (D), whereas the angle between $\vec{B}_H$ and $\vec{B}$ is the angle of dip or inclination (ϕ).

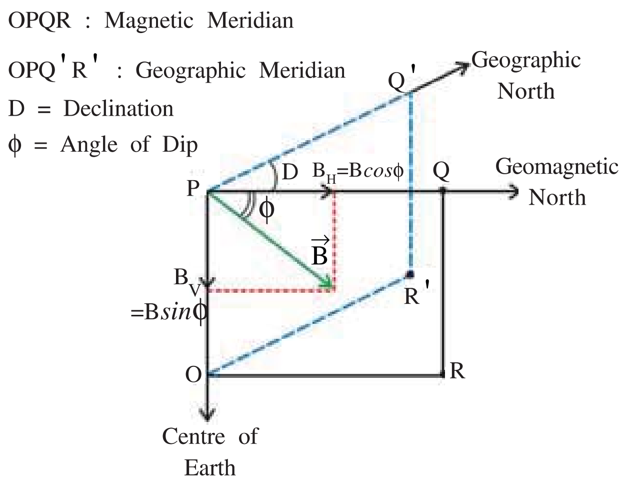


Figure 5.15 Components of Earth's Magnetic Field $\vec{B}$

The declination D, the angle of dip ϕ, and the horizontal component of Earth's field $\vec{B}_H$ are known as geomagnetic elements or the elements of Earth's magnetic field.

For the magnetic meridian OPQR of figure (5.15), we have

$$B_V = B\,sin\phi \qquad (5.9.1)$$

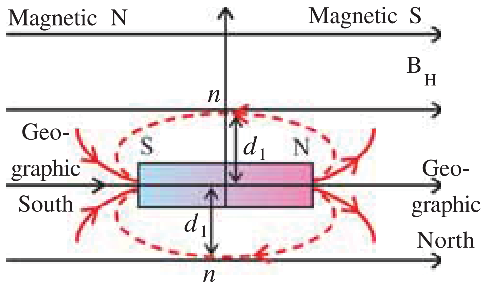
$$B_H = B\,cos\phi \qquad (5.9.2)$$

$$\therefore\ tan\ \phi = \frac{B_V}{B_H} \qquad (5.9.3)$$

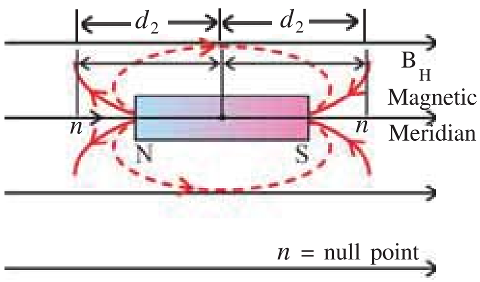
$$\text{and } B = \sqrt{B_V^{\ 2} + B_H^{\ 2}} \qquad (5.9.4)$$

Illustration 6 : A short bar magnet with magnetic dipole moment 1.6 A m^2 is kept in magnetic meridian in such a way that its north pole is in north direction. In this case, the null (neutral) point is found at a distance of 20 cm from the centre of the magnet. Find the horizontal component of the Earth's magnetic field.

Next, the magnet is kept in such a way that its magnetic north pole is in south direction. Find the positions of neutral (null) points in this case.



(a)



(b)

Solution : From the figure (a) one can observe that on the magnetic equator of the magnet, horizontal field lines of the earth's magnetic field and the magnetic field lines due to the magnet are in mutually opposite directions. Hence in this case, one finds two points on magnetic equator of the magnet at equal distance from the magnet (one above and one below) in such a way that at these points the above mentioned two magnetic fields are equal in magnitude and opposite in directions. At such points the resultant magnetic field is zero. Such points are called **neutral** or **null points.**

Here, $m = 1.6 \text{ A } m^2$

Let, the distance of neutral points from the centre of the magnet is

$d_1 = 20 \text{ cm} = 0.2 \ m$

Now the magnetic field due to a short bar magnet on its equatorial plane B_1 must equal B_H.

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m}{d_1^3} = B_H$$

$$\therefore \ B_H = \frac{10^{-7} \times 1.6}{(0.2)^3} = 2 \times 10^{-5} \text{ T}$$

However if the bar magnet is kept as in part (b) of the Figure, then it is clear that on the magnetic axis, B_H and the magnetic field due to the magnet are in mutually oposite directions. In this case the neutral points are on the axis. Let d_2 be the distance of such points from the centre of magnet, then B_2, the magnetic field on axis, must be equal to B_H,

$$\therefore \ B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{d_2^3} = B_H$$

$$\therefore \ d_2^3 = \frac{10^{-7} \cdot 2m}{B_H} = \frac{10^{-7} \times 2 \times 1.6}{2 \times 10^{-5}} = 16 \times 10^{-3}$$

$$\therefore \ d_2 = 2.52 \times 10^{-1} \ m = 2.52 \text{ cm}$$

Illustration 7 : A magnet is hung horizontally in the magnetic meridian by a wire without any twist. If the supporting wire is given a twist of 180° at the top, the magnet rotates by 30°. Now if another magnet is used, then a twist of 270° at the supporting end of wire also produces a rotation of the magnet by 30°. Compare the magnetic dipole moments of the two magnets.

Solution : If resultant twist in the wire = δ,

$$\delta_1 = 180° - 30° = 150° = 150 \times \frac{\pi}{180} \text{ rad}$$

and $\delta_2 = 270° - 30° = 240° = 240 \times \frac{\pi}{180}$ rad

If the twist–constant for the wire is k then

Rotating torque, $\tau_1 = k\delta_1$ and $\tau_2 = k\delta_2$

Here α is the angle made by the magnetic dipole moment with the magnetic meridian.

$\tau_1' = m_1 B_H \sin\alpha$

Since the second magnet is also rotated by the same angle.

$\tau_2' = m_2 B_H \sin\alpha$

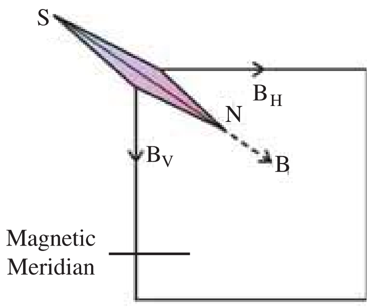
At equilibrium $\tau_1 = \tau_1'$ and $\tau_2 = \tau_2'$

$\therefore \frac{\tau_1'}{\tau_2'} = \frac{\tau_1}{\tau_2}$

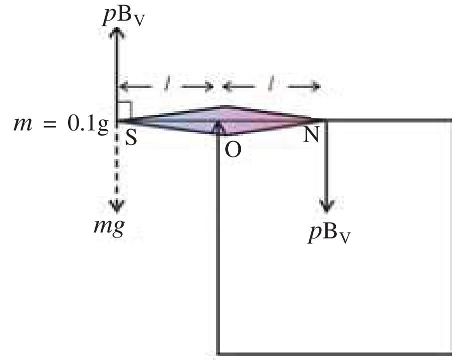
$\therefore \frac{m_1}{m_2} = \frac{\delta_1}{\delta_2} = \frac{150}{240} = \frac{5}{8}$

Illustration 8 : A magnetic needle is hung by an untwisted wire, so that it can rotate freely in the magnetic meridian. In order to keep it in the horizontal position, a weight of 0.1g is kept on one end of the needle. If the magnetic pole strength of this needle is 10 A *m*, find the value of the vertical component of the earth's magnetic field. (g = 9.8 m s^{-2})

Solution :



(a) Normal Position



(b) Position after inserting weight

Figure (a) shows the position of the magnetic needle in the magnetic meridian without any weight. In figure (b), a mass m is kept on the S–pole of the needle.

The vector sum of torques due to all forces must be zero for the equilibrium of the needle in horizontal direction.

$\therefore -pB_v(l) - pB_v(l) + m.g(l) = 0$

[The torque producing rotations in clockwise direction is taken as negative.]

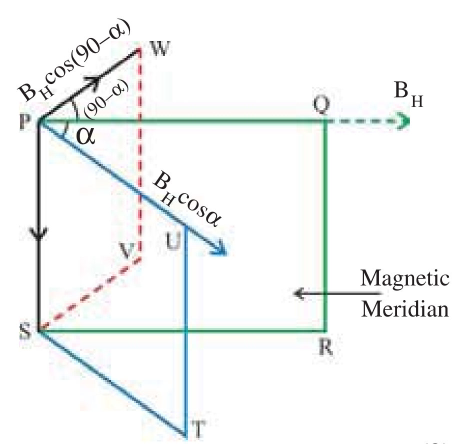
$\therefore 2pB_v = mg$

$\therefore B_v = \frac{mg}{2P} = \frac{10^{-4} \times 9.8}{2 \times 10}$ | $m = 0.1$ g $= 10^{-4}$ kg,

$\therefore B_v = 4.9 \times 10^{-5}$ T | $p = 10$ A m

Illustration 9 : As shown in figure, plane PSTU forms an angle of α and plane PSVW makes an angle of $(90° - \alpha)$ with the magnetic meridian, respectively. The value of magnetic dip angle in plane PSTU is ϕ_1 and its value in plane PSVW is ϕ_2. If the actual dip angle at the place is ϕ, show that,

$cot^2\phi = cot^2\phi_1 + cot^2\phi_2$



Solution : $tan\phi = \frac{B_V}{B_H}$ (1)

In plane PSTU horizontal component is $B_H cos\alpha$

$\therefore tan\phi_1 = \frac{B_V}{B_H cos\alpha} \Rightarrow cos\alpha = \frac{tan\phi}{tan\phi_1} = tan\phi \cdot cot\phi_1$

(from equation (1)) (2)

Similarly for plane PSVW

$sin\alpha = tan\phi \cdot cot\phi_2$ (3)

Squaring and summing the equations (2) and (3)

$cos^2\alpha + sin^2\alpha = 1 = tan^2\phi \, (cot^2\phi_1 + cot^2\phi_2)$

$\therefore cot^2\phi = cot^2\phi_1 + cot^2\phi_2$

5.10 Magnetization and Magnetic Intensity

Consider a solenoid of N turns having length l. When a current I_f is passed through it, the magnetic field produced inside the solenoid (with air or vacuum) is

$$B_0 = \mu_0 n I_f \tag{5.10.1}$$

Where $n = \frac{N}{l}$ = number of turns per unit length of solenoid

This current I_f is called **free current**. If we denote the free current per unit length by i_f then

$$i_f = nI_f \tag{5.10.2}$$

$$\therefore\ B_0 = \mu_0 i_f \tag{5.10.3}$$

Now a material whose magnetic properties are to be studied is placed inside the solenoid. Let l be the length of the material, and A be its cross-sectional area. The magnetic field B_0, present inside the solenoid due to magnetizing current i_f, magnitizes the material such that it acquires some magnetic moment, say $\vec{m}$. This magnetic moment $\vec{m}$ of the material can be considered to be produced due to an equivalent surface current loop carrying current I_b. This current is called **bound current.** The dipole moment of this current loop is

$$\vec{m} = I_b \vec{A} \tag{5.10.4}$$

where A = area of cross-section of the material = area of current loop.

The net magnetic moment per unit volume of the material is called magnetization M of the material. Thus

$$M = \frac{m}{V} = \frac{I_b A}{lA} = \frac{I_b}{l} = i_b \tag{5.10.5}$$

Here $i_b = \frac{I_b}{l}$ = bound current per unit length of the core material.

The unit of M is A $m^2\ m^{-3}$ = A m^{-1}. Here M is a vector quantity. Its direction is along $\vec{m}$.

Thus the total magnetic field inside the magnetic core material placed inside the solenoid is due to both currents i_f and i_b.

$$\therefore\ B = \mu_0 (i_f + i_b) \tag{5.10.6}$$

Using equation (5.10.5) in (5.10.6)

$$B = \mu_0 (i_f + M) \tag{5.10.7}$$

$$\therefore\ \frac{B}{\mu_0} - M = i_f \tag{5.10.8}$$

Here, $\frac{B}{\mu_0} - M$ is defined as magnetic intensity H, and its value is equal to magnetizing current, i_f. Hence

$$\frac{B}{\mu_0} - M = H = i_f \tag{5.10.9}$$

$$B = \mu_0 (H + M) \tag{5.10.10}$$

Thus, the magnetic field B induced in a substance, depends on H and M. Further, it is observed that, if H is not too much strong, then the magnetization M induced in the substance is proportional to magnetic intensity H.

$\therefore \ M = \chi_m H$ (5.10.11)

Here χ_m is a constant, called magnetic susceptibility of the material of the substance. It is a dimensionless quantity. Its value depends on the type of material and its temperature. It is a measure of how a magnetic material responds to external magnetic field. The magnetic susceptibility of some of the substances is listed in table (5.2) for information only.

Table 5.2 Magnetic Susceptibility of some Elements at 300 K (for information only)

Dimagnetic Substance	χ_m	Paramagnetic Substance	χ_m
Bismuth	-1.66×10^{-5}	Aluminium	2.3×10^{-5}
Copper	-9.8×10^{-6}	Calcium	1.9×10^{-5}
Dimond	-2.2×10^{-5}	Choromium	2.7×10^{-4}
Gold	-3.6×10^{-5}	Lithium	2.1×10^{-5}
Lead	-1.7×10^{-5}	Magnesium	1.2×10^{-5}
Mercury	-2.9×10^{-5}	Niobim	2.6×10^{-5}
Nitrogen (STP)	-5.0×10^{-9}	Oxygen (STP)	2.1×10^{-6}
Silver	-2.6×10^{-5}	Platinum	2.9×10^{-4}
Silicon	-4.2×10^{-6}	Tungsten	6.8×10^{-5}

The interpretation of equation (5.10.6) shows that, without putting magnetic material in solenoid, if the same magnetic field $[B = \mu_0 (i_f + i_b)]$ is required to be produced, then over and above the current I_f, an additional current I_m must be passed through the solenoid, such that the additional magnetizing current per unit length $nI_m = i_b$ is produced.

The substances for which χ_m is positive are called paramagnetic, for which $\vec{M}$ and $\vec{H}$ are in the same direction. The substances for which χ_m is negative are called diamagnetic, for which $\vec{M}$ and $\vec{H}$ are in opposite direction.

Substituting (5.10.11) in (5.10.10),

$B = \mu_0 [H + \chi_m H] = \mu_0 (1 + \chi_m) H = \mu H$ (5.10.12)

Where $\mu = \mu_0 (1 + \chi_m)$ is called permeability (magnetic permeability) of the material. $\frac{\mu}{\mu_0}$ is called relative permeability of the material, denoted by μ_r.

$\therefore \ \mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$ (5.10.13) which gives,

$$B = \mu_0 \mu_r H \tag{5.10.14}$$

Note : The vacuum cannot be magnetized. Hence for vacuum M = 0. Thus from equation (5.10.10), for vacuum $B = \mu_0 H$.

Illustration 10 : A solenoid has a core of material with relative permeability of 400. The current passing through the wire of solenoid is 2A. If the number of turns per cm are 10, calculate the magnitude of

(a) H, (b) B, (c) χ_m, (d) M, and (e) the additional magnetizing current I_m. (Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}).

Solution : Here $\mu_r = 400$, I = 2 A, $n = 10 \, \frac{\text{turns}}{\text{cm}} = 1000 \, \frac{\text{turns}}{\text{m}}$

(a) Magnetic intensity $H = i_f = nI = 1000 \times 2 = 2000 \text{ A m}^{-1}$

(b) Magnetic field $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 400 \times 2000 = 1.0$ T

(c) Magnetic susceptibility of the core material is

$$\chi_m = \mu_r - 1 = 400 - 1 = 399$$

(d) Magnetization

$$M = \chi_m H = 399 \times 2000 = 7.98 \times 10^5 \approx 8 \times 10^5 \text{ A m}^{-1}$$

(e) The additional magnetizing current I_m is obtained from $M = nI_m = i_b$ as

$$I_m = \frac{M}{n} = \frac{8 \times 10^5}{1000} = 800 \text{ A}$$

Illustration 11 : The region inside a current carrying torodial winding is filled with tungsten of susceptibility 6.8×10^{-5}. What is the percentage increase in the magnetic field in the presence of the material with respect to the magnetic field without it ?

Solution : The magnetic field in the current carrying torodial winding without tungsten is

$$B_0 = \mu_0 H$$

The magnetic field in the same current carrying torodial winding with tungsten is

$$B = \mu H$$

$$\therefore \quad \frac{B - B_0}{B_0} = \frac{\mu - \mu_0}{\mu_0}$$

But $\mu = \mu_0 (1 + \chi_m) \Rightarrow \frac{\mu}{\mu_0} = 1 + \chi_m \Rightarrow \frac{\mu}{\mu_0} - 1 = \chi_m \Rightarrow \frac{\mu - \mu_0}{\mu_0} = \chi_m$

Hence, $\frac{B - B_0}{B_0} = \chi_m$

$\therefore$ Percentage increase in the magnetic field in presence of tungsten is

$$\frac{B - B_0}{B_0} \times 100 = (6.8 \times 10^{-5}) \times 100 = 6.8 \times 10^{-3} \ \%$$

5.11 Magnetic Properties of Materials : Dia, Para and Ferro Magnetism

We know that each electron in an atom possess an orbital magnetic dipole moment and a spin magnetic dipole moment, that add vectorially. This type of resultant magnetic moment of each electron in an atom add vectorially, and the resultant dipole moment of each atom in the sample of a material add vectorially. If the resultant of all these dipole moments produces a magnetic field, then the material is said to be magnetic material.

The behaviour of a material in presence of an external magnetic field classifies the material as diamagnetic, paramagnetic or ferromagnetic. The classification of dia, para and ferro magnetic materials in terms of their susceptibility, relative permeability, and a small positive number ε (this 'ε' should be not be taken as permitivity of the medium) used to quantity paramagnetic material are briefly represented in Table 5.3.

Table 5.3

Diamagnetic	Parmagnetic	Ferromagnetic
$-1 \leq \chi_m < 0$	$0 < \chi_m < \varepsilon$	$\chi_m >> 1$
$0 \leq \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

5.11.1 Diamagnetic Materials : The atoms/molecules of gold, silver, copper, silicon, water and bismuth etc. do not possess permanent magnetic dipole moments. The orbital motion of the electrons and their spins are such that their total magnetic dipole moment is zero. Such materials are called diamagnetic materials.

When the diamagnetic material is placed in an external magnetic field, a net magnetic moment in a direction opposite to that of the external magnetic field is induced in each atom. Due to this, each atom of diamagnetic material experiences repulsion.

Figure 5.16 shows a bar of diamagnetic material placed in an external magnetic field $\vec{B}$. The field lines are repelled by induced magnetic field (weak) in the material, and the resultant field inside the material is reduced.

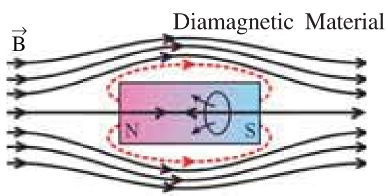
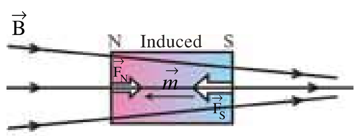


Figure 5.16 Diamagnetic Material in External Magnetic Field

As shown in figure 5.17, when the bar of diamagnetic material is placed in a non-uniform magneic field, the induced magnetic south pole is in the strong magnetic field, and the induced north pole is in the weak magnetic field. Hence, the magnetic force on the induced S-pole ($\vec{F}_S$ acting towards left) is more than the force on induced N-pole ($\vec{F}_N$ towards right). As a result the bar of diamagnetic material experiences a resultant force towards the region of weaker magnetic field. The magnetic susceptibility χ_m of diamagnetic materials is negative.



$\vec{F} = \vec{F}_S - \vec{F}_N$ (Towards Left)

Figure 5.17 Force Acting on Diamagnetic Material Placed in Non-uniform Magnetic Field

For superconductors $\chi_m = -1$ and $\mu_r = 0$. When superconductors are placed in an external magnetic field, the field lines are completely expelled. The phenomenon of perfect diamagnetism in superconductors is called the **Meissner effect,** after the name of its discoverer. Superconducting magnets can be used for running magnetically leviated superfast trains.

5.11.2 Paramagnetism : In paramagnetic material, the atoms/molecules possess permanent magnetic dipole moments. Normally, the molecules are arranged such that, their magnetic dipole moments are randomly oriented. Hence the resultant magnetic moment of the material is zero (See figure 5.18).

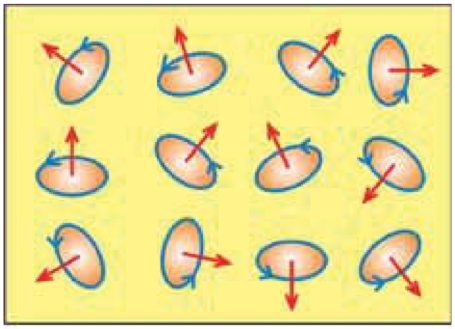


Figure 5.18 Normal Dipole Distribution in Paramagnetic Material

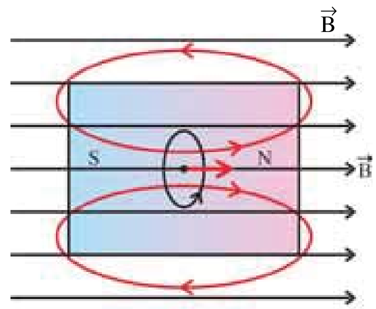


Figure 5.19 Magnetic Dipole Moment of One Dipole Shown Aligned with $\vec{B}$

When the paramagnetic material is placed in external magnetic field $\vec{B}$, these tiny dipoles try to align in the direction of $\vec{B}$. However, due to thermal agitation, all dipoles could not attain 100% alignment in the direction of $\vec{B}$.

Figure 5.19 shows the magnetic field due to the magnetic dipole aligned with $\vec{B}$. The field lines get concentrated inside the material (see figure 5.20)

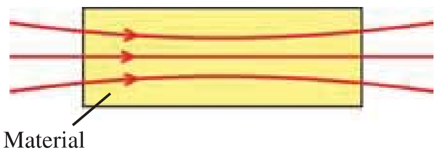


Figure 5.20 Magnetic Field Lines in Paramagnetic Material

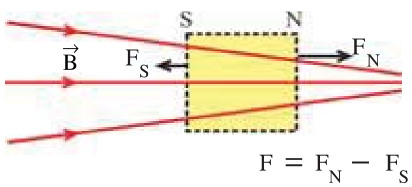


Figure 5.21 Paramagnetic Material in Non-uniform Magnetic Field

When a bar of paramagnetic material is placed in non-uniform magnetic field (See figure 5.21) the resultant north pole of the magnetized material feels strong magnetic field, whereas the south pole experiences comparatively weak magnetic field. As a result of which the resultant force ($F_N - F_S$) acts towards the stronger magnetic field (towards right) on the bar of paramagnetic material. In practice, this force is very weak.

Aluminium, sodium, calcium, oxygen at STP and copper chloride are few examples of paramagnetic materials. The magnetic susceptibility χ_m of paramagnetic materials is positive.

In 1895 Pierre Curie observed that the magnetization M of a paramagnetic material is directly proportional to the external magnetic field $\vec{B}$ and inversely proportional to its absolute temperature T, called Curie's law,

$$M = C\frac{B}{T} \tag{5.11.1}$$

Where C = Curie's constant

From equation (5.11.1)

$$M = C\frac{B}{\mu_0}\frac{\mu_0}{T} = CH\frac{\mu_0}{T}$$

$$\therefore \frac{M}{H} = \chi_m = C\frac{\mu_0}{T} \tag{5.11.2}$$

$$\therefore\ \mu_r - 1 = C\frac{\mu_0}{T} \qquad (5.11.3)$$

As we increase the applied external magnetic field or decrease the temperature of the paramagnetic material, or both, then alignment of atomic magnetic moments increase. Thus magnetization M increases. When magnetic moments of all atoms are aligned parallel to the external magnetic field, M, μ_r and χ_m become maximum. This situation is called **satuaration magnetization**. Curie's law is not obeyed after this state. If there are N atoms in volume V of the sample, each with magnetic moment $\vec{m}$, then at saturation magnetization

$$\vec{M}_{max} = \frac{N\vec{m}}{V} \qquad (5.11.4)$$

5.11.3 Ferromagnetism : The atoms of iron, cobalt, nickel and their alloys possess permanent magnetic dipole moments due to spin of electrons in outermost orbits. The atoms of such materials are arranged in such a way that over a region called **domain**, the magnetic moment of the atoms are aligned in the same direction. In unmagnetized sample, such domains having a net magnetization are randomly oriented so that the effective magnetic moment is zero (See figure 5.22).

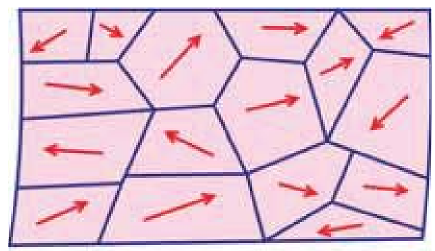


Figure 5.22 Random Arrangement of Domains

The explanation about the formation of such domains requires quantum mechanics which is beyond the scope of this book. The typical domain size is about 1 mm and the domain contains about 10^{11} atoms. The boundaries between the adjacent domains, having different orientations of magnetic moment, are called **domain walls**.

Hysteresis : The effect of an external magnetic field on ferromagnetic material is quite interesting. To understand this, consider an unmagnetized ferromagnetic material having initial magnetic field B = 0. Suppose this material is placed in a solenoid of n turns per unit length as shown in figure 5.8 (b). On passing a current through the solenoid, the magnetic field is generated, which induces magnetic moment inside the rod. Knowing the volume of the rod, we can evaluate M, the magnetic moment per unit volume. We already know that

$$\frac{B}{\mu_0} - M = i_f = H \text{ (See Equation (5.10.9))}$$

where, i_f = current passing through unit length of the solenoid.

From the values of H and M, we can evaluate B and study its variation with i_f (hence the variation of H). The graph of B versus. H can be drawn as shown in figure 5.23.

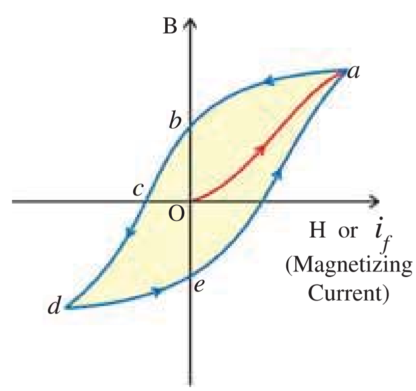


Figure 5.23 Hysteresis Loop

At the point 0 in the graph, the substance is in its normal condition, without any resultant magnetic field. As H (or i_f) is increased, B increases, but this increase is not linear. Near point *a*, B is maximized, which is the saturation magnetization condition of the rod.

One can explain the curve Oa as follows : Starting from O, as long as the value of H is small, most of the atoms, due to strong bonding with their neighbours, do not respond to the external magnetic field. But the atoms near the domain boundary are in precarious situation. Hence the domain boundaries, instead of remaining sharp, start shifting. In this situation one domain of the two adjacent domains, increases in size and the other one reduces in size. If we still keep on increasing the value of H, ultimately only one domain survives in the substance and the **saturation magnetization** is acquired near point a on the graph.

This process is not reversible. At this stage, if the current in the solenoid is reduced, we do not get back the earlier domain constituation, and when H = 0, we do not get B = 0. This means that when H is made zero, the substance retains certain magnetic moment, hence the curve ab represents the effect of reducing H.

The value of B, when H = 0, is called **retentivity** or **remanence**. Now, if the current is increased in reverse direction, then we reach at point c in the graph, the value of H for which B = 0 is called **coercivity**. At this point, the magnetic moments of the domains are again in random directions but according to some different domain structure.

If we keep on increasing the current in the reverse direction, B goes on increasing in the reverse direction and saturation magnetization is again acquired, but in opposite direction. After reaching d, if the current is reduced, the substance follows the curve de and again by reversing the current direction and increasing its value, we obtain the curve ea. This process is called **hysteresis cycle.** The area enclosed by the B-H curve represents the heat energy (in joules) lost in the sample per unit volume per cycle.

Hard ferromagnetic substances : The substances with large retentivity are called hard ferromagnetic substances. These are used in producing permanent magnets. Obviously, the hysteresis cycle for such substances is broad (See figure 5.24 (a)). Alnico (an alloy of Al, Ni, Co and Cu) is a hard ferromagnetic meterial. Hence permanent magnets are made using Alnico.

Soft Ferromagnetic Substances : The substances with small retentivity, which means the materials with narrow hysteresis cycle (See figure 5.24 (b)), are called soft ferromagnetic substances. For example soft iron; such materials are used for making electromagnets.

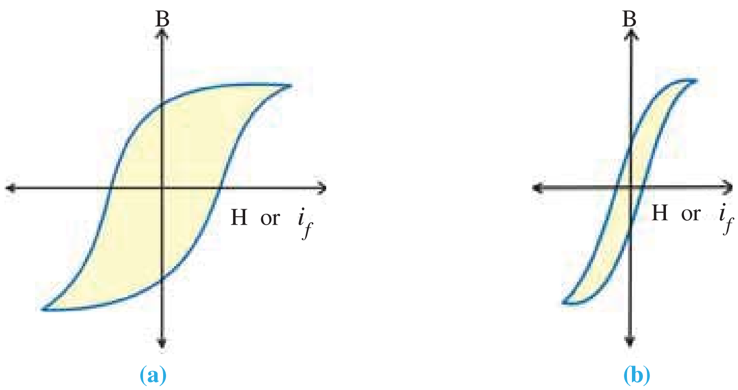


Figure 5.24 Hysteresis Loops for (a) Hard and (b) Soft Ferromagnetic Materials

Effect of Temperature : As the temperature of ferromagnetic substance is increased, the domain structure starts getting distorted. At a certain temperature depending upon the material, it is totally broken up. Each and every atomic magnetic moment attains independence from one another and the substance gets converted to a paramagnetic material.

The temperature at which a ferromagnetic substance is converted into a paramagnetic substance is called **Curie temperature T_C** of that substance. The relation between the magnetic susceptibility of the substance in the acquired paramagnetic form and the temperature T is given by

$$\chi_m = \frac{C_1}{T - T_C}, \ (T > T_C) \tag{5.11.1}$$

where, C_1 is a constant.

Finally, note that the ferromagnetic material is attracted towards the strong field region whenever it is kept in a non-uniform magnetic field.

The hysteresis loop shows that the magnetization of a ferromagnetic material depends on the history (the previous state) of the material as well as on the magnitude of applied field H. The shape and size of the hysteresis loop depends on the properties of ferromagnetic material as well as on the maximum value of applied magnetic field H.

5.12 Permanent Magnets and Electromagnets

The ferromagnetic materials which retain magnetism for a longer period of time at room temperature, are called permanent magnets. These materials have higher retentivity.

Before 400 years, the iron rods were fixed in north-south direction and hammered repeatedly to prepare magnets. Further, if one end of a magnet is continuously rubbed on a fixed steel rod only in one direction, then it acquires permanent magnetism.When a current is passed through a solenoid containing a steel rod, then the rod gets magnetized. Due to hysteresis, the rod retains magnetism even after the current is switched off. The materials like steel, hard alloys, and alnico have high retentivity and high coercivity, and hence are used to prepare permanent magnets.

Soft iron has large permeability and small retentivity, and hence is used to prepare electromagnets. For this purpose, a rod of soft iron is placed in a solenoid as a core, as shown in Figure 5.7(b). On passing a current through the solenoid, the magnetic field associated with the solenoid increases by a thousand fold. When the current through the solenoid is switched off, the associated magnetic field effectively becomes zero.

Electromagnets are used in electric bells and loudspeakers. Giant electromagnets are used in cranes to lift heavy loads made of iron or loads packed in iron containers (boggies).

In certain applications, an AC current is passed through the solenoid containing ferromagnetic material, for example in transformer cores and telephone diaphragms. The hysteresis loop of such materials must be narrow to reduce dissipation of energy in the form of heat.

Illustration 12 : A magnet has coercivity of 3×10^3 A m^{-1}. It is kept in a 10 cm long solenoid with a total of 50 turns. How much current has to be passed through the solenoid to demagnetize it ?

Solution : The value of H for which magnetization is zero is called coercivity.

For a solenoid $H = nI$

Here, $H = 3 \times 10^3$, $n = \frac{N}{l} = \frac{50}{0.1} = 500$

$$\therefore \ I = \frac{H}{n} = \frac{3 \times 10^3}{5 \times 10^2} = 6 \text{ A}$$

Illustration 13 : There are 2.0×10^{24} molecular dipoles in a paramagnetic salt. Each has dipole moment 1.5×10^{-23} A m^2 (or J T^{-1}). This salt kept in a uniform magnetic field 0.84 T is cooled to a temperature of 4.2 K. In this case the magnetization acquired is 15% of the saturation magnetization. What must be the dipole moment of this sample in magnetic field 0.98 T and at temperature of 2.8 K ? (Assume the applicability of the Curie's law).

Solution : Dipole moment of every molecular dipole = 1.5×10^{-23} A m^2

There are 2.0×10^{24} dipoles in the sample.

$\therefore$ Maximum (saturation) magnetization $= 1.5 \times 10^{-23} \times 2.0 \times 10^{24} = 30$ A m^2

But at 4.2 K, sample has 15 % of saturation magnetization

$\therefore m_1 = 30 \times 0.15 = 4.5$ A m^2

Now according to Curie's law, if m_1 is the dipole moment at T_1 and m_2 the dipole moment at T_2 then

$$\frac{m_1}{m_2} = \frac{B_1}{T_1} \times \frac{T_2}{B_2} \quad \left(\text{from } m \propto \frac{B}{T}\right)$$

Here B_1 and B_2 are applied magnetic fields

$$\therefore m_2 = m_1 \times \frac{T_1}{T_2} \times \frac{B_2}{B_1}$$

Here, $m_1 = 4.5$ A m^2, $T_1 = 4.2$ K, $T_2 = 2.8$ K, $B_1 = 0.84$ T and $B_2 = 0.98$ T

$$\therefore m_2 = \frac{4.5 \times 4.2 \times 0.98}{2.8 \times 0.84} = 7.87 \text{ A m}^2$$

SUMMARY

1. The north and south magnetic poles cannot be separated by splitting the magnet into two or more pieces. The independent magnetic monopoles does not exist.
2. The magnetic field lines do not intersect at a point.
3. The magnetic field lines of a magnet form continuous closed loops. The magnetic field lines emerge out from the magnetic north pole, reach the magnetic south pole and then passing through the magnet, reach the north pole to complete the loop.
4. The magnetic moment of a current loop of area A, carrying current I is given by $m = IA$. If there are N turns of a loop, then $m = NIA$

   If there are N turns of a loop, then $m = NIA$
5. The axial magnetic field of a current loop is given by $\vec{B}(x) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3}$
6. The orbital magnetic moment of an electron in an atom is given by $m_0 = \frac{1}{2}evr$
7. When a bar magnet is divided into two equal pieces, the pole strength p_b of each piece remains the same, but the magnetic dipole moment of each piece becomes half of the original value.
8. When a magnet of magnetic moment $\vec{m}$ is placed in external magnetic field $\vec{B}$, the torque acting on it is given by $\vec{\tau} = \vec{m} \times \vec{B}$ or $\tau = mB\sin\theta$ and has potential energy $U_B = -\vec{m} \cdot \vec{B}$
9. The Gauss's law for magnetic field is $\oint_{\text{closed surface}} \vec{B} \cdot \vec{da} = 0$ which states that "the net magnetic flux passing through any closed surface is zero.
10. **Magnetic Meridian :** An imaginary vertical plane at a place on the Earth, passing through the magnetic axis is called magnetic meridian at that place.

11. The angle between the magnetic meridian and the geographic meridian at a place on the surface of Earth is called the magnetic declination (D) at that place.

12. **Magnetic dip or inclination(ϕ) :** It is the angle (up or down) that the magnetic field of Earth makes with the horizontal at a place in magnetic meridian.
$\phi = 0^{o}$ at magnetic equator and $\phi = 90^{o}$ at geomagnetic poles.

13. The net magnetic moment per unit volume of the material is called magnetization of the material, represented by $\vec{M} = \frac{\vec{m}}{V}$.

14. The magnetic susceptibility χ_m of a material is a measure of how a magnetic material responds to external magnetic field. It is dimensionless quantity.

15. When a diamagnetic material is placed in non-uniform magnetic field, it experiences a resultant force towards the region of weak magnetic field. The magnetic susceptibility χ_m of diamagnetic material is negative.

16. When a paramagnetic material is placed in non-uniform magnetic field, it experiences a (weak) force towards strong magnetic field. The magnetic susceptibility χ_m of paramagnetic material is positive.

17. According to Curie's law, the magnetization M of a paramagnetic material is given by $M = C\frac{B}{T}$.
When magnetic moments of all atoms are aligned with external magnetic field M, χ_m and μ_r become maximum, called saturation magnetization. Curie's law is not obeyed after saturation magnetization.

18. The atoms of ferromagnetic material possess permanent magnetic dipole moment due to spin of electrons in outermost orbits. These atoms are arranged in such a way that over a region called domain, the magnetic moments of such atoms are aligned in the same direction. In unmagnetized sample, such domains having a net magnetization are randomly oriented so that the effective magnetic moment is zero.

19. The temperature at which a ferromagnetic substance is converted into a paramagnetic substance is called Curie temperature T_C of that substance. The relation between the magnetic susceptibility of the substance in the acquired form and the temperature T is
$\chi_m = \frac{C_1}{T-T_C}$, $(T > T_C)$, where C_1 = constant

20. Permanent magnets have higher retentivity and high coercivity.

21. Soft iron used to prepare electromagnets have large permeability and small retentivity.

EXERCISE

For the following statements choose the correct option from the given options :

1. A magnet of magnetic dipole moment 5.0 A m^2 is lying in a uniform magnetic field of 7×10^{-4} T such that its dipole moment vector makes an angle of 30^{o} with the field. The work done in increasing this angle from 30^{o} to 45^{o} is about .... J.
(A) 5.56×10^{-4} (B) 24.74×10^{-4} (C) 30.3×10^{-4} (D) 5.50×10^{-3}

2. A bar magnet is oscillating in Earth's magnetic field with periodic time T. If a similar magnet with the same mass and dimensions has magnetic dipole moment, which is 4 times that of this magnet, then its periodic time will be ....
(A) $\frac{T}{2}$ (B) 2T (C) T (D) 4T

3. A circular loop carrying current I is replaced by a bar magnet of equivalent magnetic dipole moment. The point on the loop is lying ....

(A) on equatorial plane of magnet

(B) on axis of the magnet

(C) A and B both

(D) except equatorial plane or axis of bar magnet

4. When a current carrying loop is replaced by an equivalent magnetic dipole

(A) the distance l between the poles is fixed.

(B) the pole strength p of each pole is fixed.

(C) the dipole moment is reversed.

(D) the product pl is fixed.

5. Let r be the distance of a point on the axis of a bar magnet from its centre. The magnetic field at r is always proportional to

(A) $\frac{1}{r^2}$ (B) $\frac{1}{r^3}$

(C) $\frac{1}{r}$ (D) not necessarily $\frac{1}{r^3}$ at all points

6. Magnetic meridian is a plane ....

(A) perpendicular to magnetic axis of Earth.

(B) perpendicular to geographic axis of Earth.

(C) passing through the magnetic axis of Earth.

(D) passing through the geographic axis.

7. At geomagnetic pole, a magnetic needle allowed to rotate in horizontal plane will

(A) stay in north-south direction only (B) stay in any position

(C) stay in east-west direction only (D) become rigid showing no movement

8. The horizontal and vertical components of magnetic field of Earth are same at some place on the surface of Earth. The magnetic dip angle at this place will be ....

(A) 30° (B) 45° (C) 0° (D) 90°

9. Inside a bar magnet, the magnetic field lines

(A) are not present

(B) are parallel to the cross-sectional area of the magnet

(C) are in the direction from N-pole to S-pole

(D) are in the direction from S-pole to N-pole

10. In non-uniform magnetic field, a diamagnetic substance experiences a resultant force
(A) from the region of strong magnetic field to the region of weak magnetic field.
(B) perpendicular to the magnetic field.
(C) from the region of weak magnetic field to the region of strong magnetic field.
(D) which is zero.

11. A straight steel wire of length l has magnetic moment m. If the wire is bent in the form of a semicircle, the new value of the magnetic dipole moment is

(A) m (B) $\frac{2m}{\pi}$ (C) $\frac{m}{2}$ (D) $\frac{m}{\pi}$

12. At a place on Earth, the horizontal component of Earth's magnetic field is $\sqrt{3}$ times its vertical component. The angle of dip at this place is ....

(A) 0 (B) $\frac{\pi}{2}$ rad (C) $\frac{\pi}{3}$ rad (D) $\frac{\pi}{6}$ rad

13. A place, where the vertical component of Earth's magnetic field is zero has the angle of dip equal to
(A) 0^o (B) 45^o (C) 60^o (D) 90^o

14. A place where the horizontal component of Earth's magnetic field is zero lies at
(A) geographic equator (B) geomagnetic equator
(C) one of the geographic poles (D) one of the geomagnetic poles

15. When a paramagnetic substance is brought near a north pole or a south pole of a bar magnet, it
(A) experiences repulsion (B) experiences attraction
(C) does not experience attraction or repulsion
(D) experiences attraction or repulsion depending upon which pole is brought near to it.

16. A magnetic needle kept on horizontal surface oscillates in Earth's magnetic field. If the temperature of this needle is raised beyond the Curie temperature of the material of the needle, then ....
(A) the periodic time of oscillation will decrease.
(B) the periodic time of oscillation will increase.
(C) the periodic time of the oscillation will not change.
(D) the needle will stop oscillating.

17. A bar magnet of length l, pole strength 'p' and magnetic moment '$\vec{m}$' is split $\frac{l}{2}$ into two equal pieces each of length. The magnetic moment and pole strength of each piece is respectively .... and ....

(A) $\vec{m}$, $\frac{p}{2}$ (B) $\frac{\vec{m}}{2}$, p (C) $\frac{\vec{m}}{2}$, $\frac{p}{2}$ (D) $\vec{m}$, p

18. Magnetization for vacuum is ....

(A) negative (B) positive (C) infinite (D) zero

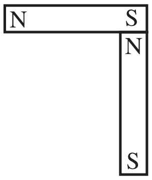
19. A bar magnet of magnetic moment $\vec{m}$ is placed in uniform magnetic field $\vec{B}$ such that $\vec{m} \parallel \vec{B}$. In this position, the torque and force acting on it are .... and .... respectively.

(A) 0, 0 (B) $\vec{m} \times \vec{B}$, mB (C) $\vec{m} \cdot \vec{B}$, mB (D) $\vec{m} \cdot \vec{B}$, $\vec{m} \times \vec{B}$

20. Relative permeability of a substance is 0.075. Its magnetic susceptibility is

(A) 0.925 (B) −0.925 (C) 1.075 (D) −1.075

21. Two similar magnets of magnetic moment m are arranged as shown in figure. The magnetic dipole moment of this combination is



(A) $2m$ (B) $\sqrt{2}\, m$ (C) $\frac{m}{\sqrt{2}}$ (D) $\frac{m}{2}$

22. A magnetic needle kept non-parallel to the magnetic field in a non-uniform magnetic field experiences

(A) a force but not a torque. (B) a torque but not a force

(C) both a force and a torque. (D) neither a force nor a torque

23. A steamer would like to move in the direction making an angle of 10^{o} south with the west. The magnetic declination at that place is 17^{o} west from the north. The steamer should move in a direction ....

(A) making an angle of 83^{o} west with the north pole of Earth.

(B) making an angle of 83^{o} east with the north pole of Earth.

(C) making an angle of 27^{o} west with the south pole of Earth.

(D) making an angle of 27^{o} east with the south pole of Earth.

24. A toroid wound with 100 turns/m of wire carries a current of 3 A. The core of toroid is made of iron having relative magnetic permeability of $\mu_r = 5000$ under given conditions. The magnetic field inside the iron is .... (Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

(A) 0.15 T (B) 0.47 T (C) 1.5×10^{-2} T (D) 1.88 T

ANSWERS

1. (A)	**2.** (A)	**3.** (A)	**4.** (D)	**5.** (D)	**6.** (C)
7. (B)	**8.** (B)	**9.** (D)	**10.** (A)	**11.** (B)	**12.** (D)
13. (A)	**14.** (D)	**15.** (B)	**16.** (D)	**17.** (B)	**18.** (D)
19. (A)	**20.** (B)	**21.** (B)	**22.** (C)	**23.** (A)	**24.** (D)

Answer the following questions in brief :

1. What happens if a bar magnet is cut into two pieces transverse to its length/along its length ?

2. Does a current carrying toroid have a north pole and a south pole ?

3. Which phase / phases of matter cannot be ferromagnetic in character ?

4. Magnetic properties of which materials are affected by temperature ?

5. What should be retentivity and coercivity of permanent magnet ?

6. What happens to a ferromagnetic material when its temperature increases above Curie temperature ?

7. What is the unit of magnetic intensity ?

8. What does the hysteresis loop represent ?

9. What are the applications of electromagnet ?
10. What could be the equation for Gauss's law of magnetism, if a monopole of polestrength p is enclosed by a surface ?
11. What happens when a paramagnetic material is placed in a non-uniform magnetic field ?
12. What is the unit of magnetic susceptibility ?
13. What is the declination for Delhi ?
14. Mention the names of diamagnetic materials.
15. Which property of soft iron makes it useful for preparing electromagnet ?

Answer the following questions :

1. Obtain an expression for axial magnetic field of a current loop in terms of its magnetic moment.
2. Explain symbolic notation for detecting north and south pole of magnetic field in a current carrying loop.
3. Obtain an expression for orbital magnetic moment of an electron rotating about the nucleus in an atom.
4. Explain in brief, the Gauss's law for magnetic fields.
5. What is a geographic meridian and a geomagnetic meridian ? What is the angle between them ?
6. Give definition of magnetic declination. How does the declination vary with latitude ? Where is it minimum ?
7. Give definition of magnetic dip. What is the dip angle at magnetic equator ? What happens to dip angle as we move towards magnetic pole from the magnetic equator ?
8. What happens when a diamagnetic material is placed in non-uniform magnetic field ? Explain with necessary Figure.
9. Discuss Curie's law for paramagnetic materials.
10. Discuss why the soft iron is suitable for preparing electromagnets.

Solve the following examples :

1. A toroidal core with 3000 turns has inner and outer radii of 11 cm and 12 cm, respectively. When a current of 0.70 A is passed, the magnetic field produced in the core is 2.5 T. Find the relative permeability of the core. ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

   [**Ans. :** 685]

2. A paramagnetic gas has 2.0×10^{26} atoms/m^3 with the atomic magnetic dipole moment of 1.5×10^{-23} A m^2 each. The gas is at 27° C. (i) Find the maximum magnetization intensity of this sample. (ii) If the gas in this problem is kept in a uniform magnetic field of 3 T, is it possible to achieve saturation magnetization ? Why ?

   [**Hint :** Thermal energy of an atom of gas is $\frac{3}{2}k_B T$, and

   Maximum potential energy of the atom = mB.

   Find the ratio of thermal energy to the maximum potential energy and give answer.]

   ($k_B = 1.38 \times 10^{-23}$ J K^{-1}) [**Ans. :** 3.0×10^3 A m^{-1}, No]

3. Two small and similar bar magnets have magnetic dipole moment of 1.0 A m^2 each. They are kept in a plane in such a way that their axes are perpendicular to each other. A line drawn through the axis of one magnet passes through the centre of other magnet. If the distance between their centers is 2 m, find the magnitude of magnetic field at the mid point of the line joining their centers.

   [**Ans. :** $\sqrt{5} \times 10^{-7}$ T]

4. A magnetic pole of bar magnet with pole-strength of 100 A m is 20 cm away from the centre of a bar magnet. Bar magnet has pole-strength of 200 A m and has a length of 5 cm. If the magnetic pole is on the axis of the bar magnet, find the force on the magnetic pole.

[**Ans. :** 2.5×10^{-2} N]

5. The work done for rotating a magnet with magnetic dipole moment *m*, by 90° from its magnetic meridian is *n* times the work done to rotate it by 60°. Find the value of *n*.

[**Ans. :** 2]

6. A magnet makes an angle of 45° with the horizontal in a plane making an angle of 30° with the magnetic meridian. Find the true value of the dip angle at the place.

[**Ans. :** tan^{-1} (0.866)]

7. An electron in an atom is revolving round the nucleus in a circular orbit of radius 5.3×10^{-11} m, with a speed of 2×10^{6} m s^{-1}. Find the resultant orbital magnetic moment and angular momentum of the electron. Take charge of electron = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31}kg. [**Ans. :** 8.48×10^{-24} Am^2, and 9.65×10^{-35} N m s]

8. The magnetic field from a current carrying loop of diameter 1 cm is 10^{-4} T at 10 cm from the centre, along the axis of the loop.
   (a) Find the magnetic moment of the loop.
   (b) Find the magnetic field at 10 cm from the centre, along the equator of the loop.

   Take $\frac{\mu_0}{4\pi} = 10^{-7}$ T m A^{-1} [**Ans. :** (a) 0.5 A m^2, (b) 5×10^{-5} T]

9. A magnet in the form of a cylindrical rod has a length of 5 cm and a diameter of 2 cm. It has a uniform magnetization of 5×10^{3} A m^{-1}. Find its net magnetic dipole moment.

[**Ans. :** 7.85×10^{-2} J T^{-1}]

10. An ionized gas consists of 5×10^{21} electrons/m^3 and the same number of ions/m^3. If the average electron kinetic energy is 6×10^{-20} J, and an average ion kinetic energy is 8×10^{-21} J, calculate the magnetization of the gas when a magnetic field of 1.0 T is applied to the gas. [**Ans. :** 340 J T^{-1} m^{-3}]

11. A closely wound solenoid of 6 cm, having 10 turns/cm and area of cross-section 3×10^{-4} m^2 carries a current of 1.0 A. Find the magnetic moment and the pole strength of the solenoid.

    [**Ans. :** Magnetic moment of solenoid along its axis = 1.8×10^{-2} A m^2, pole strength of the solenoid = 0.3 A m]

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