

1. Units, Dimensions and Measurement

1.1 Physical Quantity

A quantity which can be measured and expressed in form of laws is called a physical quantity.

$$\text{Physical quantity } (Q) = \text{Magnitude} \times \text{Unit} = n \times u$$

Where, n represents the numerical value and u represents the unit. as the unit(u) changes, the magnitude(n) will also change but product ' nu ' will remain same.

$$\text{i.e. } n u = \text{constant, or } n_1 u_1 = n_2 u_2 = \text{constant;}$$

1.2 Fundamental and Derived Units

Any unit of mass, length and time in mechanics is called a *fundamental, absolute or base unit*. Other units which can be expressed in terms of fundamental units, are called derived units

System of units: A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units.

- (1) CGS system (2) MKS system (3) FPS system.
- (4) **S.I. system:** It is known as International system of units. There are seven fundamental quantities in this system. These quantities and their units are given in the following table.

Quantity	Name of Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	Kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

Besides the above seven fundamental units two supplementary units are also defined – Radian (rad) for plane angle and Steradian (sr) for solid angle.

1.3 Practical Units

- (1) **Length:**
 - (i) 1 fermi = $1 fm = 10^{-15} m$
 - (ii) 1 X-ray unit = $1 XU = 10^{-13} m$
 - (iii) 1 angstrom = $1 \text{\AA} = 10^{-10} m = 10^{-8} cm = 10^{-7} mm = 0.1 \mu m$
 - (iv) 1 micron = $\mu m = 10^{-6} m$
 - (v) 1 astronomical unit = $1 A.U. = 1.49 \times 10^{11} m \approx 1.5 \times 10^{11} m \approx 10^8 km$
 - (vi) 1 Light year = $1 ly = 9.46 \times 10^{15} m$
 - (vii) 1 Parsec = $1 pc = 3.26 \text{ light year}$
- (2) **Mass:**
 - (i) Chandra Shekhar unit: $1 CSU = 1.4 \text{ times the mass of sun} = 2.8 \times 10^{30} kg$
 - (ii) Metric tonne: 1 Metric tonne = $1000 kg$
 - (iii) Quintal: 1 Quintal = $100 kg$

- (iv) Atomic mass unit (amu): $amu = 1.67 \times 10^{-27} \text{ kg}$ mass of proton or neutron is of the order of 1 amu

(3) **Time:**

- (i) Year: It is the time taken by earth to complete 1 revolution around the sun in its orbit.
(ii) Lunar month: It is the time taken by moon to complete 1 revolution around the earth in its orbit.

$$1 \text{ L.M.} = 27.3 \text{ days}$$

- (iii) Solar day: It is the time taken by earth to complete one rotation about its axis with respect to sun. Since this time varies from day to day, average solar day is calculated by taking average of the duration of all the days in a year and this is called Average Solar day.

$$1 \text{ Solar year} = 365.25 \text{ average solar day}$$

$$\text{or average solar day} = \frac{1}{365.25} \text{ the part of solar year}$$

- (iv) Sedrial day: It is the time taken by earth to complete one rotation about its axis with respect to a distant star.

$$1 \text{ Solar year} = 366.25 \text{ Sedrial day} = 365.25 \text{ average solar day}$$

Thus 1 Sedrial day is less than 1 solar day.

- (v) Shake: It is an obsolete and practical unit of time.

$$1 \text{ Shake} = 10^{-8} \text{ sec}$$

1.4 Dimensions of a Physical Quantity

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

1.5 Important Dimensions of Complete Physics

Mechanics

S. N.	Quantity	Unit	Dimension
(1)	Velocity or speed (v)	m/s	$[M^0 L^1 T^{-1}]$
(2)	Acceleration (a)	m/s^2	$[M^0 L T^{-2}]$
(3)	Momentum (P)	$kg \cdot m/s$	$[M^1 L^1 T^{-1}]$
(4)	Impulse (I)	Newton-sec or $kg \cdot m/s$	$[M^1 L^1 T^{-1}]$
(5)	Force (F)	Newton	$[M^1 L^1 T^{-2}]$
(6)	Pressure (P)	Pascal	$[M^1 L^{-1} T^{-2}]$
(7)	Kinetic energy (E_K)	Joule	$[M^1 L^2 T^{-2}]$
(8)	Power (P)	Watt or Joule/s	$[M^1 L^2 T^{-3}]$
(9)	Density (d)	kg/m^3	$[M^1 L^{-3} T^0]$
(10)	Angular displacement (θ)	Radian (rad.)	$[M^0 L^0 T^0]$
(11)	Angular velocity (ω)	Radian/sec	$[M^0 L^0 T^{-1}]$
(12)	Angular acceleration (α)	Radian/sec^2	$[M^0 L^0 T^{-2}]$
(13)	Moment of inertia (I)	$kg \cdot m^2$	$[M^1 L^2 T^0]$
(14)	Torque (τ)	Newton-meter	$[M^1 L^2 T^{-2}]$

S. N.	Quantity	Unit	Dimension
(15)	Angular momentum (L)	<i>Joule-sec</i>	$[M^1L^2T^{-1}]$
(16)	Force constant or spring constant (k)	<i>Newton / m</i>	$[M^1L^0T^{-2}]$
(17)	Gravitational constant (G)	$N\cdot m^2/kg^2$	$[M^{-1}L^3T^{-2}]$
(18)	Intensity of gravitational field (E_g)	N/kg	$[M^0L^1T^{-2}]$
(19)	Gravitational potential (V_g)	<i>Joule / kg</i>	$[M^0L^2T^{-2}]$
(20)	Surface tension (T)	N/m or <i>Joule / m²</i>	$[M^1L^0T^{-2}]$
(21)	Velocity gradient (V_g)	<i>Second⁻¹</i>	$[M^0L^0T^{-1}]$
(22)	Coefficient of viscosity (η)	$kg/m\cdot s$	$[M^1L^{-1}T^{-1}]$
(23)	Stress	N/m^2	$[M^1L^{-1}T^{-2}]$
(24)	Strain	No unit	$[M^0L^0T^0]$
(25)	Modulus of elasticity (E)	N/m^2	$[M^1L^{-1}T^{-2}]$
(26)	Poisson Ratio (σ)	No unit	$[M^0L^0T^0]$
(27)	Time period (T)	<i>Second</i>	$[M^0L^0T^1]$
(28)	Frequency (n)	<i>Hz</i>	$[M^0L^0T^{-1}]$

Heat

S. N.	Quantity	Unit	Dimension
(1)	Temperature (T)	<i>Kelvin</i>	$[M^0L^0T^0\theta^1]$
(2)	Heat (Q)	<i>Joule</i>	$[ML^2T^{-2}]$
(3)	Specific Heat (c)	<i>Joule / kg-K</i>	$[M^0L^2T^{-2}\theta^{-1}]$
(4)	Thermal capacity	<i>Joule / K</i>	$[M^1L^2T^{-2}\theta^{-1}]$
(5)	Latent heat (L)	<i>Joule / kg</i>	$[M^0L^2T^{-2}]$
(6)	Gas constant (R)	<i>Joule / mol-K</i>	$[M^1L^2T^{-2}\theta^{-1}]$
(7)	Boltzmann constant (k)	<i>Joule / K</i>	$[M^1L^2T^{-2}\theta^{-1}]$
(8)	Coefficient of thermal conductivity (K)	<i>Joule / m-s-K</i>	$[M^1L^1T^{-3}\theta^{-1}]$
(9)	Stefan's constant (σ)	<i>Watt / m²-K⁴</i>	$[M^1L^0T^{-3}\theta^{-4}]$
(10)	Wien's constant (b)	<i>Meter-K</i>	$[M^0L^1T^0\theta]$
(11)	Planck's constant (h)	<i>Joule-s</i>	$[M^1L^2T^{-1}]$
(12)	Coefficient of Linear Expansion (α)	<i>Kelvin⁻¹</i>	$[M^0L^0T^0\theta^{-1}]$
(13)	Mechanical eq. of Heat (J)	<i>Joule / Calorie</i>	$[M^0L^0T^0]$
(14)	Vander wall's constant (a)	<i>Newton-m⁴</i>	$[ML^5T^{-2}]$
(15)	Vander wall's constant (b)	m^3	$[M^0L^3T^0]$

Electricity

S. N.	Quantity	Unit	Dimension
(1)	Electric charge (q)	<i>Coulomb</i>	$[M^0L^0T^1A^1]$
(2)	Electric current (I)	<i>Ampere</i>	$[M^0L^0T^0A^1]$
(3)	Capacitance (C)	<i>Coulomb / volt or Farad</i>	$[M^{-1}L^{-2}T^4A^2]$
(4)	Electric potential (V)	<i>Joule / coulomb</i>	$M^1L^2T^{-3}A^{-1}$

S. N.	Quantity	Unit	Dimension
(5)	Permittivity of free space (ϵ_0)	$\frac{\text{Coulomb}^2}{\text{Newton} - \text{meter}^2}$	$[M^{-1}L^{-3}T^4A^2]$
(6)	Dielectric constant (K)	Unitless	$[M^0L^0T^0]$
(7)	Resistance (R)	<i>Volt / Ampere or ohm</i>	$[M^1L^2T^{-3}A^{-2}]$
(8)	Resistivity or Specific resistance (ρ)	<i>Ohm-meter</i>	$[M^1L^3T^{-3}A^{-2}]$
(9)	Coefficient of Self-induction (L)	$\frac{\text{volt} - \text{second}}{\text{ampere}}$ or <i>henery or ohm-second</i>	$[M^1L^2T^{-2}A^{-2}]$
(10)	Magnetic flux (ϕ)	<i>Volt-second or weber</i>	$[M^1L^2T^{-2}A^{-1}]$
(11)	Magnetic induction (B)	$\frac{\text{newton}}{\text{ampere} - \text{meter}}$ or $\frac{\text{Joule}}{\text{ampere}^2 - \text{meter}}$ or $\frac{\text{volt} - \text{second}}{\text{meter}^2}$ or <i>Tesla</i>	$[M^1L^0T^{-2}A^{-1}]$
(12)	Magnetic Intensity (H)	<i>Ampere / meter</i>	$[M^0L^{-1}T^0A^1]$
(13)	Magnetic Dipole Moment (M)	<i>Ampere-meter</i> ²	$[M^0L^2T^0A^1]$
(14)	Permeability of Free Space (μ_0)	$\frac{\text{Newton}}{\text{ampere}^2}$ or $\frac{\text{Joule}}{\text{ampere}^2 - \text{meter}}$ or $\frac{\text{Volt} - \text{second}}{\text{ampere} - \text{meter}}$ or $\frac{\text{Ohm} - \text{second}}{\text{meter}}$ or $\frac{\text{henery}}{\text{meter}}$	$[M^1L^1T^{-2}A^{-2}]$
(15)	Surface charge density (σ)	<i>Coulomb metre</i> ⁻²	$[M^0L^{-2}T^1A^1]$
(16)	Electric dipole moment (p)	<i>Coulomb – meter</i>	$[M^0L^1T^1A^1]$
(17)	Conductance (G) ($1/R$)	<i>ohm</i> ⁻¹	$[M^{-1}L^{-2}T^3A^2]$
(18)	Conductivity (σ) ($1/\rho$)	<i>ohm</i> ⁻¹ <i>meter</i> ⁻¹	$[M^{-1}L^{-3}T^3A^2]$
(19)	Current density (J)	<i>Ampere / m</i> ²	$M^0L^{-2}T^0A^1$
(20)	Intensity of electric field (E)	<i>Volt / meter, Newton / coulomb</i>	$M^1L^1T^{-3}A^{-1}$
(21)	Rydberg constant (R)	<i>m</i> ⁻¹	$M^0L^{-1}T^0$

1.6 Quantities Having Same Dimensions

S. N.	Dimension	Quantity
(1)	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
(2)	$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
(3)	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
(4)	$[M^1L^1T^{-1}]$	Momentum, impulse
(5)	$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity

(6)	$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
(7)	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
(8)	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
(9)	$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
(10)	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
(11)	$[M^0L^2T^{-2}\theta^1]$	Thermal capacity, gas constant, Boltzmann constant and entropy
(12)	$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g}$, where l = length g = acceleration due to gravity, m = mass, k = spring constant
(13)	$[M^0L^0T^1]$	$L/R, \sqrt{LC}, RC$ where L = inductance, R = resistance, C = capacitance
(14)	$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t, VIt, qV, LI^2, \frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

1.7 Application of Dimensional Analysis

- (1) To find the unit of a physical quantity in a given system of units.
- (2) To find dimensions of physical constant or coefficients.
- (3) To convert a physical quantity from one system to the other.
- (4) To check the dimensional correctness of a given physical relation: This is based on the 'principle of homogeneity'. According to this principle the dimensions of each term on both sides of an equation must be the same.
- (5) To derive new relations.

1.8 Limitations of Dimensional Analysis

- (1) If dimensions are given, physical quantity may not be unique.
- (2) Numerical constant having no dimensions cannot be deduced by the methods of dimensions.
- (3) The method of dimensions can not be used to derive relations other than product of power functions. For example,

$$s = ut + (1/2)at^2 \quad \text{or} \quad y = a \sin \omega t$$
- (4) The method of dimensions cannot be applied to derive formula consist of more than 3 physical quantities.

1.9 Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

- (1) All non-zero digits are significant.
- (2) A zero becomes significant figure if it appears between to non-zero digits.
- (3) Leading zeros or the zeros placed to the left of the number are never significant.

Example: 0.543 has three significant figures.
0.006 has one significant figures.

- (4) Trailing zeros or the zeros placed to the right of the number are significant.
Example: 4.330 has four significant figures.
 343.000 has six significant figures.
- (5) In exponential notation, the numerical portion gives the number of significant figures.
Example: 1.32×10^{-2} has three significant figures.

1.10 Rounding Off

- (1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
Example: $x = 7.82$ is rounded off to 7.8, again $x = 3.94$ is rounded off to 3.9.
- (2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.
Example: $x = 6.87$ is rounded off to 6.9, again $x = 12.78$ is rounded off to 12.8.
- (3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.
Example: $x = 16.351$ is rounded off to 16.4, again $x = 6.758$ is rounded off to 6.8.
- (4) If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.
Example: $x = 3.250$ becomes 3.2 on rounding off, again $x = 12.650$ becomes 12.6 on rounding off.
- (5) If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.
Example: $x = 3.750$ is rounded off to 3.8, again $x = 16.150$ is rounded off to 16.2.

1.11 Significant Figures in Calculation

The following two rules should be followed to obtain the proper number of significant figures in any calculation.

- (1) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places
- (2) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation.

1.12 Order of Magnitude

Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

- (1) Speed of light in vacuum = $3 \times 10^8 \text{ ms}^{-1} \approx 10^8 \text{ m/s}$ (ignoring 3 < 5) (ignoring 3 < 5)
- (2) Mass of electron = $9.1 \times 10^{-31} \text{ kg}$ 10^{-30} kg (as 9.1 > 5).

1.13 Errors of Measurement

The measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

- (1) **Absolute error:** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be $a_1, a_2, a_3, \dots, a_n$.

The arithmetic mean of these value is $a_m = \frac{a_1 + a_2 + \dots + a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise. By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

.....

$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases.

- (2) **Mean absolute error:** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$.

- (3) **Relative error or Fractional error:** Relative error or Fractional error

$$= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\overline{\Delta a}}{a_m}$$

- (4) **Percentage error:** Percentage error = $\frac{\overline{\Delta a}}{a_m} \times 100\%$

1.14 Propagation of Errors

- (1) **Error in sum of the quantities:** Suppose $x = a + b$

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x i.e. sum of a and b .

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

- (2) **Error in difference of the quantities:** Suppose $x = a - b$

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

- (3) **Error in product of quantities:** Suppose $x = a \times b$

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$

- (4) **Error in division of quantities:** Suppose $x = \frac{a}{b}$

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$

- (5) **Error in quantity raised to some power:** Suppose $x = \frac{a^n}{b^m}$

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm \left(n \frac{\Delta a}{a} + m \frac{\Delta b}{b} \right)$

□ The quantity which have maximum power must be measured carefully because it's contribution to error is maximum.

2. Kinematics

2.1 Motion in One Dimension: Position

Position of any point is completely expressed by two factors: Its distance from the observer and its direction with respect to observer.

That is why position is characterised by a vector known as position vector.

Let point P is in a xy plane and its coordinates are (x, y) . Then position vector ($\vec{r}$) of point will be $x\hat{i} + y\hat{j}$ and if the point P is in a space and its coordinates are (x, y, z) then position vector can be expressed as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

2.2 Rest and Motion

If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference: It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

Rest and motion are relative terms. It depends upon the frame of references.

2.3 Types of Motion

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
<i>e.g.</i> .. Motion of car on a straight road. Motion of freely falling body.	<i>e.g.</i> Motion of car on a circular turn. Motion of billiards ball.	<i>e.g.</i> .. Motion of flying kite. Motion of flying insect.

2.4 Particle or Point Mass

The smallest part of matter with zero dimension which can be described by its mass and position is defined as a particle.

If the size of a body is negligible in comparison to its range of motion then that body is known as a particle.

2.5 Distance and Displacement

- (1) **Distance:** It is the actual path length covered by a moving particle in a given interval of time.
 - (i) Its a scalar quantity.
 - (ii) Dimension: $[M^0L^1T^0]$
 - (iii) Unit: *metre* (S.I.)

- (2) **Displacement:** Displacement is the change in position vector *i.e.*, A vector joining initial to final position.
- (i) Displacement is a vector quantity
 - (ii) Dimension: $[M^0L^1T^0]$
 - (iii) Unit: *metre* (S.I.)
 - (iv) If $\vec{S}_1, \vec{S}_2, \vec{S}_3, \dots, \vec{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals. $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \dots + \vec{S}_n$
- (3) **Comparison between distance and displacement:**
- (i) Distance $\geq$ |Displacement|.
 - (ii) For a moving particle distance can never be negative or zero while displacement can be.
i.e., Distance > 0 but Displacement $> =$ or < 0
 - (iii) For motion between two points displacement is single valued while distance depends on actual path and so can have many values.
 - (iv) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.
 - (v) In general magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.

2.6 Speed and Velocity

- (1) **Speed:** Rate of distance covered with time is called speed.
- (i) It is a scalar quantity having symbol v .
 - (ii) Dimension: $[M^0L^1T^{-1}]$
 - (iii) Unit: *metre / second* (S.I.), *cm/second* (C.G.S.)
 - (iv) Types of speed:
 - (a) **Uniform speed:** When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed.
 - (b) **Non-uniform (variable) speed:** In non-uniform speed particle covers unequal distances in equal intervals of time.
 - (c) **Average speed:** The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} ; v_{av} = \frac{\Delta s}{\Delta t}$$
- Time average speed: When particle moves with different uniform speed $v_1, v_2, v_3 \dots$ etc in different time intervals $t_1, t_2, t_3, \dots$ etc respectively, its average speed over the total time of journey is given as
- $$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + \dots}{t_1 + t_2 + t_3 + \dots} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$
- Special case:* When particle moves with speed v_1 upto half time of its total motion and in rest time it is moving with speed v_2 then $v_{av} = \frac{v_1 + v_2}{2}$
- **Distance averaged speed:** When a particle describes different distances $d_1, d_2, d_3, \dots$ with different time intervals $t_1, t_2, t_3, \dots$ with speeds $v_1, v_2, v_3, \dots$

respectively then the speed of particle averaged over the total distance can be given as

$$\frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + \dots}{t_1 + t_2 + t_3 + \dots} = \frac{d_1 + d_2 + d_3 + \dots}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3} + \dots}$$

- (d) *Instantaneous speed*: It is the speed of a particle at particular instant. When we say “speed”, it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (*i.e.*, $\Delta t \rightarrow 0$). Thus

$$\text{Instantaneous speed } v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- (2) **Velocity**: Rate of change of position *i.e.* rate of displacement with time is called velocity.

- (i) It is a scalar quantity having symbol v .
- (ii) Dimension: $[M^0L^1T^{-1}]$
- (iii) Unit: *metre/second* (S.I.), *cm/second* (C.G.S.)
- (iv) Types

- (a) *Uniform velocity*: A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.
- (b) *Non-uniform velocity*: A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).
- (c) *Average velocity*: It is defined as the ratio of displacement to time taken by the body

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

- (d) *Instantaneous velocity*: Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

$$\text{Instantaneous velocity } \vec{v} = \lim_{t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- (v) Comparison between instantaneous speed and instantaneous velocity

- (a) instantaneous velocity is always tangential to the path followed by the particle.
- (b) A particle may have constant instantaneous speed but variable instantaneous velocity.
- (c) The magnitude of instantaneous velocity is equal to the instantaneous speed.
- (d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.
- (e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

- (vi) Comparison between average speed and average velocity

- (a) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.
- (b) Average speed or velocity depends on time interval over which it is defined.
- (c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

- (d) If after motion body comes back to its initial position then $\vec{v}_{av} = \vec{0}$ (as $\Delta\vec{r} = 0$) but $v_{av} > 0$ and finite as ($\Delta s > 0$).
- (e) For a moving body average speed can never be negative or zero (unless $t \rightarrow \infty$) while average velocity can be *i.e.* $v_{av} > 0$ while $\vec{v}_{av} =$ or < 0 .

2.7 Acceleration

The time rate of change of velocity of an object is called acceleration of the object.

- (1) It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)
- (2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
<i>e.g.</i> Uniform circular motion	<i>e.g.</i> Motion under gravity	<i>e.g.</i> Projectile motion

- (3) Dimension: $[M^0L^1T^{-2}]$
- (4) Unit: *metre / second*² (S.I.); *cm/second*² (C.G.S.)
- (5) Types of acceleration:
 - (i) *Uniform acceleration*: A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

□ If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. *e.g.* Projectile motion.
 - (ii) *Non-uniform acceleration*: A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.
 - (iii) *Average acceleration*: $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$
 - (iv) *Instantaneous acceleration* $= \vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$
 - (v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.

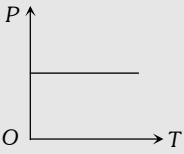
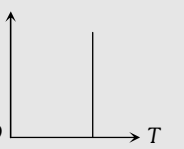
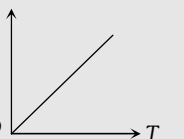
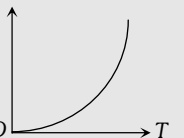
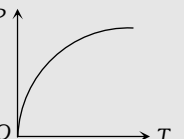
e.g. (a) In uniform circular motion $\theta = 90^\circ$ always (b) In a projectile motion θ is variable for every point of trajectory.

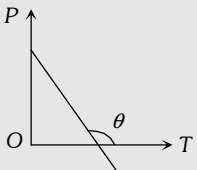
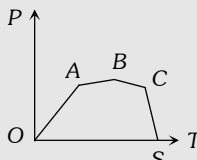
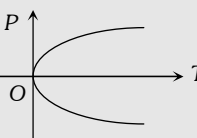
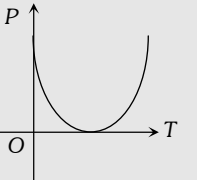
- (vi) By definition $\bar{a} = \frac{d\bar{v}}{dt} = \frac{d^2\bar{x}}{dt^2}$ [As $\bar{v} = \frac{d\bar{x}}{dt}$]
- (vii) If velocity is given as a function of position, then by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v \cdot \frac{dv}{dx} \left[\text{as } v = \frac{dx}{dt} \right]$$
- (viii) If a particle is accelerated for a time t_1 by acceleration a_1 and for time t_2 by acceleration a_2 then average acceleration is $a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$
- (ix) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.
- (x) For motion of a body under gravity, acceleration will be equal to “g”, where g is the acceleration due to gravity. Its normal value is 9.8 m/s² or 980 cm/s² or 32 feet/s².

2.8 Position Time Graph

Various position – time graphs and their interpretation

	$\theta = 0^\circ$ so $v = 0$ <i>i.e.</i> , line parallel to time axis represents that the particle is at rest.
	$\theta = 90^\circ$ so $v = \infty$ <i>i.e.</i> , line perpendicular to time axis represents that particle is changing its position but time does not change it means the particle possesses infinite velocity. Practically this is not possible.
	$\theta = \text{constant}$ so $v = \text{constant}$, $a = 0$ <i>i.e.</i> , line with constant slope represents uniform velocity of the particle.
	θ is increasing so v is increasing, a is positive. <i>i.e.</i> , line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so v is decreasing, a is negative <i>i.e.</i> , line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.

	θ constant but $> 90^\circ$ so v will be constant but negative i.e., line with negative slope represent that particle returns towards the point of reference. (negative displacement).
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions. Which is not possible.
	The graph shows that particle coming towards origin initially and after that it is moving away from origin.

Note:

- ❑ If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time.
- ❑ For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\frac{v_1}{v_2} = \frac{\tan \theta_1}{\tan \theta_2}$

2.9 Velocity Time Graph

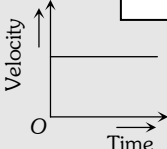
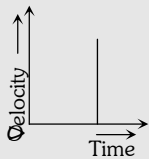
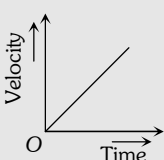
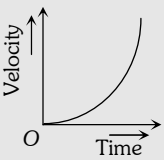
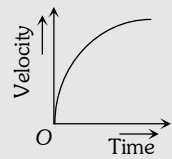
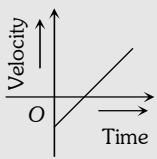
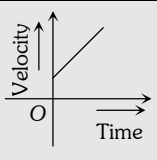
The graph is plotted by taking time t along x -axis and velocity of the particle on y -axis.

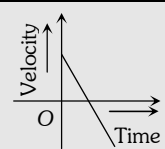
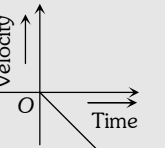
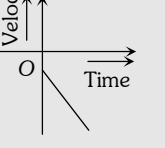
Distance and displacement: The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

Then Total distance = Addition of modulus of different area. i.e. $s = \int |v| dt$

Total displacement = Addition of different area considering their sign. i.e. $r = \int v dt$

Acceleration: It is clear that slope of velocity-time graph represents the acceleration of the particle.

Various velocity – time graphs and their interpretation	
	$\theta = 0, a = 0, v = \text{constant}$ <i>i.e.</i> , line parallel to time axis represents that the particle is moving with constant velocity.
	$\theta = 90^\circ, a = \infty, v = \text{increasing}$ <i>i.e.</i> , line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.
	$\theta = \text{constant}, \text{ so } a = \text{constant and } v \text{ is increasing uniformly with time}$ <i>i.e.</i> , line with constant slope represents uniform acceleration of the particle.
	$\theta \text{ increasing so acceleration increasing}$ <i>i.e.</i> , line bending towards velocity axis represent the increasing acceleration in the body.
	$\theta \text{ decreasing so acceleration decreasing}$ <i>i.e.</i> line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because θ is constant and $< 90^\circ$ but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and $< 90^\circ$ but initial velocity of particle is positive.

	Negative constant acceleration because θ is constant and $> 90^\circ$ but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and $> 90^\circ$ but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and $> 90^\circ$ but initial velocity of the particle is negative.

2.10 Equations of Kinematics

These are the various relations between u , v , a , t and s for the moving particle where the notations are used as:

u = Initial velocity of the particle at time $t = 0$ sec

v = Final velocity at time t sec

a = Acceleration of the particle

s = Distance travelled in time t sec

s_n = Distance travelled by the body in n^{th} sec

(1) **When particle moves with constant acceleration**

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar form

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \left(\frac{u+v}{2} \right) t$$

$$s_n = u + \frac{a}{2}(2n-1)$$

Equation of motion in vector form

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} \cdot \vec{v} - \vec{u} \cdot \vec{u} = 2\vec{a} \cdot \vec{s}$$

$$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$$

$$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

(2) **Important points for uniformly accelerated motion**

(i) If a body starts from rest and moves with uniform acceleration then distance covered by the body in t sec is proportional to t^2 (i.e. $s \propto t^2$).

So the ratio of distance covered in 1 sec, 2 sec and 3 sec is $1^2: 2^2: 3^2$ or 1: 4: 9.

(ii) If a body starts from rest and moves with uniform acceleration then distance covered by the body in n^{th} sec is proportional to $(2n-1)$ (i.e. $s_n \propto (2n-1)$).

So the ratio of distance covered in I sec, II sec and III sec is 1: 3: 5.

- (iii) A body moving with a velocity u is stopped by application of brakes after covering a distance s . If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s .

2.11 Motion of Body Under Gravity (Free Fall)

Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g .

In the absence of air resistance, it is found that all bodies fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ($h \ll R$) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) ***If a body dropped from some height (initial velocity zero)***

- (i) *Equation of motion:* Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

$$u = 0 \quad [\text{As body starts from rest}]$$

$$a = +g \quad [\text{As acceleration is in the direction of motion}]$$

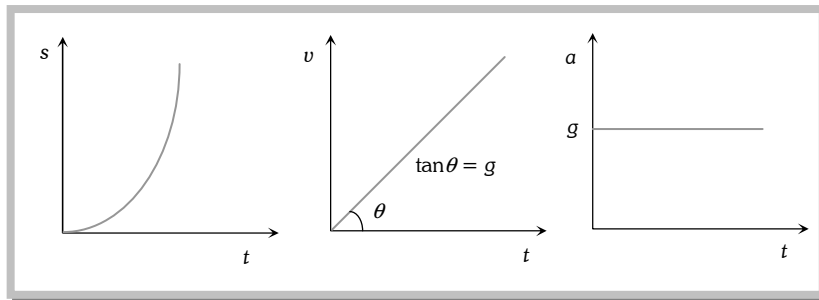
$$v = gt \quad \dots(\text{i})$$

$$h = \frac{1}{2}gt^2 \quad \dots(\text{ii})$$

$$v^2 = 2gh \quad \dots(\text{iii})$$

$$h_n = \frac{g}{2}(2n-1) \quad \dots(\text{iv})$$

- (ii) *Graph of distance velocity and acceleration with respect to time:*



(2) ***If a body is projected vertically downward with some initial velocity***

Equation of motion: $v = u + gt$

$$h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh$$

$$h_n = u + \frac{g}{2}(2n-1)$$

(3) ***If a body is projected vertically upward***

- (i) if the body is projected with velocity u and after time t it reaches up to height h then

$$v = u - gt; \quad h = ut - \frac{1}{2}gt^2; \quad v^2 = u^2 - 2gh; \quad h_n = u - \frac{g}{2}(2n-1)$$

- (ii) For maximum height
- $v = 0$

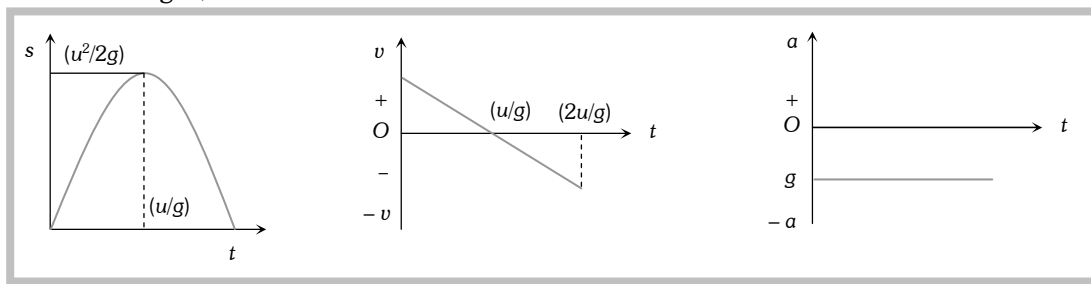
So from above equation

$$u = gt,$$

$$h = \frac{1}{2}gt^2$$

$$\text{and } u^2 = 2gh$$

- (iii) Graph of distance, velocity and acceleration with respect to time (for maximum height):



It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

- (4) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*, $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.
- (6) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance.
- (7) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
- (8) A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively then

$$t = \sqrt{t_1 t_2}$$

- (9) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

2.12 Motion with Variable Acceleration

- (i) If acceleration is a function of time

$$a = f(t), \text{ then } v = u + \int_0^t f(t) dt \text{ and } s = ut + \int \left(\int f(t) dt \right) dt$$

- (ii) If acceleration is a function of distance

$$a = f(x) \text{ then } v^2 = u^2 + 2 \int_{x_0}^x f(x) dx$$

- (iii) If acceleration is a function of velocity

$$a = f(v) \quad \text{then } t = \int_u^v \frac{dv}{f(v)} \text{ and } x = x_0 + \int_u^v \frac{v dv}{f(v)}$$

The motion of an object is called two dimensional, if two of the three co-ordinates are required to specify the position of the object in space changes *w.r.t* time.

In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun *etc.*

Two special cases of motion in two dimension are: 1. Projectile motion 2. Circular motion

PROJECTILE MOTION

2.13 Introduction

If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.

2.14 Projectile

A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile.

2.15 Assumptions of Projectile Motion

- (1) There is no resistance due to air.
- (2) The effect due to curvature of earth is negligible.
- (3) The effect due to rotation of earth is negligible.
- (4) For all points of the trajectory, the acceleration due to gravity ' g ' is constant in magnitude and direction.

2.16 Principles of Physical Independence of Motions

- (1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other. This is called the principle of physical independence of motions.
- (2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.
- (3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.
- (4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated retarded motion.

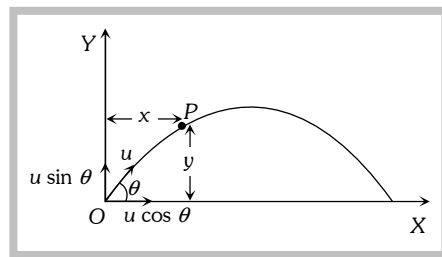
2.17 Types of Projectile Motion

(1) Oblique projectile motion (2) Horizontal projectile motion (3) Projectile motion on an inclined plane

2.18 Oblique Projectile

In projectile motion, horizontal component of velocity ($u \cos \theta$), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity ($u \sin \theta$), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

- (1) **Equation of trajectory:** A projectile thrown with velocity u at an angle θ with the horizontal.



The velocity u can be resolved into two rectangular components $u \cos \theta$ component along X-axis and $u \sin \theta$ component along Y-axis.

$$y = x \tan \theta - \frac{1}{2} \frac{gx^2}{u^2 \cos^2 \theta}$$

Note:

□ Equation of oblique projectile also can be written as

$$y = x \tan \theta \left[1 - \frac{x}{R} \right] \quad (\text{where } R = \text{horizontal range})$$

- (2) **Displacement of projectile ($\vec{r}$):** Let the particle acquires a position P having the coordinates (x, y) just after time t from the instant of projection. The corresponding position vector of the particle at time t is $\vec{r}$ as shown in the figure.

$$\vec{r} = x\hat{i} + y\hat{j} \quad \dots(i)$$

The horizontal distance covered during time t is given as

$$x = v_x t \Rightarrow x = u \cos \theta t \quad \dots(ii)$$

The vertical velocity of the particle at time t is given as

$$y = u \sin \theta t - \frac{1}{2} gt^2 \quad \dots(iii)$$

and $\phi = \tan^{-1}(y/x)$

Note:

□ The angle of elevation ϕ of the highest point of the projectile and the angle of projection θ are related to each other as $\tan \phi = \frac{1}{2} \tan \theta$

- (3) **Instantaneous velocity v :** In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.
Let v_i be the instantaneous velocity of projectile at time t direction of this velocity is along the tangent to the trajectory at point P.

$$\vec{v}_i = v_x \hat{i} + v_y \hat{j} \Rightarrow v_i = \sqrt{v_x^2 + v_y^2}$$

Direction of instantaneous velocity $\tan \alpha = \frac{v_y}{v_x} = \frac{u \sin \theta - gt}{u \cos \theta}$

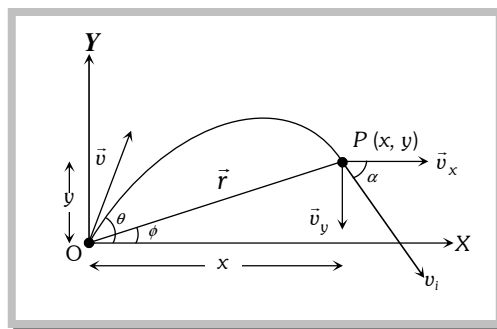
- (7) **Time of flight:** The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.
For vertical upward motion $0 = u \sin \theta - gt \Rightarrow t = (u \sin \theta / g)$

$$\text{Time of flight } T = 2t = \frac{2u \sin \theta}{g}$$

- (8) **Horizontal range:** It is the horizontal distance travelled by a body during the time of flight.

So by using second equation of motion

$$R = u \cos \theta \times T = u \cos \theta \times (2u \sin \theta / g) = \frac{u^2 \sin 2\theta}{g}$$



$$R = \frac{u^2 \sin 2\theta}{g}$$

If angle of projection is changed from θ to $\theta' = (90 - \theta)$ then range remains unchanged.

These angles are called complementary angles of projection.

(iv) **Maximum range:** For range to be maximum $\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta} \left[\frac{u^2 \sin 2\theta}{g} \right] = 0$

a projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g) .

When the range is maximum, the height H reached by the projectile

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{\max}}{4}$$

(v) **Relation between horizontal range and maximum height:**

$$R = 4H \cot \theta$$

If $R = 4H$ then $\theta = \tan^{-1}(1)$ or $\theta = 45^\circ$.

(9) **Maximum height:** It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$

$$0 = (u \sin \theta)^2 - 2gH$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

(i) $H_{\max} = \frac{u^2}{2g}$ (when $\sin^2 \theta = \max = 1$ i.e., $\theta = 90^\circ$)

i.e., for maximum height body should be projected vertically upward.

(10) Motion of a projectile as observed from another projectile is a straight line.

2.19 Horizontal Projectile

A body be projected horizontally from a certain height 'y' vertically above the ground with initial velocity u . If friction is considered to be absent, then there is no other horizontal force which can affect the horizontal motion. The horizontal velocity therefore remains constant.

(4) **Time of flight:** If a body is projected horizontally from a height h with velocity u and time taken by the body to reach the ground is T , then $T = \sqrt{\frac{2h}{g}}$

(5) **Horizontal range:** Let R is the horizontal distance travelled by the body $R = u \sqrt{\frac{2h}{g}}$

(6) If projectiles A and B are projected horizontally with different initial velocity from same height and third particle C is dropped from same point then

(i) All three particles will take equal time to reach the ground.

(ii) Their net velocity would be different but all three particle possess same vertical component of velocity.

(iii) The trajectory of projectiles A and B will be straight line w.r.t. particle C .

(7) If various particles thrown with same initial velocity but indifferent direction then

- (i) They strike the ground with same speed at different times irrespective of their initial direction of velocities.
- (ii) Time would be least for particle which was thrown vertically downward.
- (iii) Time would be maximum for particle A which was thrown vertically upward.

2.20 Projectile Motion on an Inclined Plane

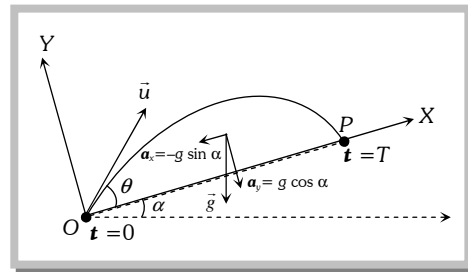
Let a particle be projected up with a speed u from an inclined plane which makes an angle α with the horizontal velocity of projection makes an angle θ with the inclined plane.

We have taken reference x -axis in the direction of plane.

Hence the component of initial velocity parallel and perpendicular to the plane are equal to $u \cos \theta$ and $u \sin \theta$ respectively i.e. $u_{||} = u \cos \theta$ and $u_{\perp} = u \sin \theta$.

The component of g along the plane is $g \sin \alpha$ and perpendicular to the plane is $g \cos \alpha$ as shown in the figure i.e. $a_{||} = -g \sin \alpha$ and $a_{\perp} = g \cos \alpha$.

Therefore the particle decelerates at a rate of $g \sin \alpha$ as it moves from O to P .



- (1) **Time of flight:** We know for oblique projectile motion $T = \frac{2u \sin \theta}{g}$ or we can say $T = \frac{2u_{\perp}}{a_{\perp}}$

$$\therefore \text{Time of flight on an inclined plane } T = \frac{2u \sin \theta}{g \cos \alpha}$$

- (2) **Maximum height:** We know for oblique projectile motion $H = \frac{u^2 \sin^2 \theta}{2g}$ or we can say

$$H = \frac{u_{\perp}^2}{2a_{\perp}}$$

$$\therefore \text{Maximum height on an inclined plane } H = \frac{u^2 \sin^2 \theta}{2g \cos \alpha}$$

- (3) **Horizontal range:** $R = \frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$

- (i) Maximum range occurs when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$

- (ii) The maximum range along the inclined plane when the projectile is thrown upwards

$$\text{is given by } R_{\max} = \frac{u^2}{g(1 + \sin \alpha)}$$

- (iii) The maximum range along the inclined plane when the projectile is thrown

$$\text{downwards is given by } R_{\max} = \frac{u^2}{g(1 - \sin \alpha)}$$

CIRCULAR MOTION

Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.

Circular motion can be classified into two types – Uniform circular motion and non-uniform circular motion.

2.21 Variables of Circular Motion

- (1) **Displacement and distance:** When particle moves in a circular path describing an angle θ during time t (as shown in the figure) from the position A to the position B , we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant. i.e., $|\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.

- (i) **Displacement:** The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position A to the position B is given by referring the figure.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta r = 2r \sin \frac{\theta}{2}$$

- (ii) **Distance:** The distance covered by the particle during the time t is given as $d = \text{length of the arc } AB = r\theta$

- (2) **Angular displacement (θ):** The angle turned by a body moving on a circle from some reference line is called angular displacement.

- Dimension = $[M^0L^0T^0]$ (as $\theta = \text{arc}/\text{radius}$).
- Units = Radian or Degree. It is some times also specified in terms of fraction or multiple of revolution.
- $2\pi\text{rad} = 360^\circ = 1 \text{ Revolution}$
- Angular displacement is a axial vector quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.

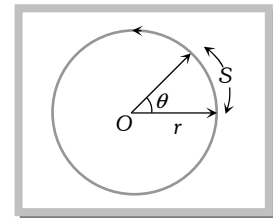
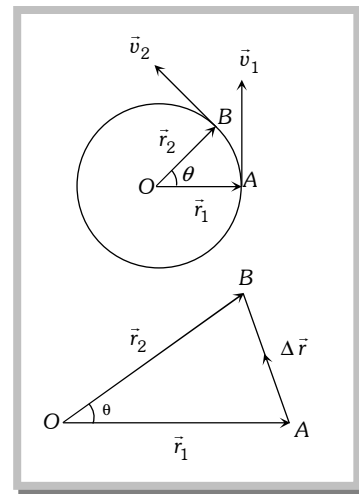
- (v) Relation between linear displacement and angular displacement $\vec{s} = \vec{\theta} \times \vec{r}$
or $s = r\theta$

- (3) **Angular velocity (ω):** Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

- (i) Angular velocity $\omega = \frac{\text{angle traced}}{\text{time taken}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

$$\therefore \omega = \frac{d\theta}{dt}$$

- Dimension: $[M^0L^0T^{-1}]$
- Units: Radians per second (rad.s^{-1}) or Degree per second.
- Angular velocity is an axial vector. Its direction is the same as that of $\Delta\theta$.
- Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$



- (vi) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.
- (4) **Change in velocity:** We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from A to B during time t as shown in figure. The change in velocity vector is given as

$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\Delta v = 2v \sin \frac{\theta}{2}$$

- Relation between linear velocity and angular velocity.

In vector form $\vec{v} = \vec{\omega} \times \vec{r}$

- (5) **Time period (T):** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.
- (6) **Frequency (n):** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.
- (i) Units: s^{-1} or hertz (Hz).
- (ii) Dimension: $[M^0 L^0 T^{-1}]$

Note:

- Relation between time period and frequency:

$$\therefore T = 1/n$$

- Relation between angular velocity, frequency and time period:

$$\omega = \frac{2\pi}{T} = 2\pi n$$

- (7) **Angular acceleration (α):** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) $\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

(ii) Units: $rad. s^{-2}$

(iii) Dimension: $[M^0 L^0 T^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{dt} = 0$

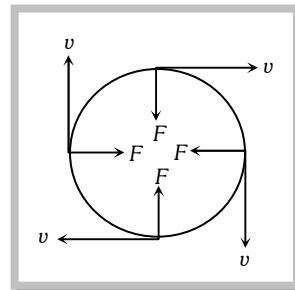
(vi) For non-uniform circular motion $\alpha \neq 0$.

2.22 Centripetal Acceleration

- (1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.
- (2) It always acts on the object along the radius towards the centre of the circular path.
- (3) Magnitude of centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r = 4\pi n^2 r = \frac{4\pi^2}{T^2} r$
- (4) Direction of centripetal acceleration: It is always the same as that of $\Delta \vec{v}$.

2.23 Centripetal Force

According to Newton's first law of motion, whenever a body moves in a straight line with uniform velocity, no force is required to maintain this velocity. But when a body moves along a circular



path with uniform speed, its direction changes continuously *i.e.* velocity keeps on changing on account of a change in direction. According to Newton's second law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.

Due to inertia, at every point of the circular path; the body tends to move along the tangent to the circular path at that point (in figure). Since every body has directional inertia, a velocity cannot change by itself and as such we have to apply a force. But this force should be such that it changes the direction of velocity and not its magnitude. This is possible only if the force acts perpendicular to the direction of velocity. Because the velocity is along the tangent, this force must be along the radius (because the radius of a circle at any point is perpendicular to the tangent at that point). Further, as this force is to move the body in a circular path, it must act towards the centre. This centre-seeking force is called the centripetal force.

Hence, centripetal force is that force which is required to move a body in a circular path with uniform speed. The force acts on the body along the radius and towards centre.

(1) Formulae for centripetal force: $F = \frac{mv^2}{r} = m\omega^2 r = m4\pi^2 n^2 r = \frac{m4\pi^2 r}{T^2}$

(2) Centripetal force in different situation

Situation	Centripetal Force
A particle tied to a string and whirled in a horizontal circle.	Tension in the string.
Vehicle taking a turn on a level road.	Frictional force exerted by the road on the tyres.
A vehicle on a speed breaker.	Weight of the body or a component of weight.
Revolution of earth around the sun	Gravitational force exerted by the sun.
Electron revolving around the nucleus in an atom.	Coulomb attraction exerted by the protons in the nucleus.
A charged particle describing a circular path in a magnetic field.	Magnetic force exerted by the agent that sets up the magnetic field.

2.24 Centrifugal Force

It is an imaginary force due to incorporated effects of inertia. Centrifugal force is a fictitious force which has significance only in a rotating frame of reference.

2.25 Work done by Centripetal Force

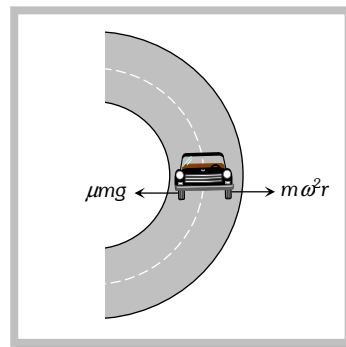
The work done by centripetal force is always zero as it is perpendicular to velocity and hence instantaneous displacement.

Example: (i) When an electron revolve around the nucleus in hydrogen atom in a particular orbit, it neither absorb nor emit any energy means its energy remains constant.

(ii) When a satellite established once in a orbit around the earth and it starts revolving with particular speed, then no fuel is required for its circular motion.

2.26 Skidding of Vehicle on a Level Road

When a vehicle turns on a circular path it requires centripetal force.



If friction provides this centripetal force then vehicle can move in circular path safely if Friction force $\geq$ Required centripetal force

$$\mu mg \geq \frac{mv^2}{r}$$

$$\therefore v_{safe} \leq \sqrt{\mu rg}$$

This is the maximum speed by which vehicle can turn in a circular path of radius r , where coefficient of friction between the road and tyre is μ .

2.27 Skidding of Object on a Rotating Platform

On a rotating platform, to avoid the skidding of an object (mass m) placed at a distance r from axis of rotation, the centripetal force should be provided by force of friction.

Centripetal force = Force of friction

$$m\omega^2 r = \mu mg$$

$$\therefore \omega_{\max} = \sqrt{(\mu g / r)},$$

Hence maximum angular velocity of rotation of the platform is $\sqrt{(\mu g / r)}$, so that object will not skid on it.

2.28 Bending of a Cyclist

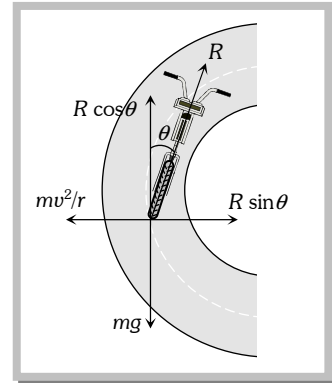
A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track, while going round a curve. Consider a cyclist of weight mg taking a turn of radius r with velocity v . In order to provide the necessary centripetal force, the cyclist leans through angle θ inwards as shown in figure.

$$R \sin \theta = \frac{mv^2}{r} \quad \dots(i)$$

$$\text{and} \quad R \cos \theta = mg \quad \dots(ii)$$

Dividing equation (i) by (ii), we have

$$\tan \theta = \frac{v^2}{rg} \quad \dots(iii)$$



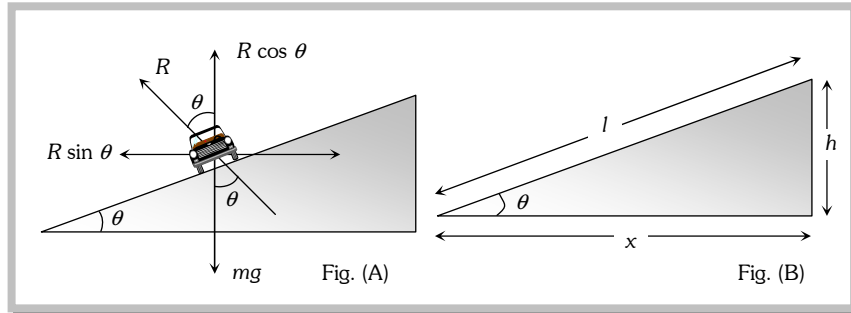
Note:

- For the same reasons, an ice skater or an aeroplane has to bend inwards, while taking a turn.

2.29 Banking of a Road

For getting a centripetal force cyclist bend towards the centre of circular path but it is not possible in case of four wheelers.

Therefore, outer bed of the road is raised so that a vehicle moving on it gets automatically inclined towards the centre.



$$\tan \theta = \frac{v^2}{rg}$$

$$\text{or } \tan \theta = \frac{\omega^2 r}{g} = \frac{v\omega}{rg} \quad \dots (iv) \quad [\text{As } v = r\omega]$$

If l = width of the road, h = height of the outer edge from the ground level then from the figure (B)

$$\tan \theta = \frac{h}{x} = \frac{h}{l} \quad \dots (v) \quad [\text{since } \theta \text{ is very small}]$$

$$\square \text{ Maximum safe speed on a banked frictional road } v = \sqrt{\frac{rg(\mu + \tan \theta)}{1 - \mu \tan \theta}}$$

2.30 Overturning of Vehicle

When a car moves in a circular path with speed more than maximum speed then it overturns and its inner wheel leaves the ground first

Weight of the car = mg

Speed of the car = v

Radius of the circular path = r

Distance between the centre of wheels of the car = $2a$

Height of the centre of gravity (G) of the car from the road level = h

The maximum speed of a car without overturning on a flat road is given by $v = \sqrt{\frac{gra}{h}}$.

2.31 Non-Uniform Circular Motion

If the speed of the particle in a horizontal circular motion changes with respect to time, then its motion is said to be non-uniform circular motion.

using $\vec{v} = \vec{\omega} \times \vec{r} \quad \dots (i)$

the resultant acceleration of the particle at P has two component accelerations

(1) **Tangential acceleration:** $\vec{a}_t = \vec{\alpha} \times \vec{r}$

It acts along the tangent to the circular path at P in the plane of circular path.

(2) **Centripetal (Radial) acceleration:** $\vec{a}_c = \vec{\omega} \times \vec{v}$

It is also called centripetal acceleration of the particle at P .

It acts along the radius of the particle at P .

The magnitude of centripetal acceleration is given by

$$|\vec{a}_c| = |\vec{\omega} \times \vec{v}| = \omega v \sin 90^\circ = \omega v = \omega(\omega r) = \omega^2 r = v^2 / r$$

□ Here a_t governs the magnitude of $\vec{v}$ while $\vec{a}_c$ its direction of motion.

2.32 Equations of Circular Motion

For accelerated motion	For retarded motion
$\omega_2 = \omega_1 + \alpha t$	$\omega_2 = \omega_1 + \alpha t$
$\theta = \omega_1 t + \frac{1}{2} \alpha t^2$	$\theta = \omega_1 t - \frac{1}{2} \alpha t^2$
$\omega_2^2 = \omega_1^2 + 2\alpha \theta$	$\omega_2^2 = \omega_1^2 - 2\alpha \theta$
$\theta_n = \omega_1 + \frac{\alpha}{2}(2n-1)$	$\theta_n = \omega_1 - \frac{\alpha}{2}(2n-1)$

Where

ω_1 = Initial angular velocity of particle

ω_2 = Final angular velocity of particle

α = Angular acceleration of particle

θ = Angle covered by the particle in time t

θ_n = Angle covered by the particle in n^{th} second

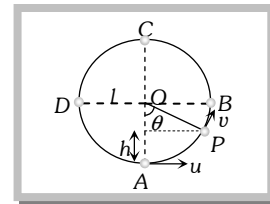
2.33 Motion in Vertical Circle

This is an example of non-uniform circular motion. In this motion body is under the influence of gravity of earth.

- (1) **Velocity at any point on vertical loop:** If u is the initial velocity imparted to body at lowest point then. Velocity of body at height h is given by

$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gl(1 - \cos \theta)}$$

where l is the length of the string



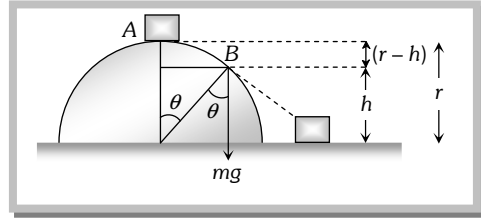
- (2) **Tension at any point on vertical loop:** Tension at general point P , $T = mg \cos \theta + \frac{mv^2}{l}$
- (3) **Various conditions for vertical motion:**

Velocity at lowest point	Condition
$u_A > \sqrt{5gl}$	Tension in the string will not be zero at any of the point and body will continue the circular motion.
$u_A = \sqrt{5gl}$,	Tension at highest point C will be zero and body will just complete the circle.
$\sqrt{2gl} < u_A < \sqrt{5gl}$,	Particle will not follow circular motion. Tension in string become zero somewhere between points B and C whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory.
$u_A = \sqrt{2gl}$	Both velocity and tension in the string becomes zero between A and B and particle will oscillate along semi-circular path.
$u_A < \sqrt{2gl}$	velocity of particle becomes zero between A and B but tension will not be zero and the particle will oscillate about the point A.

(6) **Various quantities for a critical condition in a vertical loop at different positions:**

Quantity	Point A	Point B	Point C	Point D	Point P
Linear velocity (v)	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{3gl}$	$\sqrt{gl(3+2\cos\theta)}$
Angular velocity (ω)	$\sqrt{\frac{5g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}(3+2\cos\theta)}$
Tension in String (T)	$6mg$	$3mg$	0	$3mg$	$3mg(1+\cos\theta)$
Kinetic Energy (KE)	$\frac{5}{2}mgl$	$\frac{3}{2}mgl$	$\frac{1}{2}mgl$	$\frac{3}{2}mgl$	$\frac{mv^2}{2} - 5mg = 0$
Potential Energy (PE)	0	mgl	$2mgl$	mgl	$Mgl(1-\cos\theta)$
Total Energy (TE)	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$

(7) **Motion of a block on frictionless hemisphere:** A small block of mass m slides down from the top of a frictionless hemisphere of radius r . The component of the force of gravity ($mg \cos \theta$) provides required centripetal force but at point B its circular motion ceases and the block loses contact with the surface of the sphere.



For point B , by equating the forces, $mg \cos \theta = \frac{mv^2}{r}$ (i)

by law of conservation of energy

Total energy at point A = Total energy at point B the block loses contact at the height of $\frac{2}{3}r$ from the ground.

3. Newton's Laws of Motion

3.1 Point Mass

3.2 Inertia

- (1) Inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.
- (2) Two bodies of equal mass possess same inertia because it is a factor of mass only.

3.3 Linear Momentum

- (1) Linear momentum of a body is the quantity of motion contained in the body.
- (2) It is measured as the product of the mass of the body and its velocity *i.e.*, Momentum = mass $\times$ velocity.

If a body of mass m is moving with velocity $\vec{v}$ then its linear momentum $\vec{p}$ is given by

$$\vec{p} = m\vec{v}$$

- (3) It is a vector quantity and its direction is the same as the direction of velocity of the body.
- (4) Units: $kg\text{-}m/sec$ [S.I.], $g\text{-}cm/sec$ [C.G.S.]
- (5) Dimension: $[MLT^{-1}]$

3.4 Newton's First Law

A body continue to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

- (1) If no net force acts on a body, then the velocity of the body cannot change *i.e.* the body cannot accelerate.
- (2) Newton's first law defines inertia and is rightly called the law of inertia. Inertia are of three types:
 Inertia of rest, Inertia of motion, Inertia of direction

3.5 Newton's Second Law

- (1) The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.
- (2) If a body of mass m , moves with velocity $\vec{v}$ then its linear momentum can be given by $\vec{p} = m\vec{v}$

and if force $\vec{F}$ is applied on a body, then

$$\text{or} \quad \vec{F} = \frac{d\vec{p}}{dt} \quad (K = 1 \text{ in C.G.S. and S.I. units})$$

$$\therefore \quad \vec{F} = m\vec{a} \quad (\text{Force} = \text{mass} \times \text{acceleration})$$

3.6 Force

- (1) Force is an external effect in the form of a push or pulls which:
 - (i) Produces or tries to produce motion in a body at rest.
 - (ii) Stops or tries to stop a moving body.

- (iii) Changes or tries to change the direction of motion of the body.
- (2) Dimension: Force = mass $\times$ acceleration
 $[F] = [M][LT^{-2}] = [MLT^{-2}]$
- (3) Units: Absolute units: (i) *Newton* (S.I.) (ii) *Dyne* (C.G.S)
 Gravitational units: (i) Kilogram-force (M.K.S.) (ii) Gram-force (C.G.S)
- (4) $\vec{F} = m\vec{a}$ formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.
- (5) If m is not constant $\vec{F} = \frac{d}{dt}(m\vec{v}) = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt}$
- (6) No force is required to move a body uniformly along a straight line.
 $\vec{F} = m\vec{a} \quad \therefore \vec{F} = 0$ (As $a = 0$)
- (7) When force is written without direction then positive force means repulsive while negative force means attractive.
- (8) Out of so many natural forces nuclear force is strongest while gravitational force weakest.
- (9) **Central force:** If a position dependent force is always directed towards or away from a fixed point it is said to be central otherwise non-central.
- (10) **Conservative or non conservative force:** If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non conservative.
Example: Conservative force: Gravitational force, electric force, elastic force.
 Non conservative force: Frictional force, viscous force.
- (11) **Common forces in mechanics:**
- Weight:** Weight of an object is the force with which earth attracts it. ($W = mg$)
 - Reaction or Normal force:** When a body is placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. Then force is called 'Normal force' or 'Reaction'.
 - Tension:** The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.
 - Spring force:** Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by $F = -Kx$; where x is the change in length and K is the spring constant (unit N/m).

3.7 Equilibrium of Concurrent Force

- If all the forces working on a body are acting on the same point, then they are said to be concurrent.
- A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.
- The condition for the equilibrium of a body is that the vector sum of all the forces acting on the body must be zero.

3.8 Newton's Third Law

To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

If $\vec{F}_{AB}$ = force exerted on body A by body B (Action) and $\vec{F}_{BA}$ = force exerted on body B by body A (Reaction).

Then according to Newton's third law of motion $\vec{F}_{AB} = -\vec{F}_{BA}$

Example: (i) A book lying on a table exerts a force on the table which is equal to the weight of the book. This is the force of action.

(ii) Swimming is possible due to third law of motion.

(iii) When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction)

3.9 Frame of Reference

(1) A frame in which an observer is situated and makes his observations is known as his 'Frame of reference'.

It is associated with a co-ordinate system.

(2) Frame of reference are of two types: (i) Inertial frame of reference (ii) Non-inertial frame of reference.

(i) *Inertial frame of reference:*

(a) A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.

(b) In inertial frame of reference Newton's laws of motion holds good.

(c) Ideally no inertial frame exist in universe. For practical purpose a frame of reference may be considered as inertial if it's acceleration is negligible with respect to the acceleration of the object to be observed.

Example: The lift at rest, lift moving (up or down) with constant velocity,

(ii) *Non inertial frame of reference:*

(a) Accelerated frame of references are called non-inertial frame of reference.

(b) Newton's laws of motion are not applicable in non-inertial frame of reference.

Example: Car moving in uniform circular motion, lift which is moving upward or downward with some acceleration, plane which is taking off.

3.10 Impulse

(1) When a large force works on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case we measure the total effect of force.

(2) Impulse of a force is a measure of total effect of force.

$$(3) \quad \vec{I} = \int_{t_1}^{t_2} \vec{F} dt .$$

(4) Impulse is a vector quantity and its direction is same as that of force.

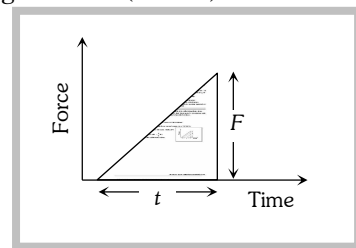
(5) Dimension: $[MLT^{-1}]$

(6) Units: *Newton-second* or $Kg-m-s^{-1}$ (S.I.) and *Dyne-second* or $gm-cm s^{-1}$ (C.G.S.)

(7) Force-time graph: Impulse is equal to the area under $F-t$ curve.

$$I = \text{Area between curve and time axis} = \frac{1}{2} F t$$

(8) If F_{av} is the average magnitude of the force then



$$I = \int_{t_1}^{t_2} F dt = F_{av} \int_{t_1}^{t_2} dt = F_{av} \Delta t$$

- (9) From Newton's second law $\vec{F} = \frac{d\vec{p}}{dt}$

or $\int_{t_1}^{t_2} \vec{F} dt = \int_{p_1}^{p_2} d\vec{p} \Rightarrow \vec{I} = \vec{p}_2 - \vec{p}_1 = \Delta\vec{p}$

i.e. The impulse of a force is equal to the change in momentum.

This statement is known as *Impulse momentum theorem*.

- (10) *Examples:* Hitting, kicking, catching, jumping, diving, collision etc.

In all these cases an impulse acts. $I = \int F dt = F_{av} \cdot \Delta t = \Delta p = \text{constant}$

So if time of contact Δt is increased, average force is decreased (or diluted) and vice-versa.

- (i) In catching a ball a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt.
- (ii) China wares are wrapped in straw or paper before packing.

3.11 Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

- (1) According to this law for a system of particles $\vec{F} = \frac{d\vec{p}}{dt}$

In the absence of external force $\vec{F} = 0$ then $\vec{p} = \text{constant}$

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots = \text{constant}$.

- (2) Law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.
- (3) Practical applications of the law of conservation of linear momentum
 - (i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.
 - (ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.
 - (iii) *Recoiling of a gun:* For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected.
 - (iv) *Rocket propulsion:* The initial momentum of the rocket on its launching pad is zero. When it is fired from the launching pad, the exhaust gases rush downward at a high speed and to conserve momentum, the rocket moves upwards.

Let m_0 = initial mass of rocket,
 m = mass of rocket at any instant 't' (instantaneous mass)
 m_r = residual mass of empty container of the rocket
 u = velocity of exhaust gases,
 v = velocity of rocket at any instant 't' (instantaneous velocity)
 $\frac{dm}{dt}$ = rate of change of mass of rocket = rate of fuel consumption

= rate of ejection of the fuel.

(a) Thrust on the rocket: $F = -u \frac{dm}{dt} - mg$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

$$F = -u \frac{dm}{dt} \text{ (if effect of gravity is neglected)}$$

(b) Acceleration of the rocket: $a = \frac{u}{m} \frac{dm}{dt} - g$ and if effect of gravity is neglected $a = \frac{u}{m} \frac{dm}{dt}$

(c) Instantaneous velocity of the rocket: $v = u \log_e \left(\frac{m_0}{m} \right) - gt$ and if effect of gravity is neglected $v = u \log_e \left(\frac{m_0}{m} \right) = 2.303u \log_{10} \left(\frac{m_0}{m} \right)$

(d) Burnt out speed of the rocket: $v_b = v_{\max} = u \log_e \left(\frac{m_0}{m_r} \right)$

The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.

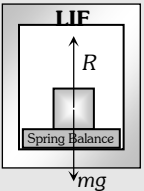
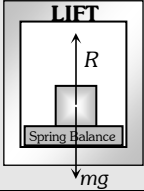
3.12 Free Body Diagram

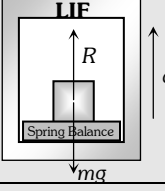
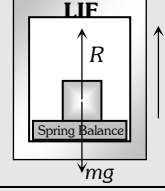
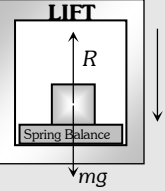
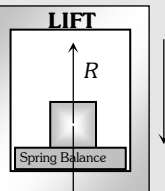
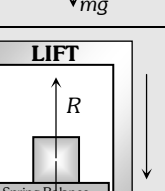
In this diagram the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.

3.13 Apparent Weight of a Body in a Lift

When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg .

This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. The reaction exerted by the surface of contact on the body is the *apparent weight* of the body.

Condition	Figure	Velocity	Acceleration	Reaction	Conclusion
Lift is at rest		$v = 0$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift moving upward or downward with constant velocity		$v = \text{constant}$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight

Lift accelerating upward at the rate of 'a'		$v = \text{variable}$	$a < g$	$R - mg = ma$ $\therefore R = m(g + a)$	Apparent weight > Actual weight
Lift accelerating upward at the rate of 'g'		$v = \text{variable}$	$a = g$	$R - mg = mg$ $R = 2mg$	Apparent weight = 2 Actual weight
Lift accelerating downward at the rate of 'a'		$v = \text{variable}$	$a < g$	$mg - R = ma$ $\therefore R = m(g - a)$	Apparent weight < Actual weight
Lift accelerating downward at the rate of 'g'		$v = \text{variable}$	$a = g$	$mg - R = mg$ $R = 0$	Apparent weight = Zero (weightlessness)
Lift accelerating downward at the rate of $a(>g)$		$v = \text{variable}$	$a > g$	$mg - R = ma$ $R = mg - ma$ $R = -ve$	Apparent weight negative means the body will rise from the floor of the lift and stick to the ceiling of the lift.

3.14 Acceleration of Block on Horizontal Smooth Surface

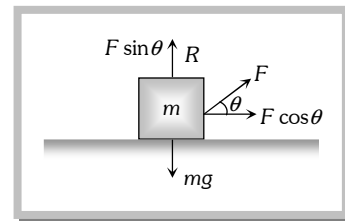
- (1) When a pull is acting at an angle (θ) to the horizontal (upward)

$$R + F \sin \theta = mg$$

$$\Rightarrow R = mg - F \sin \theta$$

$$\text{and } F \cos \theta = ma$$

$$\therefore a = \frac{F \cos \theta}{m}$$



3.15 Acceleration of Block on Smooth Inclined Plane

- (1) When inclined plane is at rest

Normal reaction $R = mg \cos \theta$

Force along a inclined plane

$$F = mg \sin \theta$$

$$ma = mg \sin \theta$$

$$\therefore a = g \sin \theta$$

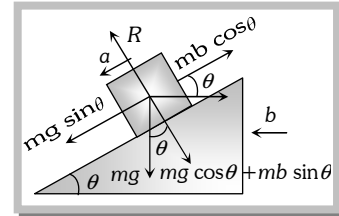
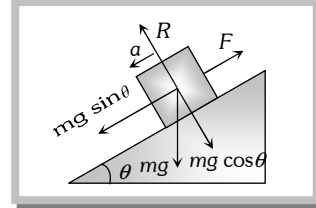
- (2) When a inclined plane given a horizontal acceleration 'b'

Since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.

Normal reaction $R = mg \cos \theta + mb \sin \theta$

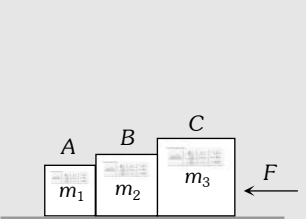
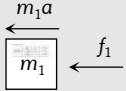
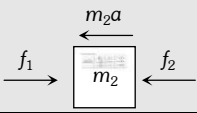
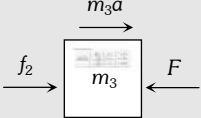
$$\text{and } ma = mg \sin \theta - mb \cos \theta$$

$$\therefore a = g \sin \theta - b \cos \theta$$

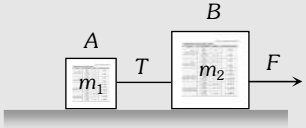
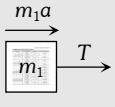
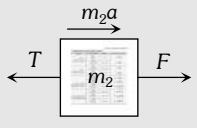
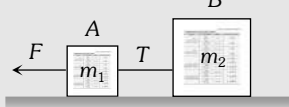
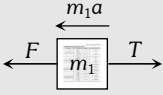
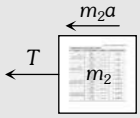
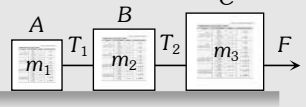
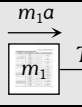
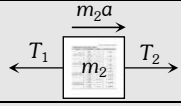
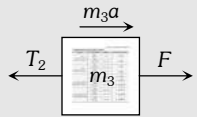
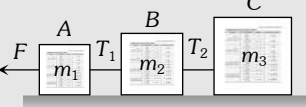
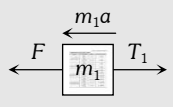
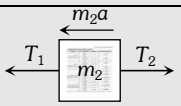
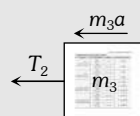


3.16 Motion of Blocks in Contact

Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3) F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

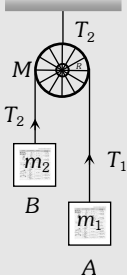
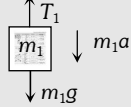
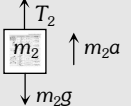
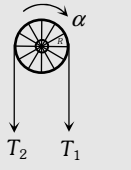
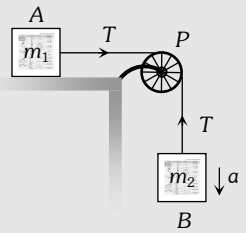
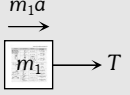
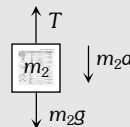
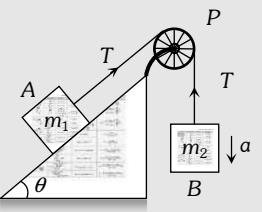
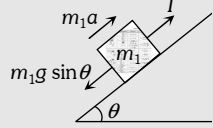
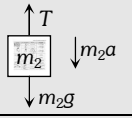
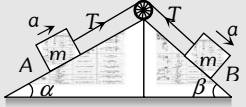
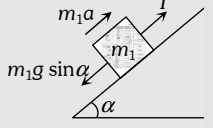
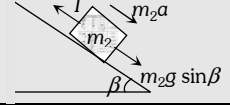
		$f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$ $f_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$ $f_2 = \frac{(m_1 + m_2) F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2 a$	
		$F - f_2 = m_3 a$	

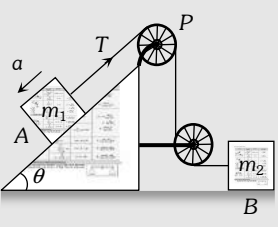
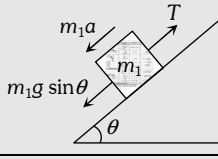
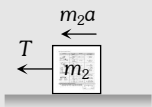
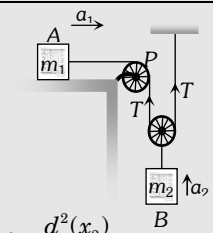
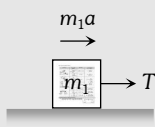
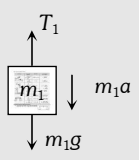
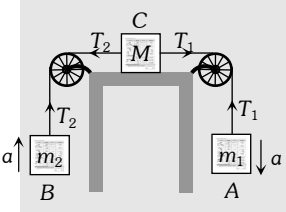
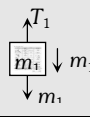
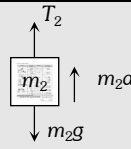
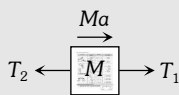
3.17 Motion of Blocks Connected by Mass Less String

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$ $T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_2 a$	
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$ $T = \frac{m_2 F}{m_1 + m_2}$
		$T = m_2 a$	
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$ $T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$ $T_2 = \frac{(m_1 + m_2) F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	
		$F - T_2 = m_3 a$	
		$F - T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$ $T_1 = \frac{(m_2 + m_3) F}{m_1 + m_2 + m_3}$ $T_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	
		$T_2 = m_3 a$	

3.18 Motion of Connected Block Over a Pulley

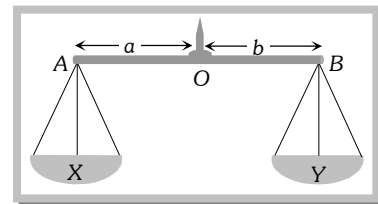
Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1 m_2}{m_1 + m_2} g$ $T_2 = \frac{4m_1 m_2}{m_1 + m_2} g$ $a = \left[\frac{m_2 - m_1}{m_1 + m_2} \right] g$
		$m_2 a = m_2 g - T_1$	
		$T_2 = 2T_1$	
		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1 [m_2 + m_3]}{m_1 + m_2 + m_3} g$ $T_2 = \frac{2m_1 m_3}{m_1 + m_2 + m_3} g$ $T_3 = \frac{4m_1 [m_2 + m_3]}{m_1 + m_2 + m_3} g$ $a = \frac{[(m_2 + m_3) - m_1] g}{m_1 + m_2 + m_3}$
		$m_2 a = m_2 g + T_2 - T_1$	
		$m_3 a = m_3 g - T_2$	
		$T_3 = 2T_1$	

Condition	Free body diagram	Equation	Tension and acceleration
When pulley have a finite mass M and radius R then tension in two segments of string are different 		$m_1 a = m_1 g - T_1$	$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{M}{2}} g$ $T_1 = \frac{m_1 \left[2m_2 + \frac{M}{2} \right]}{m_1 + m_2 + \frac{M}{2}} g$ $T_2 = \frac{m_2 \left[2m_1 + \frac{M}{2} \right]}{m_1 + m_2 + \frac{M}{2}} g$
		$m_2 a = T_2 - m_2 g$	
		Torque $= (T_1 - T_2) R = I \alpha$ $(T_1 - T_2) R = I \frac{a}{R}$ $(T_1 - T_2) R = \frac{1}{2} M R^2 \frac{a}{R}$ $T_1 - T_2 = \frac{M a}{2}$	
		$T = m_1 a$	$a = \frac{m_2}{m_1 + m_2} g$ $T = \frac{m_1 m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	
		$m_1 a = T - m_1 g \sin \theta$	$a = \left[\frac{m_2 - m_1 \sin \theta}{m_1 + m_2} \right] g$ $T = \frac{m_1 m_2 (1 + \sin \theta)}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	
		$T - m_1 g \sin \alpha = m_1 a$	$a = \frac{(m_2 \sin \beta - m_1 \sin \alpha)}{m_1 + m_2} g$ $T = \frac{m_1 m_2 (\sin \alpha + \sin \beta)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin \beta - T$	

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 g \sin \theta - T = m_1 a$	$a = \frac{m_1 g \sin \theta}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{2m_1 m_2}{4m_1 + m_2} g$
 As $\frac{d^2(x_2)}{dt^2} = \frac{1}{2} \frac{d^2(x_1)}{dt^2}$ $\therefore a_2 = \frac{a_1}{2}$ $a_1 = \text{acceleration of block A}$ $a_2 = \text{acceleration of block B}$		$T = m_1 a$	$a_1 = a = \frac{2m_2 g}{4m_1 + m_2}$
		$m_2 \frac{a}{2} = m_2 g - 2T$	$a_2 = \frac{m_2 g}{4m_1 + m_2}$
		$m_1 a = m_1 g - T_1$	$a = \frac{(m_1 - m_2)}{[m_1 + m_2 + M]} g$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1(2m_2 + M)}{[m_1 + m_2 + M]} g$
		$T_1 - T_2 = Ma$	$T_2 = \frac{m_2(2m_2 + M)}{[m_1 + m_2 + M]} g$

3.19 Spring Balance and Physical Balance

- (1) **Spring balance:** When its upper end is fixed with rigid support and body of mass m hung from its lower end.



Spring is stretched and the weight of the body can be measured by the reading of spring balance $R = W = mg$.

- (2) **Physical balance:** In physical balance actually we compare the mass of body in both the pans. Here we does not calculate the absolute weight of the body.

Here X and Y are the mass of the empty pan.

- (i) Perfect physical balance:

Weight of the pan should be equal *i.e.* $X = Y$

and the needle must in middle of the beam *i.e.* $a = b$.

Effect of frame of reference: If the physical balance is perfect then there will be no effect of frame of reference (either inertial or non-inertial)

- (ii) False balance: When the masses of the pan are not equal then balance shows the error in measurement. False balance may be of two types:

- (a) If the beam of physical balance is horizontal (when the pans are empty) but the arms are not equal

$$X > Y \text{ and } a < b$$

In this physical balance if a body of weight W is placed in pan X then to balance it we have to put a weight W_1 in pan Y .

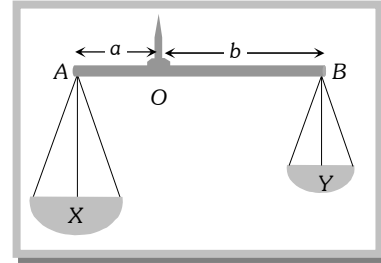
Now if the pans are changed then to balance the body we have to put a weight W_2 in pan X .

$$\text{True weight } W = \sqrt{W_1 W_2}$$

- (b) If the beam of physical balance is not horizontal (when the pans are empty) and the arms are equal

$$\text{i.e. } X > Y \text{ and } a = b$$

$$\text{True weight } W = \frac{W_1 + W_2}{2}$$



3.20 Friction :Introduction

If we slide or try to slide a body over a surface the motion is resisted by a bonding between the body and the surface. This resistance is represented by a single force and is called friction.

The force of friction is parallel to the surface and opposite to the direction of intended motion.

3.21 Types of Friction

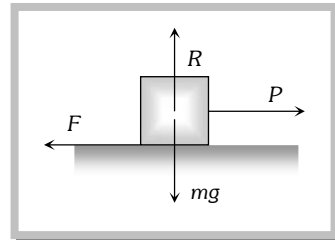
- (1) **Static friction:** The opposing force that comes into play when objects are at rest.

- (i) In this case static friction $F = P$.

- (ii) Static friction is a self-adjusting force because it changes itself in accordance with the applied force.

- (2) **Limiting friction:** The maximum value of static friction upto which body does not move is called limiting friction.

- (i) The magnitude of limiting friction between any two bodies in contact is directly proportional to the normal reaction between them.



$$F_1 \propto R \text{ or } F_1 = \mu_s R$$

- (ii) Direction of the force of limiting friction is always opposite to the direction in which one body is at the verge of moving
- (iii) Coefficient of static friction:
 - (a) μ_s is called coefficient of static friction.
 - (b) Dimension: $[M^0 L^0 T^0]$
 - (c) Unit: It has no unit.
 - (d) Value of μ_s lies in between 0 and 1
 - (e) Value of μ depends on material and nature of surfaces in contact.
 - (f) Value of μ does not depend upon apparent area of contact.

- (3) **Kinetic or dynamic friction:** If the applied force sets the body in motion, the friction opposing the motion is called kinetic friction.

- (i) Kinetic friction depends upon the normal reaction.

$$F_k \propto R \text{ or } F_k = \mu_k R \text{ where } \mu_k \text{ is called the coefficient of kinetic friction}$$

- (ii) Kinetic friction is always lesser than limiting friction $F_k < F_l \therefore \mu_k < \mu_s$

Thus we require more force to start a motion than to maintain it against friction. This is because when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of the other surface.

- (iv) Types of kinetic friction: (a) Sliding friction (b) Rolling friction

- Rolling friction is directly proportional to the normal reaction (R) and inversely proportional to the radius (r) of the rolling cylinder or wheel.

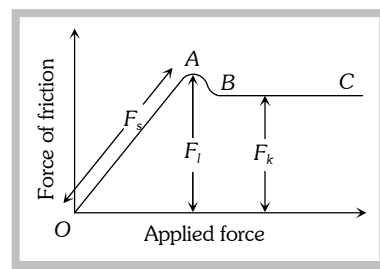
$$F_{\text{rolling}} = \mu_r \frac{R}{r}$$

μ_r is called coefficient of rolling friction. It would have the dimensions of length and would be measured in *metre*.

- Rolling friction is often quite small as compared to the sliding friction.
- In rolling the surfaces at contact do not rub each other.
- The velocity of point of contact with respect to the surface remains zero all the times.

3.22 Graph Between Applied Force and Force of Friction

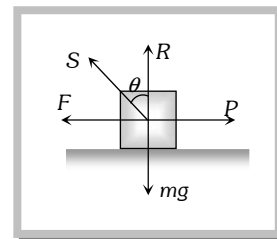
- (1) Part OA = static friction (F_s).
- (2) At point A = limiting friction (F_l).
- (3) Beyond A , the force of friction is seen to decrease slightly.
The portion BC = kinetic friction (F_k).
- (4) As the portion BC of the curve is parallel to x -axis therefore kinetic friction does not change with the applied force.



3.23 Angle of Friction

Angle of friction may be defined as the angle which the resultant of limiting friction and normal reaction makes with the normal reaction.

By definition angle θ is called the angle of friction $\tan \theta = \frac{F}{R}$



$$\therefore \tan \theta = \mu \quad \left[\text{As we know } \frac{F}{R} = \mu \right]$$

$$\text{or} \quad \theta = \tan^{-1}(\mu)$$

3.24 Angle of Repose

Angle of repose is defined as the angle of the inclined plane with horizontal such that a body placed on it is just begins to slide.

If α is called the angle of repose.

$\alpha = \theta$ i.e. angle of repose = angle of friction.

4. Work, Energy and Power

4.1 Introduction

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

4.2 Work Done by a Constant Force

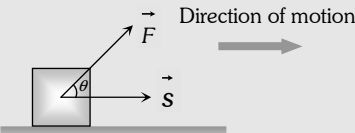
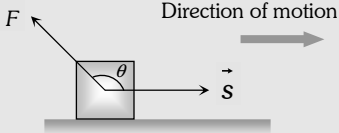
Let a constant force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s

Then work done by the force in displacing the body through a distance s is given by

$$W = (F \cos \theta) s = Fs \cos \theta \quad W = (F \cos \theta) s = Fs \cos \theta$$

or
$$W = \vec{F} \cdot \vec{s}$$

4.3 Nature of Work Done

Positive work	Negative work
Positive work means that force (or its component) is parallel to displacement  $0^\circ \leq \theta < 90^\circ$ The positive work signifies that the external force favours the motion of the body.	Negative work means that force (or its component) is opposite to displacement i.e.  $90^\circ < \theta \leq 180^\circ$ The negative work signifies that the external force opposes the motion of the body.

4.4 Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F} \cdot d\vec{s}$

The total work done in going from A to B is $W = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B (F \cos \theta) ds$

Area under force displacement curve with proper algebraic sign represents work done by the force.

4.5 Dimension and Units of Work

Dimension: As work = Force $\times$ displacement

$$\begin{aligned} \therefore [W] &= [\text{Force}] \times [\text{Displacement}] \\ &= [MLT^{-2}] \times [L] = [ML^2T^{-2}] \end{aligned}$$

Units: The units of work are of two types 1 Joule = 1 Newton $\times$ 1 metre (SI unit) Erg [C.G.S.] 1Erg = 1 Dyne $\times$ 1 cm.

4.6 Work Depends on Frame of Reference

With change of frame of reference (inertial) force does not change while displacement may change. So the work done by a force will be different in different frames.

Examples: If a person is pushing a box inside a moving train, the work done in the frame of train will be $\vec{F} \cdot \vec{s}$ while in the frame of earth will be $\vec{F} \cdot (\vec{s} + \vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground.

4.7 Energy

The energy of a body is defined as its capacity for doing work.

- (1) It is a scalar quantity.
- (2) Dimension: $[ML^2T^{-2}]$ it is same as that of work or torque.
- (3) Units: *Joule* [S.I.], *erg* [C.G.S.]
 Practical units: *electron volt* (eV), *Kilowatt hour* (KWh), *Calories* (Cal)
 Relation between different units: $1 \text{ Joule} = 10^7 \text{ erg}$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joule}$$

$$1 \text{ KWh} = 3.6 \times 10^6 \text{ Joule}$$

$$1 \text{ Calorie} = 4.18 \text{ Joule}$$

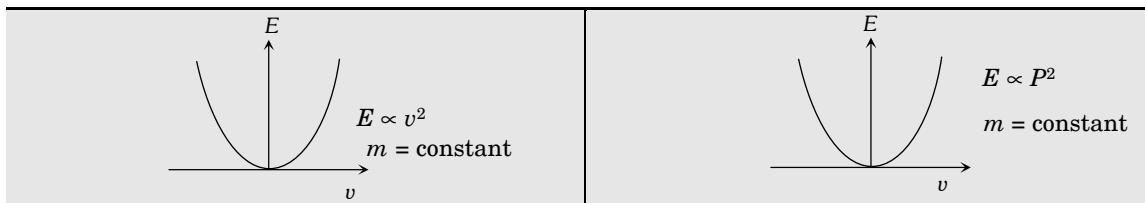
- (4) Mass energy equivalence: The relation between the mass of a particle m and its equivalent energy is given as $E = mc^2$ where c = velocity of light in vacuum.

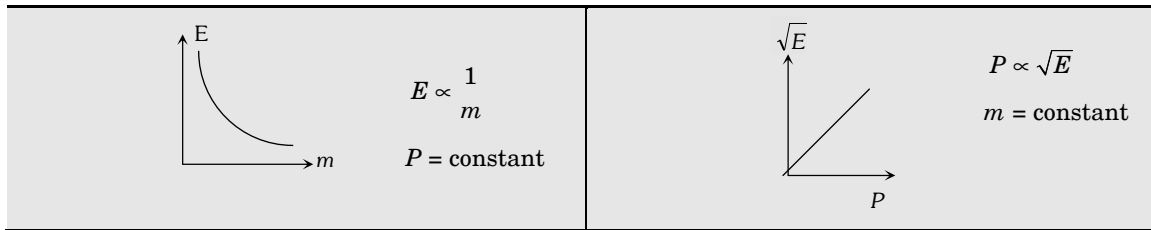
4.8 Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

Let m = mass of the body, v = velocity of the body $W = \frac{1}{2}mv^2$

- (1) **Kinetic energy depends on frame of reference:** The kinetic energy of a person of mass m , sitting in a train moving with speed v , is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.
- (2) **Work-energy theorem:** It states that work done by a force acting on a body is equal to the change produced in the kinetic energy of the body.
 This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).
- (3) **Relation of kinetic energy with linear momentum:** As we know Momentum $P = \frac{2E}{v} = \sqrt{2mE}$.
- (4) **Various graphs of kinetic energy**





4.9 Stopping of Vehicle by Retarding Force

- (1) **Comparison of stopping distance and time for two vehicles:** Two vehicles of masses m_1 and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same retarding force (F).

The ratio of their stopping distances $\frac{x_1}{x_2} = \frac{E_1}{E_2} = \frac{m_1 v_1^2}{m_2 v_2^2}$ and the ratio of their stopping time

$$\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{m_1 v_1}{m_2 v_2}$$

If vehicles possess same velocities		
$v_1 = v_2$	$\frac{x_1}{x_2} = \frac{m_1}{m_2}$	$\frac{t_1}{t_2} = \frac{m_1}{m_2}$
If vehicle possess same kinetic momentum		
$P_1 = P_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = \left(\frac{P_1^2}{2m_1} \right) \left(\frac{2m_2}{P_2^2} \right) = \frac{m_2}{m_1}$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = 1$
If vehicle possess same kinetic energy		
$E_1 = E_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = 1$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{\sqrt{2m_1 E_1}}{\sqrt{2m_2 E_2}} = \sqrt{\frac{m_1}{m_2}}$

4.10 Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types: Elastic potential energy, Electric potential energy and Gravitational potential energy *etc.*

- (1) **Change in potential energy:** Change in potential energy between any two points is defined in the terms of the work done by the force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = - \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = -W \quad \text{.....(i)}$$

- (2) **Potential energy curve:** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve. Negative gradient of the potential energy gives force.

$$\therefore -\frac{dU}{dx} = F$$

(5) **Types of equilibrium:** If net force acting on a particle is zero, it is said to be in equilibrium.

For equilibrium $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types:

Stable	Unstable	Neutral
When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$
$\frac{d^2U}{dx^2} = \text{positive}$ i.e. rate of change of $\frac{dU}{dx}$ is positive.	$\frac{d^2U}{dx^2} = \text{negative}$ i.e. rate of change of $\frac{dU}{dx}$ is negative.	$\frac{d^2U}{dx^2} = 0$ i.e. rate of change of $\frac{dU}{dx}$ is zero.
<i>Example:</i> A marble placed at the bottom of a hemispherical bowl.	<i>Example:</i> A marble balanced on top of a hemispherical bowl.	<i>Example:</i> A marble placed on horizontal table.

4.11 Elastic Potential Energy

- (1) **Restoring force and spring constant:** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x , then a restoring force is produced in the spring to bring it to the normal position.

According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.

$$\text{i.e.} \quad \vec{F} \propto -\vec{x}$$

$$\text{or} \quad \vec{F} = -k\vec{x} \quad \dots(i)$$

where k is called spring constant.

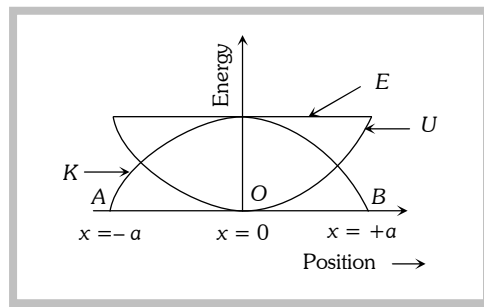
- (2) **Expression for elastic potential energy:**

$$\therefore \text{Elastic potential energy } U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$$

Note:

- If spring is stretched from initial position x_1 to final position x_2 then work done =

$$\text{Increment in elastic potential energy} = \frac{1}{2}k(x_2^2 - x_1^2)$$



- (3) **Energy graph for a spring:** It mean kinetic energy changes parabolically *w.r.t.* position but total energy remain always constant irrespective to position of the mass.

4.12 Work Done in Pulling the Chain Against Gravity

A chain of length L and mass M is held on a frictionless table with $(1/n)^{\text{th}}$ of its length hanging over the edge.

$$W = \frac{MgL}{2n^2}$$

4.13 Velocity of Chain While Leaving the Table

$$\therefore \text{Velocity of chain } v = \sqrt{gL \left(1 - \frac{1}{n^2} \right)}$$

4.14 Law of Conservation of Energy

- (1) **Law of conservation of energy:** For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.
- (2) **Law of conservation of total energy:** If the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical *etc.*, which is conserved.

4.15 Power

Power of a body is defined as the rate at which the body can do the work.

$$\text{Average power } (P_{\text{av.}}) = \frac{\Delta W}{\Delta t} = \frac{W}{t} . \text{ Instantaneous power } (P_{\text{inst.}}) = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} \quad [\text{As } dW = \vec{F} \cdot d\vec{s}]$$

$$P_{\text{inst}} = \vec{F} \cdot \vec{v} .$$

i.e. power is equal to the scalar product of force with velocity.

- (1) Dimension: $[P] = [ML^2T^{-3}]$

- (2) Units: *Watt* or *Joule/sec* [S.I.]

Practical units: *Kilowatt (kW)*, *Mega watt (MW)* and *Horse power (hp)*

Relations between different units: $1 \text{ watt} = 1 \text{ Joule / sec} = 10^7 \text{ erg / sec}$

$$1 \text{ hp} = 746 \text{ Watt}$$

- (3) The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan \theta$

- (4) Area under power time curve gives the work done as $P = \frac{dW}{dt}$

$$\therefore W = \int P dt$$

$$\therefore W = \text{Area under } P\text{-}t \text{ curve}$$

4.16 Collision

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch

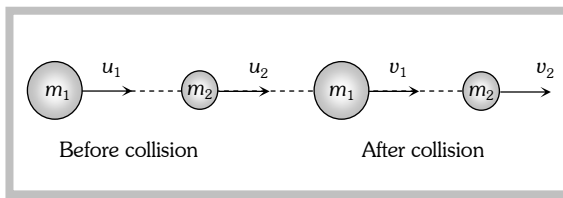
(3) **Types of collision:** (i) On the basis of conservation of kinetic energy.

Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to be inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
$(KE)_{\text{final}} = (KE)_{\text{initial}}$	Here kinetic energy appears in other forms. In some cases $(KE)_{\text{final}} < (KE)_{\text{initial}}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{\text{final}} > (KE)_{\text{initial}}$ such as when internal energy stored in the colliding particles is released.	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
<i>Examples:</i> (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	<i>Examples:</i> (1) Collision between two billiard balls. (2) Collision between two automobile on a road. In fact all majority of collision belong to this category.	<i>Example:</i> Collision between a bullet and a block of wood into which it is fired. When the bullet remains embedded in the block.

4.17 Perfectly Elastic Head on Collision

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.

According to law of conservation of momentum and conservation of kinetic energy.



Note:

- The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution.

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \text{or} \quad v_2 - v_1 = e(u_1 - u_2)$$

- For perfectly elastic collision $e = 1$ $\therefore v_2 - v_1 = u_1 - u_2$ (As shown in eq. (vi))
- For perfectly inelastic collision $e = 0$ $\therefore v_2 - v_1 = 0$ or $v_2 = v_1$

It means that two body stick together and move with same velocity.

- For inelastic collision $0 < e < 1$ $\therefore v_2 - v_1 = e(u_1 - u_2)$

In short we can say that e is the degree of elasticity of collision and it is dimension less quantity.

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \frac{2m_2 u_2}{m_1 + m_2} \quad \dots\dots(\text{vii})$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) u_2 + \frac{2m_1 u_1}{m_1 + m_2} \quad \dots\dots(\text{viii})$$

- When two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

- (2) **Kinetic energy transfer during head on elastic collision.** Fractional decrease in kinetic energy

$$\frac{\Delta K}{K} = \frac{4m_1 m_2}{(m_1 - m_2)^2 + 4m_1 m_2} \quad \dots\dots(\text{iv})$$

Note:

- Greater the difference in masses less will be transfer of kinetic energy and vice versa
 □ Transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal.

4.18 Perfectly Elastic Oblique Collision

Collision is said to be elastic oblique if after collision directions of Bodies are not along a straight line.

4.19 Head on Inelastic Collision

- (1) **Velocity after collision:** Let two bodies A and B collide inelastically and coefficient of restitution is e .

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2} \right) u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2} \right) u_2$$

Similarly

$$v_2 = \left(\frac{(1+e)m_1}{m_1 + m_2} \right) u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2} \right) u_2$$

- (2) **Ratio of velocities after inelastic collision:** A sphere of mass m moving with velocity u hits in elastically with another stationary sphere of same mass.

$$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

- (3) **Loss in kinetic energy:** Loss $(\Delta K) = \frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) (1-e^2)(u_1 - u_2)^2$

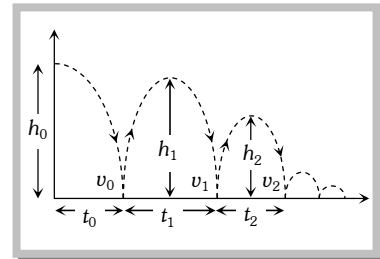
4.20 Rebounding of Ball After Collision With Ground

If a ball is dropped from a height h on a horizontal floor, then it strikes with the floor with a speed.

$$v_0 = \sqrt{2gh_0}$$

and it rebounds from the floor with a speed

$$v_1 = e v_0 = e\sqrt{2gh_0}$$



$$\left[\text{As } e = \frac{\text{velocity after collision}}{\text{velocity before collision}} \right]$$

- (1) **Height of the ball after n^{th} rebound:** The height after n^{th} rebound will be

$$\therefore h_n = e^{2n} h_0$$

- (3) **Total distance travelled by the ball before it stops bouncing:** $H = h_0 \left[\frac{1+e^2}{1-e^2} \right]$

- (4) **Total time taken by the ball to stop bouncing**

$$\therefore T = \left(\frac{1+e}{1-e} \right) \sqrt{\frac{2h_0}{g}}$$

5. Rotational Motion

5.1 Introduction

Rigid body: A rigid body is a body that can rotate with all the parts locked together and without any change in its shape.

5.2 Centre of Mass

Centre of mass of a system is a point that moves as though all the mass were concentrated there and all external forces were applied there.

- (1) **Position vector of centre of mass for n particle system:** If a system consists of n particles of masses $m_1, m_2, m_3, \dots, m_n$, whose positions vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ respectively then position vector of centre of mass

$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

If two masses are equal *i.e.* $m_1 = m_2$, then position vector of centre of mass $\vec{r} = \frac{\vec{r}_1 + \vec{r}_2}{2}$

- (2) **Important points about centre of mass**

- (i) The position of centre of mass is independent of the co-ordinate system chosen.
- (ii) The position of centre of mass depends upon the shape of the body and distribution of mass.
- (iii) In symmetrical bodies in which the distribution of mass is homogenous, the centre of mass coincides with the geometrical centre or centre of symmetry of the body.

Centre of mass of Cone or pyramid lie on the axis of the cone at point distance $\frac{3h}{4}$

from the vertex where h is the height of cone

- (iv) The centre of mass changes its position only under the translatory motion. There is no effect of rotatory motion on centre of mass of the body.
- (v) If the origin is at the centre of mass, then the sum of the moments of the masses of the system about the centre of mass is zero *i.e.* $\sum m_i \vec{r}_i = 0$.
- (vi) If a system of particles of masses $m_1, m_2, m_3, \dots$ move with velocities $v_1, v_2, v_3, \dots$ then the velocity of centre of mass $v_{cm} = \frac{\sum m_i v_i}{\sum m_i}$.
- (vii) If a system of particles of masses $m_1, m_2, m_3, \dots$ move with accelerations $a_1, a_2, a_3, \dots$ then the acceleration of centre of mass $A_{cm} = \frac{\sum m_i a_i}{\sum m_i}$.

(viii) If $\vec{r}$ is a position vector of centre of mass of a system then velocity of centre of mass

$$\vec{v}_{cm} = \frac{d\vec{r}}{dt}$$

(ix) Acceleration of centre of mass $\vec{A}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{d^2\vec{r}}{dt^2}$

(x) Force on a rigid body $\vec{F} = M \vec{A}_{cm} = M \frac{d^2\vec{r}}{dt^2}$

(xi) For an isolated system external force on the body is zero $\vec{F} = M \frac{d}{dt} \left(\vec{v}_{cm} \right) = 0$

$$\Rightarrow \vec{v}_{cm} = \text{constant}.$$

i.e., centre of mass of an isolated system moves with uniform velocity along a straight-line path.

5.3 Angular Displacement

It is the angle described by the position vector $\vec{r}$ about the axis of rotation.

$$\text{Angular displacement } (\theta) = \frac{\text{Linear displacement } (s)}{\text{Radius } (r)}$$

(1) Unit : radian

(2) Dimension : $[M^0 L^0 T^0]$

(3) Vector form $\vec{S} = \vec{\theta} \times \vec{r}$

i.e., angular displacement is a vector quantity whose direction is given by right hand rule. It is also known as axial vector. For anti-clockwise sense of rotation direction of θ is perpendicular to the plane, outward and along the axis of rotation and vice-versa.

(4) 2π radian = $360^\circ = 1$ revolution.

5.4 Angular Velocity

The angular displacement per unit time is defined as angular velocity.

If a particle moves from P to Q in time Δt , $\omega = \frac{\Delta\theta}{\Delta t}$ where $\Delta\theta$ is the angular displacement.

(1) Instantaneous angular velocity $\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

(2) Unit : *Radian/sec*

(3) Dimension : $[M^0 L^0 T^{-1}]$ which is same as that of frequency.

(4) Vector form $\vec{v} = \vec{\omega} \times \vec{r}$ [where $\vec{v}$ = linear velocity, $\vec{r}$ = radius vector] $\vec{\omega}$ is a axial vector, whose direction is normal to the rotational plane and its direction is given by right hand screw rule.

(5) $\omega = \frac{2\pi}{T} = 2\pi n$ [where T = time period, n = frequency]

5.5 Angular Acceleration

The rate of change of angular velocity is defined as angular acceleration.

If particle has angular velocity ω_1 at time t_1 and angular velocity ω_2 at time t_2 then,

$$\text{Angular acceleration } \vec{\alpha} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$$

$$(1) \text{ Instantaneous angular acceleration } \vec{\alpha} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}.$$

$$(2) \text{ Unit : rad/sec}^2$$

$$(3) \text{ Dimension : } [M^0 L^0 T^{-2}].$$

$$(4) \text{ If } \alpha = 0, \text{ circular or rotational motion is said to be uniform.}$$

$$(5) \text{ Relation between angular acceleration and linear acceleration } \vec{a} = \vec{\alpha} \times \vec{r}.$$

$$(6) \text{ It is an axial vector whose direction is along the change in direction of angular velocity i.e. normal to the rotational plane, outward or inward along the axis of rotation (depends upon the sense of rotation).}$$

5.6 Equations of Linear Motion and Rotational Motion

Rotational Motion	
If angular acceleration is 0, $\omega = \text{constant}$ and $\theta = \omega t$	
If angular acceleration $\alpha = \text{constant}$ then	
(i)	$\theta = \frac{(\omega_1 + \omega_2)}{2} t$
(ii)	$\alpha = \frac{\omega_2 - \omega_1}{t}$
(iii)	$\omega_2 = \omega_1 + \alpha t$
(iv)	$\theta = \omega_1 t + \frac{1}{2} \alpha t^2$
(v)	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$
(vi)	$\theta_{nth} = \omega_1 + (2n-1) \frac{\alpha}{2}$
If acceleration is not constant, the above equation will not be applicable. In this case	
(i)	$\omega = \frac{d\theta}{dt}$

$$(ii) \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$(iii) \omega d\omega = \alpha d\theta$$

5.7 Moment of Inertia

Moment of inertia plays the same role in rotational motion as mass plays in linear motion. It is the property of a body due to which it opposes any change in its state of rest or of uniform rotation.

- (1) Moment of inertia of a particle $I = mr^2$; where r is the perpendicular distance of particle from rotational axis.
- (2) Moment of inertia of a body made up of number of particles (discrete distribution)

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$
- (3) Moment of inertia of a continuous distribution of mass, $dI = dm r^2$ i.e., $I = \int r^2 dm$
- (4) Dimension : $[ML^2T^0]$
- (5) S.I. unit : kgm^2 .
- (6) Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation.
- (7) Moment of inertia is a tensor quantity.

5.8 Radius of Gyration

Radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis, where if whole mass of the body were concentrated, the body shall have the same moment of inertia as it has with the actual distribution of mass.

When square of radius of gyration is multiplied with the mass of the body gives the moment of inertia of the body about the given axis.

$$I = Mk^2 \text{ or } k = \sqrt{\frac{I}{M}}$$

Here k is called radius of gyration.

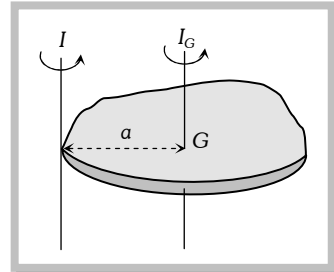
$$\therefore k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}}$$

Note:

- For a given body inertia is constant whereas moment of inertia is variable.

5.9 Theorem of Parallel Axes

Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through centre of mass of the body I_g and



Ma^2 where M is the mass of the body and a is the perpendicular distance between the two axes.

$$I = I_g + Ma^2$$

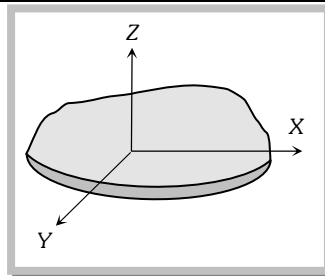
5.10 Theorem of Perpendicular Axes

According to this theorem the sum of moment of inertia of a plane lamina about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis perpendicular to the plane of lamina and passing through the point of intersection of first two axes.

$$I_z = I_x + I_y$$

Note:

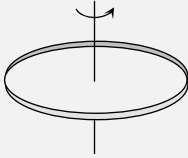
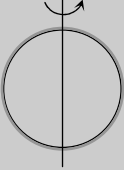
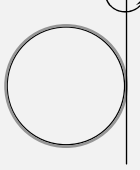
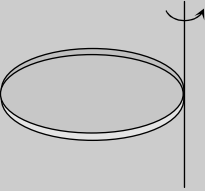
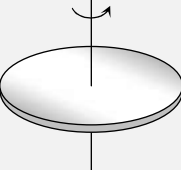
- In case of symmetrical two-dimensional bodies as moment of inertia for all axes passing through the centre of mass and in the plane of body will be same so the two axes in the plane of body need not be perpendicular to each other.

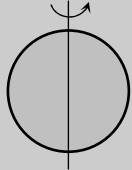
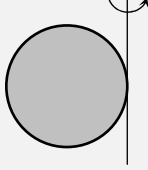
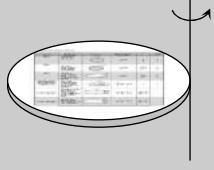
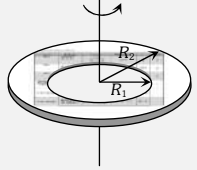
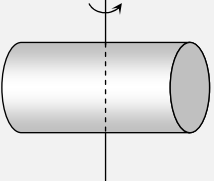
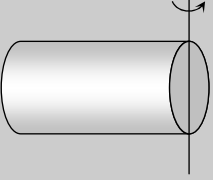


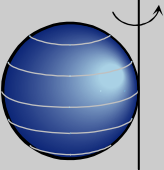
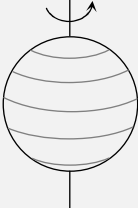
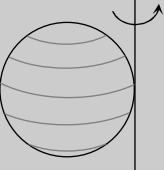
5.12 Analogy between Translatory Motion and Rotational Motion

Translatory motion		Rotatory motion	
Mass	(m)	Moment of Inertia	(I)
Linear momentum	$P = mv$ $P = \sqrt{2mE}$	Angular Momentum	$L = I\omega$ $L = \sqrt{2IE}$
Force	$F = ma$	Torque	$\tau = I\alpha$
Kinetic energy	$E = \frac{1}{2}mv^2$ $E = \frac{P^2}{2m}$		$E = \frac{1}{2}I\omega^2$ $E = \frac{L^2}{2I}$

5.13 Moment of Inertia of Some Standard Bodies about Different Axes

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Ring (Cylindrical shell)	About an axis passing through C.G. and perpendicular to its plane		MR^2	R	1
Ring	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2
Disc (Solid cylinder)	About an axis passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R_2
Disc	About its Diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Annular disc inner radius = R_1 and outer radius = R_2	Passing through the centre and perpendicular to the plane		$\frac{M}{2}[R_1^2 + R_2^2]$	—	—
Solid cylinder	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{12} + \frac{R^2}{4}}$	
Solid cylinder	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{3} + \frac{R^2}{4}}$	

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Solid Sphere	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
Solid sphere	About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
Spherical shell	About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
Spherical shell	About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$
Long thin rod	About an axis passing through its centre of mass and perpendicular to the rod.		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	

5.14 Torque

If the particle rotating in xy plane about the origin under the effect of force $\vec{F}$ and at any instant the position vector of the particle is $\vec{r}$ then,

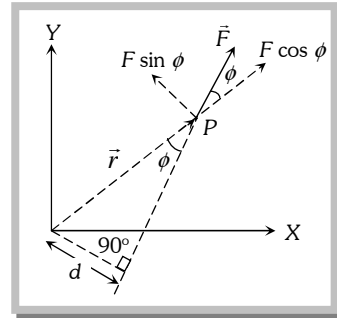
$$\text{Torque } \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r F \sin \phi$$

[where ϕ is the angle between the direction of $\vec{r}$ and $\vec{F}$]

- (1) Torque is an axial vector. *i.e.*, its direction is always perpendicular to the plane containing vector $\vec{r}$ and $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

i.e. Torque = Force $\times$ Perpendicular distance of line of action of force from the axis of rotation.
Torque is also called as moment of force and d is called moment or lever arm.

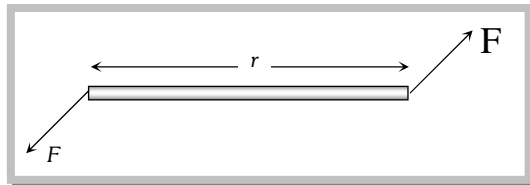


- (2) Unit : *Newton-metre* (M.K.S.) and *Dyne-cm* (C.G.S.)
(3) Dimension : $[ML^2T^{-2}]$.
(4) A body is said to be in rotational equilibrium if resultant torque acting on it is zero *i.e.* $\Sigma \vec{\tau} = 0$.
(5) Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translatory motion *i.e.*, torque is rotational analogue of force. This all is evident from the following correspondences between rotatory and translatory motion.

Rotatory Motion	Translatory Motion
$\vec{\tau} = I \vec{\alpha}$	$\vec{F} = m \vec{a}$
$W = \int \vec{\tau} \cdot d\vec{\theta}$	$W = \int \vec{F} \cdot d\vec{s}$
$P = \vec{\tau} \cdot \vec{\omega}$	$P = \vec{F} \cdot \vec{v}$
$\vec{\tau} = \frac{d\vec{L}}{dt}$	$\vec{F} = \frac{d\vec{P}}{dt}$

5.15 Couple

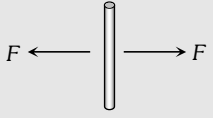
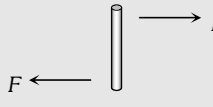
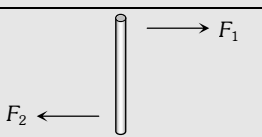
- (1) A couple is defined as combination of two equal but oppositely directed force not acting along the same line. The effect of couple is known by its moment of couple or torque by a couple $\vec{\tau} = \vec{r} \times \vec{F}$.



(2) Work done by torque in twisting the wire $W = \frac{1}{2} C \theta^2$.

Where $\tau = C \theta$; C is known as twisting coefficient or couple per unit twist.

5.16 Translatory and Rotatory Equilibrium

Forces are equal and act along the same line.		$\Sigma F = 0$ and $\Sigma \tau = 0$	Body will remain stationary if initially it was at rest.
Forces are equal and does not act along the same line.		$\Sigma F = 0$ and $\Sigma \tau \neq 0$	Rotation <i>i.e.</i> spinning.
Forces are unequal and act along the same line.		$\Sigma F \neq 0$ and $\Sigma \tau = 0$	Translation <i>i.e.</i> slipping or skidding.
Forces are unequal and does not act along the same line.		$\Sigma F \neq 0$ and $\Sigma \tau \neq 0$	Rotation and translation both <i>i.e.</i> rolling.

5.17 Angular Momentum

The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If $\vec{P}$ is the linear momentum of particle and $\vec{r}$ its position vector from the point of rotation then angular momentum.

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = r P \sin \phi \hat{n}$$

Angular momentum is an axial vector *i.e.* always directed perpendicular to the plane of rotation and along the axis of rotation.

- (1) S.I. Unit : $kg \cdot m^2 \cdot s^{-1}$ or $J \cdot sec$.
- (2) Dimension : $[ML^2T^{-1}]$ and it is similar to Planck's constant (h).
- (3) Angular momentum = (Linear momentum) $\times$ (Perpendicular distance of line of action of force from the axis of rotation)
- (4) In vector form $\vec{L} = I \vec{\omega}$
- (5) From $\vec{L} = I \vec{\omega} \therefore \frac{d\vec{L}}{dt} = I \frac{d\vec{\omega}}{dt} = I \vec{\alpha} = \vec{\tau}$

[Rotational analogue of Newton's second law]

- (6) If a large torque acts on a particle for a small time then 'angular impulse' of torque is

$$\text{given by } \vec{J} = \int \vec{\tau} dt = \vec{\tau}_{av} \int_{t_1}^{t_2} dt$$

$\therefore$ Angular impulse = Change in angular momentum

5.18 Law of Conservation of Angular Momentum

If the net external torque on a particle (or system) is zero then $\frac{d\vec{L}}{dt} = 0$

$$\text{i.e. } \vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots = \text{constant.}$$

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As $L = I\omega$ so if $\vec{\tau} = 0$ then $I\omega = \text{constant.}$

5.19 Work, Energy and Power for Rotating Body

- (1) **Work** : If the body is initially at rest and angular displacement is $d\theta$ due to torque then work done on the body.

$$W = \int \tau d\theta$$

- (2) **Kinetic energy** : The energy, which a body has by virtue of its rotational motion is called rotational kinetic energy.

Rotational kinetic energy	Analogue to translatory kinetic energy
$K_R = \frac{1}{2} I \omega^2$	$K_T = \frac{1}{2} m v^2$
$K_R = \frac{1}{2} L \omega$	$K_T = \frac{1}{2} P v$
$K_R = \frac{L^2}{2I}$	$K_T = \frac{P^2}{2m}$

- (3) **Power**: Rate of change of kinetic energy is defined as power

In vector form Power = $\vec{\tau} \cdot \vec{\omega}$

5.20 Slipping, Spinning and Rolling

- (1) **Slipping** : When the body slides on a surface without rotation then its motion is called slipping motion.

In this condition friction between the body and surface $F = 0$.

Body possess only translatory kinetic energy $K_T = \frac{1}{2}mv^2$.

- (2) **Spinning** : When the body rotates in such a manner that its axis of rotation does not move then its motion is called spinning motion. In this condition axis of rotation of a body is fixed.

In spinning, body possess only rotatory kinetic energy $K_R = \frac{1}{2}I\omega^2$.

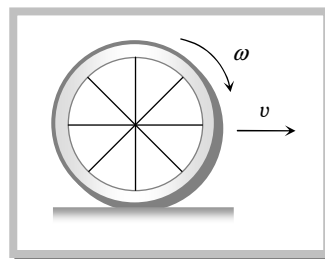
- (3) **Rolling** : If in case of rotational motion of a body about a fixed axis, the axis of rotation also moves, the motion is called combined translatory and rotatory.

Example : (i) Motion of a wheel of cycle on a road.
(ii) Motion of football rolling on a surface.

In this condition friction between the body and surface $F \neq 0$.

Body possesses both translational and rotational kinetic energy.

Net kinetic energy = (Translatory + Rotatory) kinetic energy.



5.21 Rolling Without Slipping

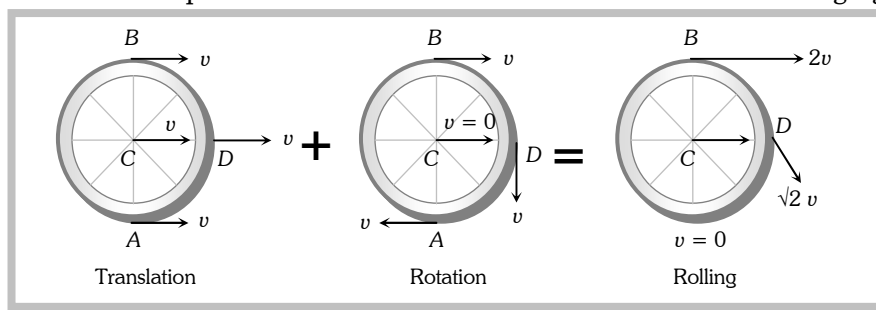
In case of combined translatory and rotatory motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.

Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact.

Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity ω . [$v = R\omega$]

Linear velocity of different points in rolling : In case of rolling, all points of a rigid body have same angular speed but different linear speed.

Let A, B, C and D are four points then their velocities are shown in the following figure.



5.22 Rolling on an Inclined Plane

When a body of mass m and radius R rolls down on inclined plane of height ' h ' and angle of inclination θ , it loses potential energy. However it acquires both linear and angular speeds and hence, gain kinetic energy of translation and that of rotation.

(1) Velocity at the lowest point : $v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$

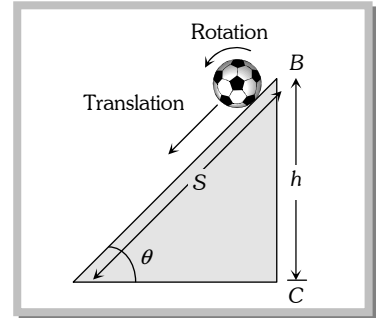
(2) Acceleration in motion : From equation $v^2 = u^2 + 2aS$

By substituting $u = 0$, $S = \frac{h}{\sin \theta}$ and $v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$ we get

$$a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$$

(3) Time of descent : From equation $v = u + at$

By substituting $u = 0$ and value of v and a from above expressions $t = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g} \left[1 + \frac{k^2}{R^2} \right]}$



5.25 Motion of Connected Mass

A point mass is tied to one end of a string which is wound round the solid body [cylinder, pulley, disc]. When the mass is released, it falls vertically downwards and the solid body rotates unwinding the string

m = mass of point-mass, M = mass of a rigid body

R = radius of a rigid body, I = moment of inertia of rotating body

(1) **Downwards acceleration of point mass** $a = \frac{g}{1 + \frac{I}{mR^2}}$

(2) **Tension in string** $T = mg \left[\frac{I}{I + mR^2} \right]$

(3) **Velocity of point mass** $v = \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}}$

(4) **Angular velocity of rigid body** $\omega = \sqrt{\frac{2mgh}{I + mR^2}}$

