7

ALTERNATING CURRENT

2.1 Introduction

Earlier, we have discussed about D.C. voltage and D.C. current. We also studied in Chapter-1 about the A.C. dynamo or the generator which is the device used for the production of A.C. voltage. In this chapter we will discuss about alternating current (A.C.). We use A.C voltage in our home, office or industries.

In this chapter we will analyse some simple A.C. circuits and then will study about an electrical device. A.C. voltage and current are taken as varying according to the sin ωt or cos ωt function. We should remember that they are not varying according to only sine or cosine functions. In future you will learn that they can change periodically with time in many other ways.

2.2 A.C. Circuit with Series Combination of Inductor, Capacitor and Resistor (L-C-R A.C. series circuit)

As shown in the figure 2.1, an inductor (L) having zero ohmic resistance, a capacitor with capacitance (C) and a resistor (R) with zero inductance are joined in series with the source of A.C. voltage.

Here the voltage from the source varies with time according to $V = V_m \cos\omega t$. (2.1.1)

At some time t, let the current passing in the circuit be = I

Charge deposited on the capacitor = Q

The rate of change of current = $\frac{dI}{dt}$

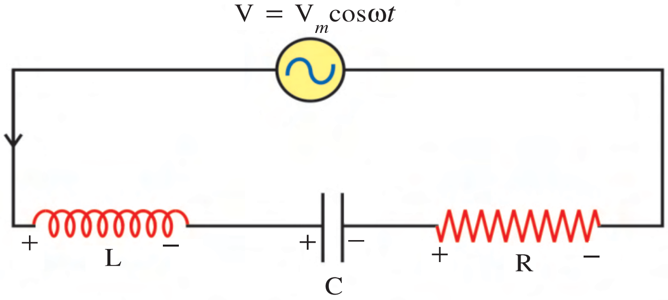


Figure 2.1 L-C-R A.C. Series Circuit

As a result, the potential difference between two ends of inductor is $V_L = L\frac{dI}{dt}$.

Potential difference between two ends of capacitor is $V_C = \frac{Q}{C}$.

The potential difference between two ends of resistor is $V_R = IR$.

According to Kirchoff's second law,

$V_L + V_C + V_R = V$.

$$L\frac{dI}{dt} + \frac{Q}{C} + IR = V_m \cos\omega t \qquad (2.2.2)$$

But, $I = \frac{dQ}{dt}$ and $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

$$\therefore L\frac{d^2Q}{dt^2} + \frac{Q}{C} + \frac{dQ}{dt}R = V_m \cos\omega t$$

$$\therefore \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = \frac{V_m}{L}\cos\omega t \qquad (2.2.3)$$

This is the differential equation for the charge Q, for this A.C. circuit. It resembles with the equatio,

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F_0}{m}\sin\omega t \qquad (2.2.4)$$

For the forced oscillations in mechanics which you studied in standard 11. This equation contains mechanical quantities while in the differential equation of LCR series A.C. circuit there are electrical quantities. Equations (2.2.3) and (2.2.4) are differential equations of similar type. As these equations have cosine and sine functions they are harmonic functions.

By comparing the above equations the equivalence between the mechanical quantities and electrical quantities can be seen in the following Table 2.1 :

Table 2.1 Equivalence between the Mechanical and Electrical Quantities

Number	Mechanical Quantity	Electrical Quantity
1	Displacement (y)	Electric charge (Q)
2	Velocity $\left(\frac{dy}{dt} = v\right)$	Electric current $\left(\frac{dQ}{dt} = I\right)$
3	Resistive coefficient (b)	Resistance R
4	Mass (m)	Inductance (L)
5	Force constant (k)	Inverse of capacitance $\left(\frac{1}{C}\right)$
6	Angular frequency $\left(\sqrt{\frac{k}{m}}\right)$	Angular Frequency $\left(\sqrt{\frac{1}{LC}}\right)$
7	Periodic Force	Periodic Voltage

Equation (2.2.3) is the differential equation for electric charge Q, in the A.C. circuit. The time-dependent function of Q, which satisfies the equation (2.2.3) is called the solution of equation (2.2.3). To obtain such solution complex functions are used. (Complex number and complex function are explained in the Appendix-A at the end of the chapter. It is only for information.)

2.3 Solution of the Differential Equation of Q for L-C-R Series A.C. circuit

Equation (2.2.2) can be written as $\frac{dI}{dt} + \frac{R}{L}I + \frac{1}{LC}\int I dt = \frac{V_m}{L}\cos\omega t$. (2.3.1)

Here we have taken $Q = \int I dt$

The solution of the above equation can be obtained by using complex number. Since $\cos\omega t$ is the real part of complex number $e^{j\omega t}$ the real part of solution which we shall obtain, will become the solution of equation (2.3.1). Moreover electric current I will have to be taken as a complex number. Expressing current I by complex current i,

$$\frac{di}{dt} + \frac{R}{L}i + \frac{1}{LC}\int i dt = \frac{V_m}{L}e^{j\omega t} \qquad (2.3.2)$$

We should remember that R, L and C are only real numbers.

On the right hand side of equation (2.3.2) there is a harmonic function of time, hence the complex current I, would also be a harmonic function of time. Hence the solution of equation (2.3.2) can be written as,

$$i = i_m e^{j\omega t} \qquad (2.3.3)$$

$$\therefore \frac{di}{dt} = i_m j\,\omega\, e^{j\omega t} \qquad (2.3.4)$$

$$\text{and, } \int i\, dt = \frac{i_m e^{j\omega t}}{j\omega} \qquad (2.3.5)$$

Using equations (2.3.3), (2.3.4) and (2.3.5) in equation (2.3.2), we get

$$i_m j\,\omega\, e^{j\omega t} + \frac{R}{L} i_m e^{j\omega t} + \frac{1}{LC}\frac{i_m e^{j\omega t}}{j\omega} = \frac{V_m}{L}e^{j\omega t}$$

$$\therefore i_m\left(j\omega + \frac{R}{L} + \frac{1}{j\omega LC}\right) = \frac{V_m}{L}$$

Multiplying both the sides by L and writing $\frac{1}{j} = \frac{j}{j^2} = -j$, we get

$$i_m\left(j\omega L + R - \frac{j}{\omega C}\right) = V_m$$

$$\therefore\ i_m = \frac{V_m}{R + j\omega L - \frac{j}{\omega C}} \qquad (2.3.6)$$

Substituting this value of i_m in equation (2.3.3), we get,

$$i = \frac{V_m e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \qquad (2.3.7)$$

This equation shows the relation between complex current i and complex voltage $V_m e^{j\omega t}$ and has the same form as the equation $I = \frac{V}{R}$ which represents Ohm's law. Thus Ohm's law is obeyed by the instantaneous voltage and current.

From this, it can be seen that whatever effect is produced by the resistance R on the current, similar effects produced by the inductor and the capacitor are obtained respectively by $j\omega L$ and $\frac{-j}{\omega C}$. That is, $j\omega L$ and $\frac{-j}{\omega C}$ can be called the effective resistances of the inductor and the capacitor. $j\omega L$ is called the inductive reactance and $\frac{-j}{\omega C}$ is called the capacitive reactance of the capacitor. Their symbols are Z_L and Z_C respectively. Their magnitudes are respectively ωL and $\frac{j}{\omega C}$ and their symbols are X_L and X_C. Thus,

$$Z_L = j\omega L \tag{2.3.8}$$

$$X_L = \omega L \tag{2.3.9}$$

$$Z_C = \frac{-j}{\omega C} \tag{2.3.10}$$

$$X_C = \frac{1}{\omega C} \tag{2.3.11}$$

The summation of Z_L, Z_C and R is called the impedance (Z) of the present series circuit. Its unit is Ohm.

$$\therefore\ Z = R + Z_L + Z_C \tag{2.3.12}$$

$$\therefore\ Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{2.3.13}$$

Now equation (2.3.7) can be written as under :

$$i = \frac{V_m e^{j\omega t}}{Z} = \frac{\text{Voltage}}{\text{Effective resistance (Z)}} \tag{2.3.14}$$

This equation is Ohm's law with complex current, complex voltage and the impedance . Note that the impedance is also complex.

Now taking $Z = |Z|e^{j\delta}$, [See equation in Appendix-A]

$$i = \frac{V_m e^{j\omega t}}{|Z|e^{j\delta}} \tag{2.3.15}$$

$$= \frac{V_m}{|Z|} e^{j(\omega t - \delta)} = \frac{V_m}{|Z|}[\cos(\omega t - \delta) + j\sin(\omega t - \delta)] \tag{2.3.16}$$

$$\text{where, } |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{2.3.17}$$

$$\text{Now, } I = R_e(i) \tag{2.3.18}$$

$$\therefore\ I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V_m \cos(\omega t - \delta)}{|Z|} \tag{2.3.19}$$

In this circuit the current varies with time according to the equation (2.3.19) while the voltage varies according to $V = V_m \cos\omega t$. This shows that the current in the circuit lags in phase behind the voltage by δ. This fact is shown in the figure 2.2.

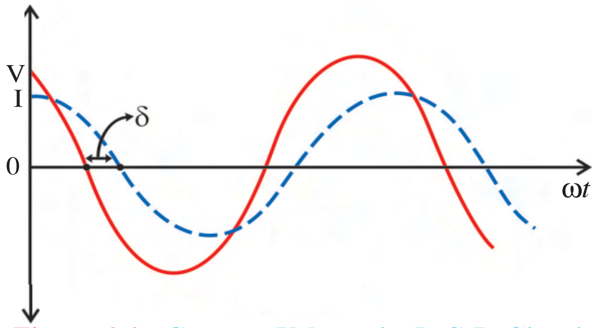


Figure 2.2 Current Voltage in L-C-R Circuit

The equation showing the complex impedance Z of the circuit is

$$Z = R + j\omega L - \frac{j}{\omega C} \qquad (2.3.20)$$

The real part of this complex number is R, shown on the real axis in the figure 2.3. The imaginary part is shown on the imaginary axis.

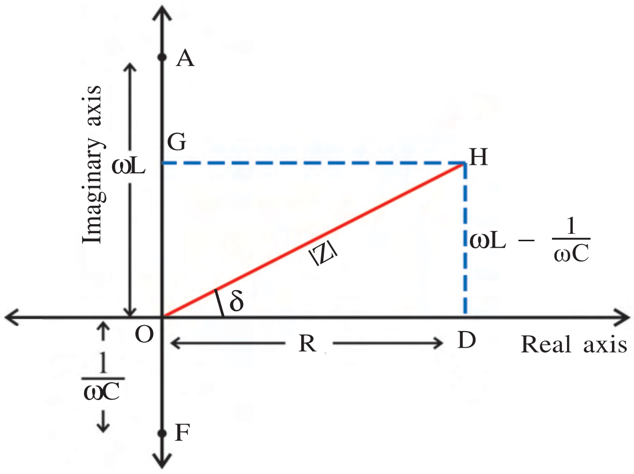


Figure 2.3 Geometrical Representation of Z

From this the point H showing the complex number Z is obtained. The impedance Z, given by equation (2.3.13) is represented in the complex plane in figure 2.3 as under,

In figure $OD = R$, $OA = \omega L$ and $OF = \frac{1}{\omega C}$

$\therefore\ OG = \omega L - \frac{1}{\omega C}$ = imaginary part of Z.

$OH = Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

From figure (2.3),

$$|Z| = \sqrt{OD^2 + DH^2} \qquad (2.3.21)$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad (2.3.22)$$

The phase difference δ is obtained from,

$$\therefore\ \tan\delta = \frac{HD}{OD} = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} \qquad (2.3.23)$$

Thus showing the impedance in the complex plane, values of δ and $|Z|$ can be easily found out geometrically. Moreover, since the values of ω, L, C and R are known using equations (2.3.22) and (2.3.23) respectively, values of $|Z|$ and δ can be obtained. Hence, the equation showing the relation between the current and the voltage can be written.

To find the impedance of the given circuit, we can use the same laws for $j\omega L$ and $-\frac{j}{\omega C}$ as those used for series and parallel combination of resistances R.

For different circuits the relations between the current and the voltage can be obtained with the help of above geometrical arrangements.

2.4 Different cases of A.C. Circuits

(1) A.C. Circuit with only Resistance : In LCR circuit when inductor (L) and capacitor (C) are absent, it will be a circuit with only resistance. In the equation

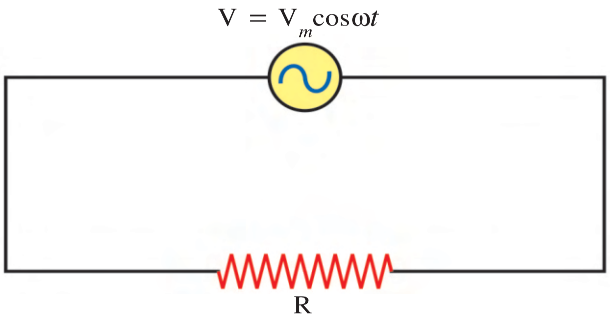


Figure 2.4 A.C. Circuit with only R

$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$ for LCR circuit, by substituting $\omega L = 0$ and $\frac{1}{\omega C} = 0$, we get $|Z| = R$ for this circuit and the value of δ from equation (2.3.23) would be zero. Thus, the equation (2.3.19) showing the relation between the current and voltage would be of the form.

$$I = \frac{V_m \cos \omega t}{R} \tag{2.4.1}$$

Thus, we can see that in the A.C. circuit with only resistance the phases of current and voltage are equal.

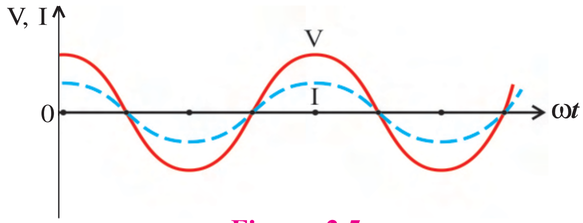


Figure 2.5

(2) A.C. Circuit with only Inductor : As seen earlier, a circuit with only inductor means in LCR circuit capacitor C and resistance R are absent. For this circuit $Z = j\omega L$ and $|Z| = \omega L = X_L$ (because $\frac{1}{\omega C} = 0$ and $R = 0$).

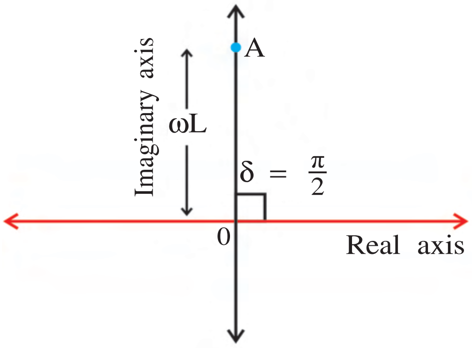


Figure 2.6

Z is shown by point A in the complex plane in figure 2.6.

Here OA makes an angle of $\frac{\pi}{2}$ with the real axis, which shows that $\delta = \frac{\pi}{2}$ and $OA = \omega L = |Z|$. Substituting values of $|Z|$ and δ in equation (2.3.19).

$$I = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{\omega L} = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{X_L} \tag{2.4.2}$$

This shows that current is lagging behind the voltage in phase by $\frac{\pi}{2}$.

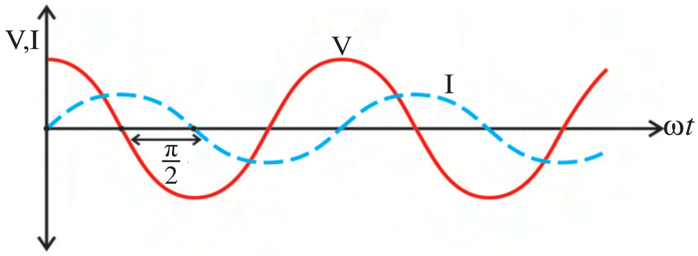


Figure 2.7

(3) A.C. Circuit with only Capacitor : In this case only capacitor is present, hence $Z = -\frac{j}{\omega C}$ and $|Z| = \frac{1}{\omega C} = X_C$ is shown by point F in the complex plane in figure 2.8. It is clear from the figure 2.8. that $\delta = -\frac{\pi}{2}$. Thus, the relation between the current and the voltage would be

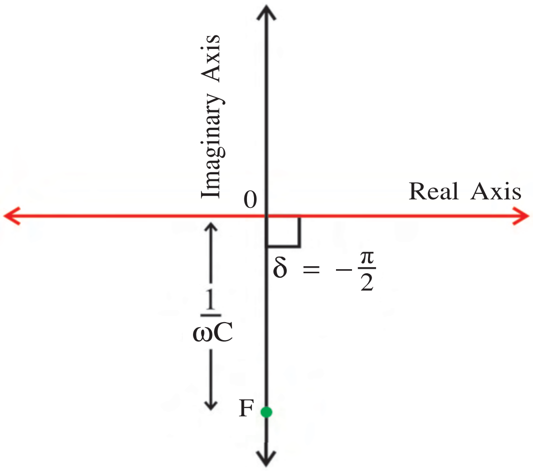


Figure 2.8

$$I = \frac{V_m \cos\left(\omega t + \frac{\pi}{2}\right)}{\left(\frac{1}{\omega C}\right)} = \frac{V_m \cos\left(\omega t + \frac{\pi}{2}\right)}{\frac{1}{\omega C}} \qquad (2.4.3)$$

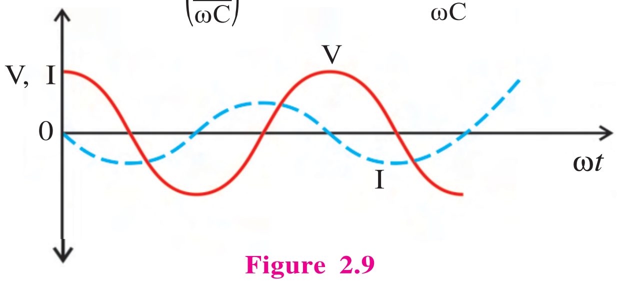


Figure 2.9

Thus, in A.C. circuit with capacitor, only current leads the voltage in phase by $\frac{\pi}{2}$. This fact is shown in the figure 2.9.

(4) A.C. Circuit with R and L Joined in Series : For this circuit, $Z = R + jX_L = R + j\omega L$ and $|Z| = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$. In the figure 2.10, Z is shown by point H in the complex plane. From the figure, it is clear that

$$\tan\delta = \frac{\omega L}{R}$$

$$\therefore\ \delta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right) \qquad (2.4.4)$$

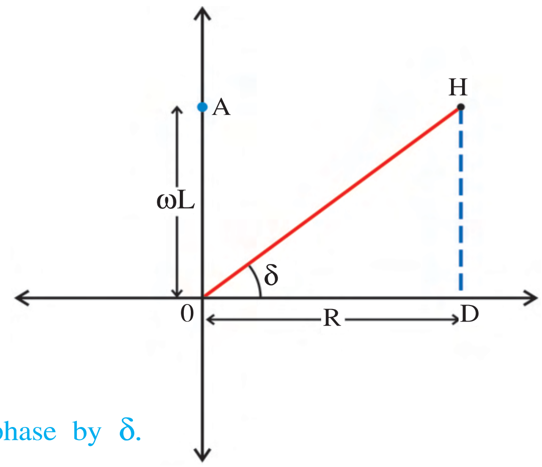


Figure 2.10

In this circuit current lags behind the voltage in phase by δ.

Here, the electric current $I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (\omega L)^2}}$ (2.4.5)

(5) A.C. Circuit with R and C Joined Series : For this circuit $Z = R - \frac{j}{\omega C} = R - jX_C$

$$\therefore\ |Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + X_C^2}$$

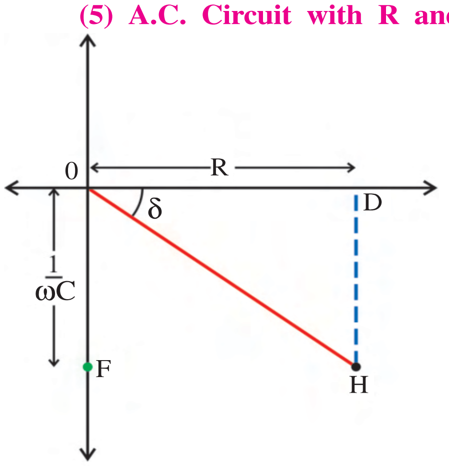


Figure 2.11

This Z is shown in figure 2.11 by point H. Here, as shown in the figure, δ is negative and has magnitude given by

$$\delta = \tan^{-1}\left(\frac{1}{\omega CR}\right) = \tan^{-1}\left(\frac{X_C}{R}\right) \qquad (2.4.6)$$

In this case the electric current leads the voltage in phase by δ. Here electric current

$$I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + (X_C)^2}} \tag{2.4.7}$$

(6) A.C. Circuit with L and C in Series : For this circuit $Z = j\omega L - \frac{j}{\omega C} = jX_L - jX_C$

$$\therefore |Z| = \omega L - \frac{1}{\omega C} = X_L - X_C$$

Assuming $\omega L > \frac{1}{\omega C}$, Z obtained here is shown in the complex plane by point G. Here, $\delta = \frac{\pi}{2}$. Thus, in L–C series circuit if $\omega L > \frac{1}{\omega C}$, the current lags behind the voltage in phase by $\frac{\pi}{2}$. (If $\omega L < \frac{1}{\omega C}$ then ? Think yourself)

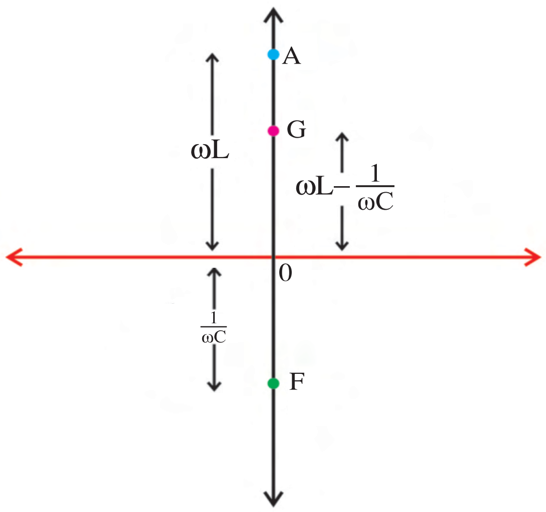


Figure 2.12

(7) A.C. Circuit with Parallel Combination of L and C, and R in Series with this Combination : A circuit with L and C joined in parallel and R in series with this combination is as shown in the figure 2.13.

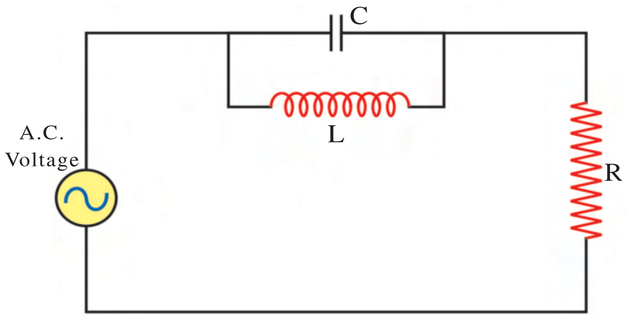


Figure 2.13

The effective impedance Z of this circuit can be obtained as follows using the laws of series-parallel combinations. If the impedance of the parallel combination of L and C is Z_1,

$$\frac{1}{Z_1} = \frac{1}{Z_C} + \frac{1}{Z_L} = \frac{1}{\frac{-j}{(\omega C)}} + \frac{1}{j\omega L} = j\left(\omega C - \frac{1}{\omega L}\right) \tag{2.4.8}$$

$$\therefore Z = \frac{1}{j\left(\omega C - \frac{1}{\omega L}\right)} = -\frac{j}{\left(\omega C - \frac{1}{\omega L}\right)} \tag{2.4.9}$$

Moreover, R and Z_1 are in series

$$\therefore Z = R + Z_1$$

$$\therefore Z = R - \frac{j}{\left(\omega C - \frac{1}{\omega L}\right)} \tag{2.4.10}$$

Assuming $\omega L > \frac{1}{\omega C}$, Z can be represented as shown in the figure 2.14.

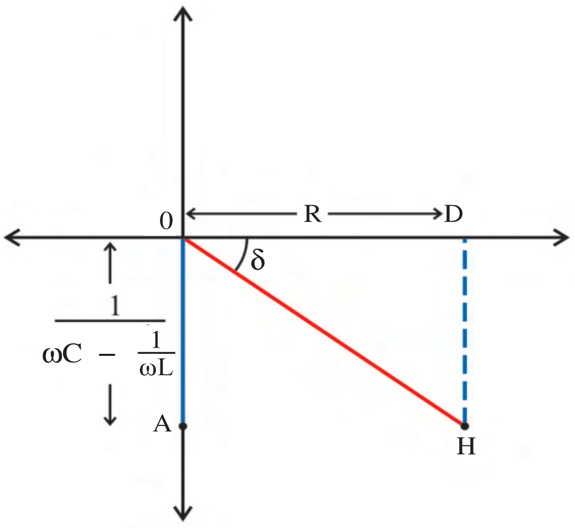


Figure 2.14

From equation (2.4.10),

$$|Z| = \sqrt{R^2 + \frac{1}{\left(\omega C - \frac{1}{\omega L}\right)^2}} \qquad (2.4.11)$$

From equations (2.3.19) and (2.4.12), we get electric current,

$$I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + \frac{1}{\left(\omega C - \frac{1}{\omega L}\right)^2}}} \qquad (2.4.12)$$

This equation shows the relation between the current and the voltage in the present circuit.

Here, $\tan\delta = \frac{HD}{OD} = \frac{1}{R\left(\omega C - \frac{1}{\omega L}\right)}$ (2.4.13)

2.5 r.m.s. Values of Voltage and Current

Till now we have seen equations like, $V = V_m \cos\omega t$ and $I = I_m \cos(\omega t \pm \delta)$ for voltage and current respectively. Here, V and I continuously vary with time periodically. In this condition by joining a simple voltmeter or ammeter properly in the circuit, it is not possible to measure voltage or current. If we try to find the average values of A.C. voltage or A.C. current, we get zero, because sine or cosine function appears in their formulae. You know that the average value of sine or cosine function over an interval of one period is zero. That is,

$$< V > = V_m \left[\frac{1}{T}\int_0^T \cos\omega t\, dt\right] = 0$$

In practice, specially designed A.C. voltmeter and A.C. ammeter are used to measure A.C. voltage and A.C. current. These meters give r.m.s. (root mean square) value of A.C. voltage and A.C. current.

Root means square (r.m.s) of a quantity means the square root of the mean (average) of the squares of that quantity. In the present case the average of the square is taken over an interval of one periodic time*. To obtain r.m.s. value of $V = V_m \cos\omega t$, we should get the average of V^2 over one periodic time and then find the square root of it.

$$\text{Average } V^2 = < V^2 > = < V_m^2 \cos^2\omega t > \qquad (2.5.1)$$

$$= V_m^2 \left\langle \frac{1+\cos 2\omega t}{2} \right\rangle = V_m^2 \left\langle \frac{1}{2} + \frac{\cos 2\omega t}{2} \right\rangle$$

Foot note : *If $f(t)$ is a function of time (t), the average value of this function over an interval of time T, is given by $\langle f(t)\rangle = \frac{1}{T}\int_0^T f(t)\,dt$.

$$= V_m^2 \left\langle \frac{1}{2} \right\rangle + \frac{V_m^2}{2}\left(\frac{1}{T}\int_0^T \cos 2\omega t.dt\right)$$

But, $\left\langle \frac{1}{2} \right\rangle = \frac{1}{2}$ and $\frac{1}{T}\int_0^T \cos 2\omega t.dt = 0$

$$\therefore \ <V_m^2> = \frac{V_m^2}{2} \qquad (2.5.2)$$

$$\therefore \ V_{rms} = \sqrt{<V^2>} = \frac{V_m}{\sqrt{2}} \qquad (2.5.3)$$

Similarly, $I_{rms} = \frac{I_m}{\sqrt{2}}$ (2.5.4)

2.6 Series Resonance

In order to understand the phenomenon of resonance in L-C-R series circuit, consider equation (2.3.19).

$$I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\therefore \ I = I_m \cos(\omega t - \delta)$$

where, $I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

From equation (2.5.4)

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{\frac{V_m}{\sqrt{2}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\therefore \ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V_{rms}}{|Z|} \qquad (2.6.1)$$

Equation (2.6.1) shows that if we go on changing the values of angular frequency ω of the voltage, then the values of I_{rms} will also go on changing and at one definite value $\omega = \omega_0$, we will get

$$\omega_0 L = \frac{1}{\omega_0 C} \qquad (2.6.2)$$

and in this condition $|Z|$ becomes minimum and I_{rms} becomes maximum.

$$I_{rms} = \frac{V_{rms}}{R} = I_{rms}(\max) \qquad (2.6.3)$$

Thus, for a definite angular frequency (ω_0) of the voltage, value of r.m.s. current becomes maximum. This is called the series resonance in L-C-R A.C. series circuit.

From equation (2.6.2)

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{2.6.4}$$

From this $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Here ω_0 is called the natural angular frequency or the resonant angular frequency and f_0 is called resonant frequency of L-C-R A.C. series circuit.

Here, note that resonance is produced when the reactive component of impedence $\left(\omega L - \frac{1}{\omega C}\right)$ becomes zero; that is the imaginary part of impedence becomes zero.

In the figure 2.15, graphs of I_{rms} against ω for L-C-R series circuit are shown for two values of R ($R_1 < R_2$), which are called resonance curves. From the figure, it is clear that resonance curve is sharper for smaller value of R.

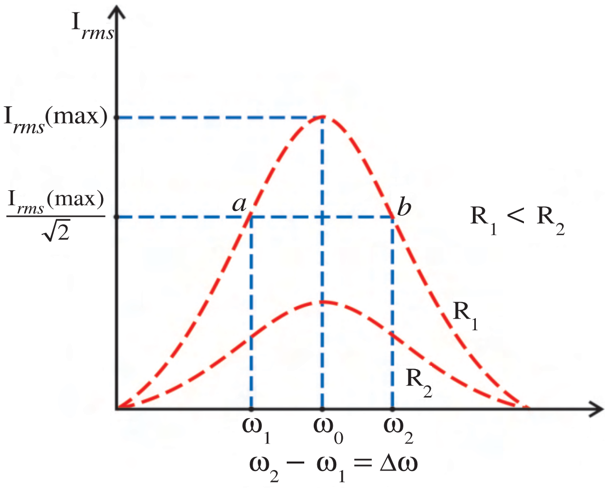


Figure 2.15 Resonance Curves L-C-R Series Circuit

Q-factor : The sharpness of the L-C-R resonance curve is measured by a quantity called the Q-factor.

In the circuit, the maximum power is proportional to the square of maximum value of rms current $[I_{rms}(\max)]^2$. When I_{rms} becomes $\frac{I_{rms}(\max)}{\sqrt{2}}$, the value of power becomes half of its maximum value. The value of $\frac{I_{rms}(\max)}{\sqrt{2}}$ corresponding to this power is shown in the figure 2.15. From the figure, it is clear that for this value of current two angular frequencies ω_1 and ω_2 are found.

$(\omega_2 - \omega_1)$ is called half-power bandwidhth $(\Delta\omega)$.

From this discussion, it is clear that as the half-power band-width is smaller when the sharpness of resonance curve is more. To understand this fact, Q-factor is defined by the following formula :

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} \tag{2.6.5}$$

Here, it is clear that as the Q-factor is larger the sharpness of the curve is more.

Moreover,

$$\Delta\omega = \frac{R}{L} \tag{2.6.6}$$

(Derivation of this formula is given in Appendix-B at the end of this chapter, only for information). Substituting this value of $\Delta\omega$, in equation (2.6.5) we get,

$$Q = \frac{\omega_0 L}{R} \tag{2.6.7}$$

But, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\therefore \ Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{2.6.8}$$

From this formula it can be seen that, Q-factor depends on the values of the all circuit-components.

From the value of the Q-factor, we can now infer how is the tuning of the circuit and also its selectivity.

Resonance circuit is used to select (or to tune) the desired frequency, out of many frequencies incident on the antenna of radio or TV. in order to change desired frequency, the arrangement is made to change either L or C or both. Here, note that resonance cannot be obtained in RL and RC circuit.

Illustration 1 : An A.C. Source of 230 V is connected in series with a 8.0 mH inductor, 80 μF capacitor and a 400 Ω resistor. Calculate (1) The resonant frequency (2) The impedance of the circuit and the value of the current at the resonant frequency (3) The *rms* value of the voltage across the components of the above circuit.

Solution :

(1) The resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

$$\therefore \ f = \frac{1}{(2)\,(3.14)\,\sqrt{8 \times 10^{-3} \times 80 \times 10^{-6}}} = \frac{1}{6.28 \times 8 \times 10^{-4}} = 199 \text{ Hz}$$

(2) $|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + (X_L - X_C)^2}$

$X_L = \omega L = 2\pi f L = (2)\ (3.14)\ (199)\ (8 \times 10^{-3}) = 10\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{(2)(3.14)(199)(80 \times 10^{-6})} = 10\ \Omega$

At resonance $X_L = X_C$

$\therefore\ |Z| = R = 400\ \Omega$

At resonance current in the circuit $I = \frac{V}{R} = \frac{230}{400} = 0.575$ A

(3) Potential difference between two ends of the inductor

$V_L = I_{rms} X_L = (0.575)\ (10) = 5.75$ volt

Similarly potential difference across the capacitor.

$V_C = I_{rms} X_C = (0.575)\ (10) = 5.75$ volt

and potential difference across the resitance

$V_R = I_{rms} R = (0.575)\ (400) = 230$ volt

Illustration 2 : For which value of ω will the impedance of figure 2.13 be maximum ? What will be the value of I_{rms} in the above case ? What will be the maximum value of impedance ?

Solution : As per the equation (2.4.12)

$$|Z| = \left[R^2 + \frac{1}{\left(\omega C - \frac{1}{\omega L}\right)^2} \right]^{\frac{1}{2}}$$

When the term $\left(\omega C - \frac{1}{\omega L}\right)^2$ becomes minimum, the $|Z|$ term becomes maximum.

$$\therefore\ \omega C - \frac{1}{\omega L} = 0$$

$$\therefore\ \omega = \frac{1}{\sqrt{LC}}$$

$$\therefore\ |Z| = \text{infinite}$$

$$\therefore\ I_{rms} = \frac{V}{|Z|} = 0$$

Illustration 3 : The A.C. voltage and the current in an L-C-R A.C. series circuit are given by the following expression. $V = 200\sqrt{2}\cos(3000t - 55°)$ V, $I = 10\sqrt{2}\cos(3000t - 10°)$ A. Calculate the impedance and the resistance of the above circuit.

Solution : Phase difference between current in the circuit and voltage is 45°.

$$\therefore\ \tan\delta = \tan 45° = 1$$

Now for L-C-R series circuit $\tan\delta = \dfrac{\omega L - \frac{1}{\omega C}}{R}$

$$\therefore\ \frac{\omega L - \frac{1}{\omega C}}{R} = 1$$

$$\therefore\ R = \omega L - \frac{1}{\omega C}$$

$$\therefore\ \text{Impedance } |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + R^2} = R\sqrt{2}$$

$$\therefore\ |Z| = \frac{V_m}{I_m} = \frac{200\sqrt{2}}{10\sqrt{2}} = 20\ \Omega$$

$$\therefore\ R\sqrt{2} = 20$$

$$\therefore\ R = 14.14\ \Omega$$

Illustration 4 : An electric current has both A.C. and D.C. components. The value of the D.C. component is equal to 12 A while the A.C. component is given as I = 9 sinωt A. Determine the formula for the resultant current and also calculate the value of I_{rms}.

Solution : Resultant current (at any instant of time) will be I = 12 + 9 sinωt (1)

Now, $I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\langle 12 + 9\sin\omega t\rangle^2} = \sqrt{\langle 144 + 216\sin\omega t + 81\sin^2\omega t\rangle}$

Here, the average is taken over a time interval equal to the periodic time.

$\therefore\ I_{rms} = \sqrt{\langle 144\rangle + 216\langle \sin\omega t\rangle + 81\langle \sin^2\omega t\rangle}$

Now $\langle 144\rangle = 144$, $216\langle \sin\omega t\rangle = 0$ and $81\langle \sin^2\omega t\rangle = 81 \times \frac{1}{2} = 40.5$

$\therefore\ I_{rms} = \sqrt{144 + 40.5} = 13.58$ A

Illustration 5 : Calculate the resultant inductance of two inductors L_1 and L_2 when they are connected in parallel in A.C. circuit.

Solution : Let Z_{L_1} and Z_{L_2} be the inductive reactance of the two coils. Since they are connected in parallel, the resultant reactance will be,

$$Z = \frac{Z_{L_1} Z_{L_2}}{Z_{L_1} + Z_{L_2}} = \frac{(j\omega L_1) \times (j\omega L_2)}{j\omega L_1 + j\omega L_2} \qquad (1)$$

If the resultant inductance is equal to L, then Z = $j\omega$L.

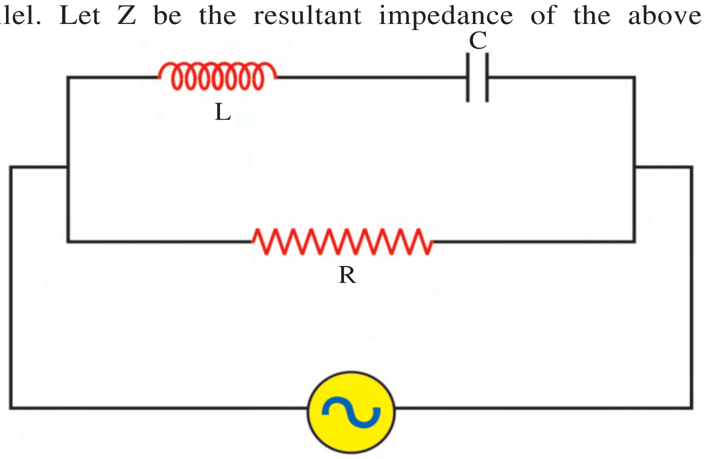
$$j\omega L = \frac{j\omega^2 L_1 L_2}{\omega L_1 + \omega L_2}$$

$$\therefore\ L = \frac{L_1 L_2}{L_1 + L_2}$$

Illustration 6 : Calculate the impedance Z of the given circuit.

Solution : Let Z_1 be the effective impedance of L and C in series, then $Z_1 = Z_L + Z_C$.

Now Z_1 and R are connected in parallel. Let Z be the resultant impedance of the above parallel connection, then



$$Z = \frac{Z_1 R}{Z_1 + R} = \frac{(Z_L + Z_C)R}{Z_L + Z_C + R}$$

$$= \frac{j\left(\omega L - \frac{1}{\omega C}\right)R}{j\left(\omega L - \frac{1}{\omega C}\right) + R}$$

$$= \frac{j(X_L - X_C)R}{j(X_L - X_C) + R}$$

Multiplying the complex numbers in numerator and the denominator with their respective complex conjugate,

$$\therefore \ |Z| = \left\{\frac{-Rj(X_L - X_C) \times Rj(X_L - X_C)}{\{j(X_L - X_C) + R\}\{R - j(X_L - X_C)\}}\right\}^{\frac{1}{2}}$$

$$\therefore \ |Z| = \left[\frac{R^2(X_L - X_C)^2}{R^2 + (X_L - X_C)^2}\right]^{\frac{1}{2}}$$

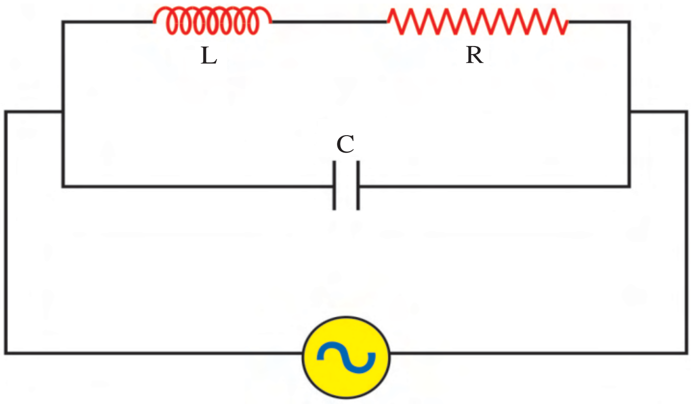
In the above expression when $X_L = X_C$, $|Z| = 0$ (Resonance can be obtained.)

Illustration 7 : Obtain the resonance angular frequency for the circuit shown in the figure.

Solution : Let Z_1 be the resultant impedance of the inductor L and resistor R.

$Z_1 = R + jX_L = R + j\omega L$

Let Z be the resultant impedance of the above circuit, hence



$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_C}$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$$\left[\because \ \frac{1}{-\frac{1}{\omega C}j} = -\frac{\omega C}{j} = j\omega C\right]$$

(Multiplying and dividing the first term on the right hand side by $R - j\omega L$.)

$$\frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$= \frac{R + j(\omega CR^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

$$\therefore \ Z = \frac{R^2 + \omega^2 L^2}{R + j(\omega CR^2 + \omega^3 L^2 C - \omega L)}$$

The imaginary part (coefficient of j) in the denominator of the above equation should be zero, for Z to be maximum.

$$\therefore \ \omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

$$\therefore \ \omega^2 L^2 C = L - CR^2$$

$$\therefore \ \omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \ \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Illustration 8 : The series combination of R(Ω) and capacitor C(F) is connected to an A.C. source of V volts and angular frequency ω. If the angular frequency is reduced to $\frac{\omega}{3}$, the current is found to be reduced to one-half without changing the value of the voltage. Determine the ratio of the capacitive reactance and the resistance.

Solution : First case : (We shall indicate the r.m.s. value of I and V as I and V for the sake of convenience.)

$$I = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad \therefore \quad I^2 = \frac{V^2}{R^2 + Xc^2} \qquad (1)$$

Second case :

$$\frac{I}{2} = \frac{V}{\sqrt{R^2 + \frac{9}{\omega^2 C^2}}} \quad \therefore \quad \frac{I^2}{4} = \frac{V^2}{R^2 + 9Xc^2} \qquad (2)$$

Dividing equation (1) by (2), we have,

$$4 = \frac{R^2 + 9Xc^2}{R^2 + Xc^2}$$

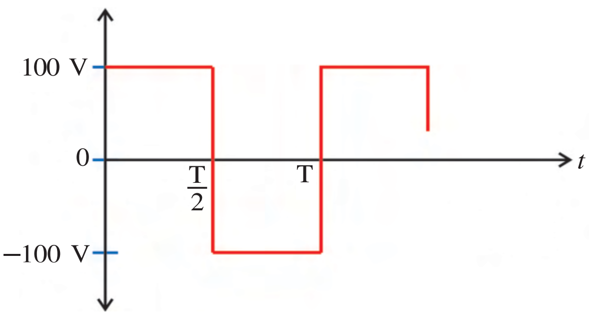
$$\therefore \ 4R^2 + 4Xc^2 = R^2 + 9Xc^2$$

$$\therefore \ 5Xc^2 + 4R^2 = R^2$$

$$\therefore \ \frac{Xc}{R} = \sqrt{\frac{3}{5}}$$

Illustration 9 : The maximum value of an A.C. voltage is equal to 100 V for a square wave shown in the figure. Calculate the rms value of the voltage.

Solution : $V_{rms} = \left[\frac{1}{T}\int_0^T V^2(t)dt\right]^{\frac{1}{2}}$



$$= \left[\frac{1}{T}\left\{\int_0^{\frac{T}{2}}(100)^2 dt + \int_{\frac{T}{2}}^{T}(-100)^2 dt\right\}\right]$$

$$= \left[\frac{1}{T}\left\{10^4\left(\frac{T}{2} - 0\right) + 10^4\left(T - \frac{T}{2}\right)\right\}\right]^{\frac{1}{2}} = \left[\frac{1}{T}\left\{10^4\frac{T}{2} + 10^4\frac{T}{2}\right\}\right]^{\frac{1}{2}} = \left[10^4\right]^{\frac{1}{2}} = 100 \text{ V}$$

Illustration 10 : The medium wave broadcast signal in a radio can be tuned from 600 kHz to 1200 kHz. If the effective value of the inductance of an inductor connected in the L-C circuit is 100 mH, find the range of the variable capacitor.

Solution : L = 100 mH, f_{max} = 1200 kHz, f_{min} = 600 kHz

For Tuning (means for resonance) frequency $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

$$\therefore \ 4\pi^2 f^2 = \frac{1}{LC}$$

$$\therefore \ C = \frac{1}{4\pi^2 f^2 L} \quad \therefore \ C_{max} = \frac{1}{4\pi^2 f^2_{min} L}, \quad \therefore \ C_{min} = \frac{1}{4\pi^2 f^2_{max} L}$$

$$\therefore \; C_{max} = \frac{1}{(4)(3.14)^2 (600 \times 10^3)^2 \; (100 \times 10^{-3})}$$

$$= 0.7 \times 10^{-12} \text{ F}$$

$$= 0.7 \text{ pF}$$

Similarly $C_{min} = \dfrac{1}{(4)(3.14)^2 (1200 \times 10^3)^2 \; (100 \times 10^{-3})}$

$$= 0.176 \times 10^{-12} \text{ F}$$

$$= 0.176 \text{ pF}$$

Thus, the range of the variable capacitor is from 0.176 pF to 0.7 pF.

2.7 Phasor Method

Addition of harmonic functions can be easily done with the method of phasor. To understand what a phasor is, consider a harmonic function.

$$I = I_m \cos(\omega t + \delta) \qquad (2.7.1)$$

A vector with magnitude I_m is drawn from the origin of coordinate system in X–Y plane, as shown in figure 2.16, which makes an angle with X-axis equal to phase $(\omega t + \delta)$. From the figure 2.16, the following points are clear.

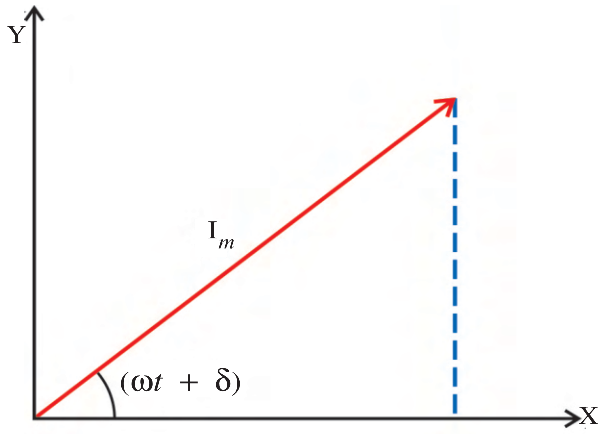


Figure 2.16

(1) The phase $(\omega t + \delta)$ changes with time. It means the angle made by the vector I_m with X-axis in figure 2.16 changes with time. Thus the vector drawn here is not steady but rotates in X–Y plane with an angular frequency ω. Such a vector is called the rotating vector. Such a rotating vector is the phaser or the rotor.

Here, note that I is scalar only. We are merely representing it as a rotating vector.

(2) At $t = t$ time, the x-component of this vector is $I_m \cos(\omega t + \delta)$, which gives the instantaneous value of I. If we want to add functions like

$I_1 \cos(\omega t + \delta_1)$, $I_2 \cos(\omega t + \delta_2)$ .... etc., now it becomes easier. We should draw phasors for all functions at time and then add their X-components, algebraically. Thus, the final algebra becomes simple.

(3) There is one more advantage of this method. If we take y-component of this function, it is $\cos\left[\frac{\pi}{2} - (\omega t + \delta)\right] = \sin(\omega t + \delta)$. Thus by considering Y-components of the vectors we can also deal with sine functions in the similar manner.

(4) Suppose we want to add two harmonic functions,

$$I_1 = I_{1m} \cos(\omega t + \delta_1) \qquad (2.7.2)$$

$$I_2 = I_{2m} \cos(\omega t + \delta_2) \qquad (2.7.3)$$

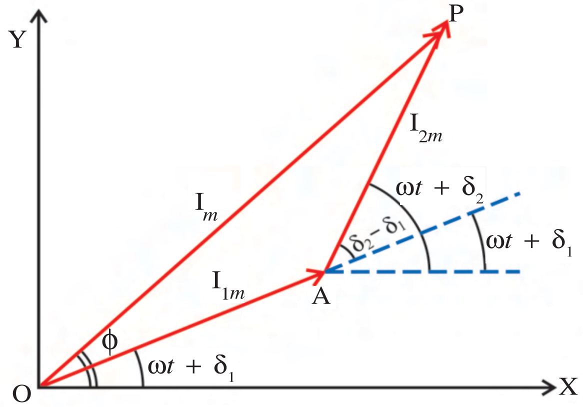


Figure 2.17

To find the resultant function, vectors representing I_{1m} and I_{2m} are drawn as shown in the figure 2.17 and then their vector addition is found. For this draw vector in the direction making angle of $(\omega t + \delta_1)$ with X-axis. Now, from the end point A of this vector draw a vector I_{2m} in the direction making an angle of $(\omega t + \delta_2)$ with X-axis. To find the resultant vector from starting point O of I_{1m} draw a vector to the end point P of I_{2m}.

From the geometry of the figure 2.17, it is clear that the vector representing I at time t, represents the resultant function obtained at that time, of the two given harmonic functions. Its amplitude is I_m (= OP) (to scale) and its phase at time t is ϕ. We can also get the functional form of I from the law of triangle of vectors.

From the figure the angle between the two vectors representing two harmonic functions. I_1 and I_2 is $(\delta_2 - \delta_1)$

Now, $I_m^2 = I_{1m}^2 + I_{2m}^2 + 2I_{1m}I_{2m}\cos(\delta_2 - \delta_1)$

Let the phase difference between the functions I_1 and I_2 be $(\delta_2 - \delta_1) = \delta$

$\therefore\ I_m^2 = I_{1m}^2 + I_{2m}^2 + 2I_{1m}I_{2m}\cos\delta$

Thus we get the resultant function also. We should remember here, that the magnitudes of the veacors representing I_1, I_2 and I are respectively I_{1m}, I_{2m} and I_m.

2.8 Use of Phasor Method in an A.C. Circuit

This method can be very easily used for obtaining the phase relation between applied voltage and current.

A.C. Circuit Containing only Resistance : For circuit containing only resistance phase difference between applied voltage V and current I is zero hence phasor for voltage and current will be in the same direction as shown in the figure 2.18. (It may be noted that the phasor of I can be taken in any arbitrary direction. The phasor of V can be drawn by knowing the phase difference between V and I).

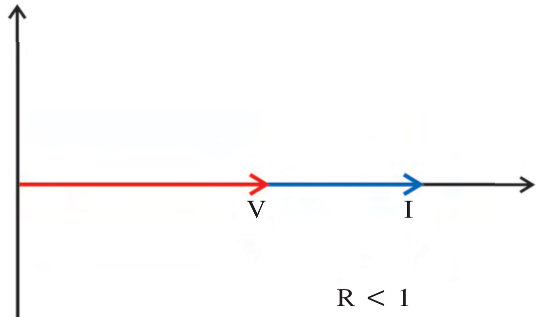


Figure 2.18
Circuit Comprising only Resistor

Circuit Containing only Inductor : We have already studied algebraically this circuit in section (2.4). In this circuit current I lags the voltage V by $\frac{\pi}{2}$ rad in phase angle or the voltage leads current by a phase angle of $\frac{\pi}{2}$. If I is represented along the X-direction then the phasor representation of V will be along the positive Y-direction. As shown in figure 2.19.

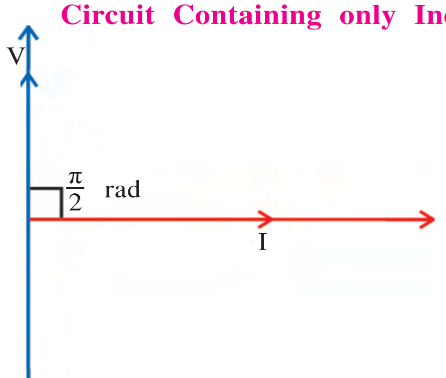


Figure 2.19 Circuit Comprising only Inducter

A.C. Circuit Containing only Capacitor :

In the circuit the current I leads the voltage V by a phase of $\frac{\pi}{2}$ or the voltage lags the current by a phase of $\frac{\pi}{2}$. The phasor diagram for this circuit is as shown in figure 2.20.

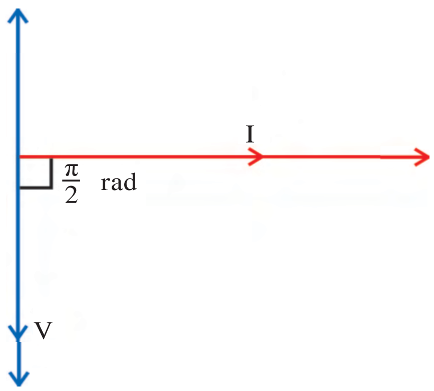


Figure 2.20 Circuit Comprising only Capacitor

L-C-R Series A.C. Circuit : From the above information the phasor diagram for each component of L-C-R series circuit will be as shown in the figure 2.21.

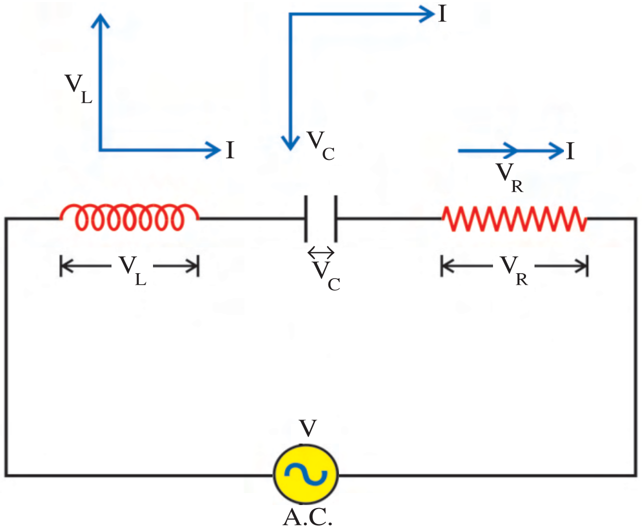


Figure 2.21 Series Circuit

In the present case as L, C and R are in series, the current passing through each component will be same. If applied voltage is V then,

$$\vec{V} = \vec{V_L} + \vec{V_C} + \vec{V_R} \tag{2.8.1}$$

Where V_L, V_C and V_R are potential difference between two ends of inductor, capacitor and resistance respectively. Suppose the phasor of current I is shown in X-direction, then the phase diagrams for each components will be as shown in figure 2.22.

It is obvious from the figure 2.22 that

$V^2 = (V_L - V_C)^2 + V_R^2$

If I_m is the maximum value of the current

$V_R = I_m R$, $V_L = I_m X_L$ and $V_C = I_m X_C$

$\therefore \; V^2 = I_m^2 (X_L - X_C)^2 + I_m^2 R^2$

$\therefore \; V = I_m \sqrt{(X_L - X_C)^2 + R^2}$

As we have taken the maximum current

$V = V_m$

$$\therefore \; I_m = \frac{V_m}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{V_m}{|Z|} \tag{2.8.2}$$

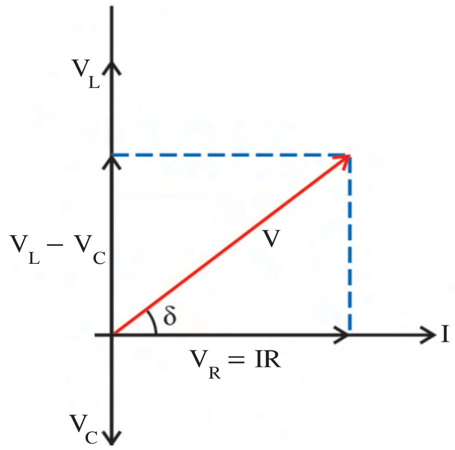


Figure 2.22

Now from figure the angle between voltage phasor and current phasor is δ. In the present case (when $V_L > V_C$) current lags the voltage by phase angle δ. If $V_L < V_C$ then current leads the voltage by phase angle δ.

From figure,

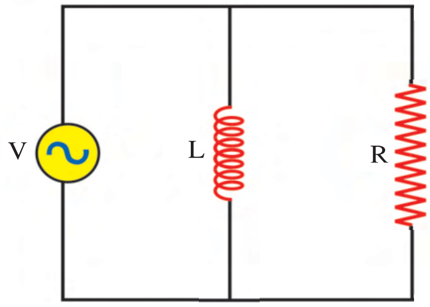
$$\tan\delta = \frac{V_L - V_C}{V_R}$$

$$= \frac{I_m X_L - I_m X_C}{I_m R}$$

$$\therefore \tan\delta = \frac{X_L - X_C}{R} \qquad (2.8.3)$$

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad (2.8.4)$$

Illustration 11 : An inductor L and resistor R are connected in parallel with an A.C. source of V volt. Determine the total current I in terms of X_L and R. Also determine the phase difference between the current and the voltage. Use the method of phasor diagram.

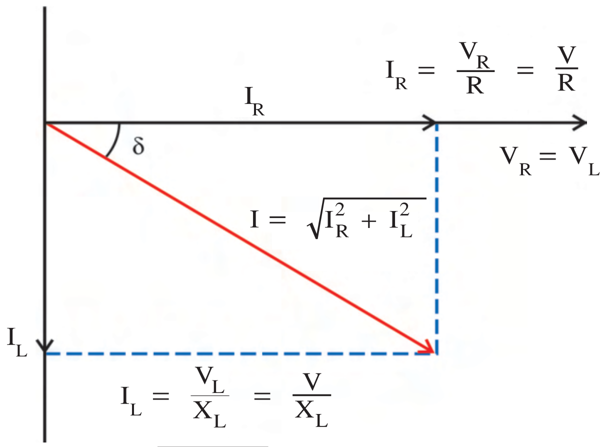


Solution : Here R and L are parallel. Hence the potential difference across them is equal. On representing the phasor of this voltage on X-axis and with reference to it representing the current phasor, the situation will be as shown in the figure.

(1) Here phasor I_R and phasor V_R are in same phase.

$\therefore I_R = \frac{V_R}{R}$ and it is in the X-direction as shown in the figure.

(2) Moreover the current in the inductor is lagging behind the voltage V_L in phase by $\frac{\pi}{2}$. Hence I_L will be on the negative Y-axis as shown in the figure.



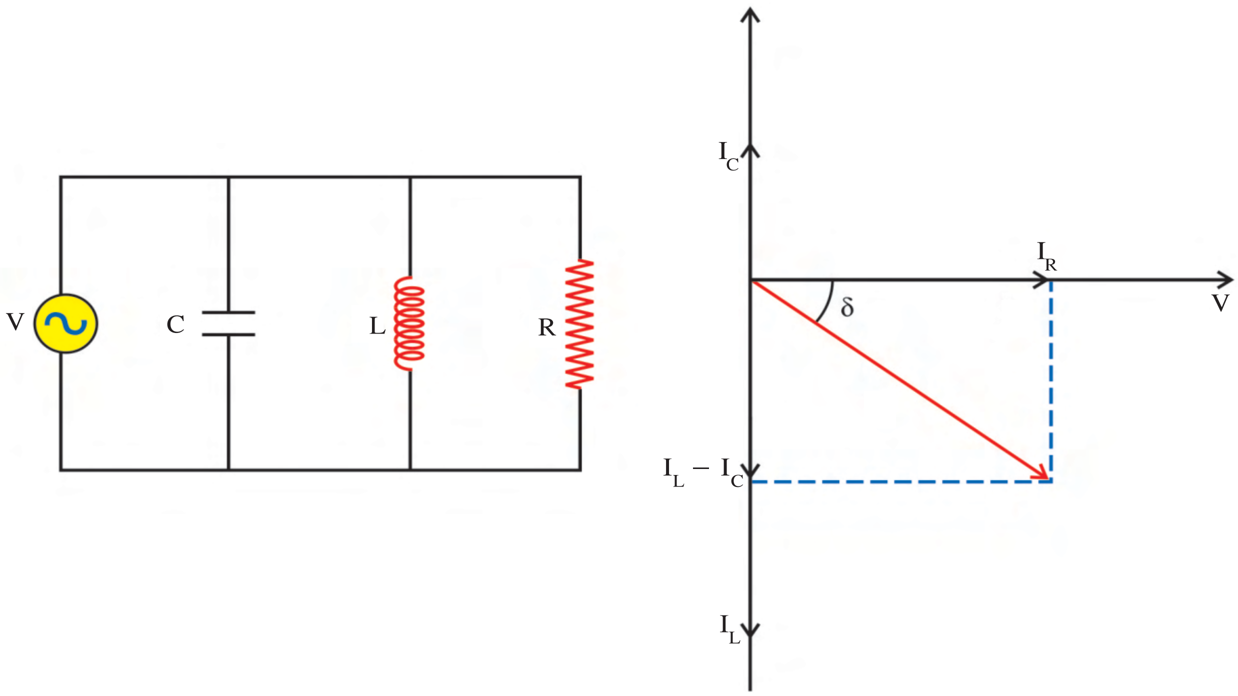
$\therefore I_L = \frac{V_L}{X_L}$. From the figure $I = \sqrt{I_R^2 + I_L^2} = V = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}$

$$\tan\delta = \frac{I_L}{I_R} = \frac{V}{X_L}\frac{R}{V} = \frac{R}{X_L}$$

$$\therefore \delta = \tan^{-1}\frac{R}{X_L}$$

Illustration 12 : Derive the expression for the total current flowing in the circuit using the phasor diagram.

Solution : The phasor diagram of the voltage and current is as shown in figure. In order to obtain the total current, we shall have to consider the addition of the currents. From the diagram we have,



$$I = \sqrt{I_R^{\,2} + (I_L - I_C)^2}$$

But, $I_R = \frac{V}{R}$; $I_L = \frac{V}{X_L}$ and $I_C = \frac{V}{X_C}$

$$\therefore \; I = V\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

From the figure, we have, $\tan\delta = \dfrac{I_L - I_C}{I_R} = \dfrac{\frac{1}{X_L} - \frac{1}{X_C}}{\frac{1}{R}}$

$$\therefore \; \tan\delta = R\left(\frac{1}{X_L} - \frac{1}{X_C}\right)$$

2.9 L-C Oscillations

If the two ends of a charged capacitor (C) are connected by conductor or a resistor, the capacitor gets discharged and energy stored in the capacitor (i.e. the energy stored in the electric field between the two plates) is dissipated in the form of joule heat.

Now let us think what happens when the two plates of charged capacitor are connected with the inductor (L) having very low resistance which can be neglected (ideally zero). Such a circuit is shown in the figure 2.23 which is known as L-C circuit. Here, initially (that $t = 0$) the capacitor is in charged condition, hence we can think of the situation as follows :

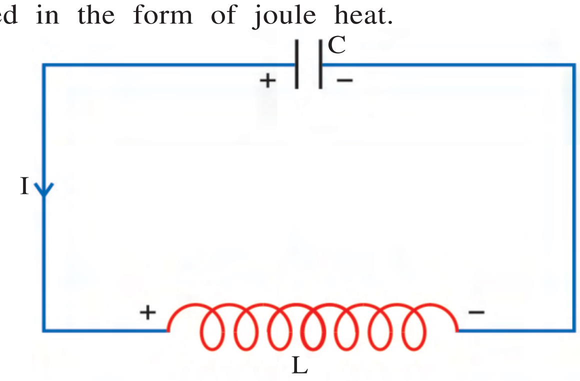


Figure 2.23

Suppose at $t = 0$ the charge on the capacitor is Q_0 and current in the circuit is zero. Here, it is assumed that the capacitor is brought in the circuit at time $t = 0$. The moment at which inductor is joined in the circuit the charge on the capacitor starts decreasing (i.e. capacitor starts discharging) and current starts in the circuit.

Due to the discharging of the capacitor suppose at time $t = t$ the charge on the capacitor = Q and curent in the circuit = I.

Hence applying Kirchoff's second law to this circuit at time $t = t$.

$$-L\frac{dI}{dt} + \frac{Q}{C} = 0$$

But, $I = -\frac{dQ}{dt}$ ($\therefore$ Charge on the capacitor dereases.)

$$\therefore\ L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

$$\therefore\ \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \qquad (2.9.1)$$

This equation is analogus to the differential equation $\frac{d^2y}{dt^2} = -\omega_0^2 y$ of the simple harmonic motion. Here, the role of charge Q is similar to the displacement y and in place of ω_0^2 the term $\frac{1}{LC}$ is appearing.

Thus, the solution of the equation in our case is $Q = Q_m\sin(\omega_0 t + \phi)$ (2.9.2)

Here Q_m and ϕ are the constants of the solution and can be determined from the initial conditions as follows.

When $t = 0$, $Q = Q_0$ on substituting these values in equation (2.9.2).

$$Q_0 = Q_m\sin\phi \qquad (2.9.3)$$

Differentiating equation (2.9.2) with respect to time

$$I = \frac{dQ}{dt} = Q_m\omega_0\cos(\omega_0 t + \phi)$$

But at $t = 0$, $I = 0$

$$\therefore\ 0 = Q_m\omega_0\cos\phi \qquad (2.9.4)$$

Here Q_m and ω_0 are non zero.

$$\therefore \cos\phi = 0$$

$$\therefore\ \phi = \frac{\pi}{2} \qquad (2.9.5)$$

substituting this value of ϕ in equation (2.9.3)

$$Q_m = Q_0 \qquad (2.9.6)$$

Using equation (2.9.5) and (2.9.6) in equation (2.9.2)

$$Q = Q_0\sin(\omega_0 t + \frac{\pi}{2})$$

$$\therefore \quad Q = Q_0 \cos\omega_0 t \qquad (2.9.7)$$

This equation shows that the charge on the capacitor changes periodically. Moreover, from this equation

$$I = \frac{dQ}{dt} = -Q_0\omega_0 \sin\omega_0 t \qquad (2.9.8)$$

It can be seen from the above equation that the current I in the circuit (i.e. current in the inductor) is also changing periodically.

At time $t = 0$ charge on the capacitor is maximum and current in the inductor is zero. In this situation the intensity of the electric field produced between the plates of the capacitor is maximum and energy stored (associated with electricfield) $\left(U_E = \frac{1}{2}\frac{Q^2}{C}\right)$ is also maximum. At this time ($t = 0$) the current in the inductor is zero, there is no magnetic field associated with it. Hence no energy is associated with inductor.

As time passes, the charge on the capacitor decreases and as a result of this energy (U_E) associated with electric field also decreases. As this charge is passing through the inductor, the current (I) in the inductor increased and as a result of this the magnetic field associated with it and the energy ($U_B = \frac{1}{2}LI^2$) associated with magnetic field also increases.

Thus, the energy stored in electric field of capacitor is transformed in the energy stored in the magnetic field associated with inductor.

When the charge on the capacitor becomes zero (ie. $Q = 0$), the current (I) in the inductor becomes maximum. At this time the total energy stored in the electric field is transferred in to magnetic field.

After this time the charge on the capacitor increase but the polarity of the two plates is reversed. With this polarity charge on the capacitor becomes maximum and this process continues periodically and the initial situation (the situation at time $t = 0$) is established. This process is repeated continuously. In short the electric charge oscillates between the two plates of the capacitor via inductor. This phenomenon is called oscillations in L-C circuit or L-C oscillations.

During this oscillation the electric field associated with capacitor and energy (U_E) associated with it and the magnetic field associated with inductor and energy (U_B) associated with it are shown in the figure 2.24. with the different time intervals of a periodic time of oscillations. ($t = 0$, $t < \frac{T}{4}$, $t = \frac{T}{4}$, $\frac{T}{4} < t < \frac{T}{2}$, $t = \frac{T}{2}$, $\frac{T}{2} < t < \frac{3}{4}T$, $t = \frac{3}{4}T < t < T$ and $t = T$.)

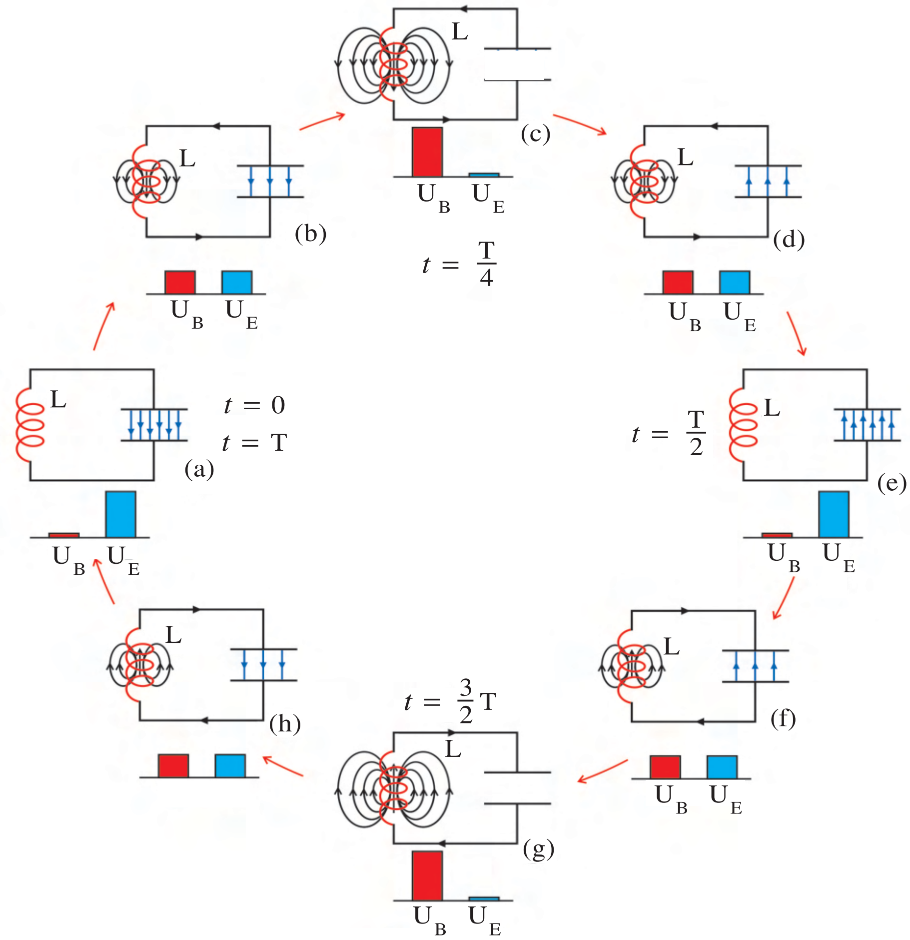


Figure 2.24 L-C Oscillations (For Information Only)

Here the electric field associated with capacitor and magnetic field associated with inductor changes with time. These changing electric and magnetic fields radiates electromagnetic radiation. Due to continuous emission of electromagnetic radiation, energy of circuit decreases gradually. Thus oscillating charge emits electromagnetic waves. This L-C circuit also called tank circuit. If the energy is provided in the L-C circuit which equals energy emitted then the continuous emission of the electromagnetic wave is obtained.

Illustration 13 : Show that for free L-C oscillations, the sum of energy stored in capacitor and energy stored in inductor is constant at any instant of time.

Solution : Let Q_0 be the initial (at time $t = 0$) on the capacitor (C). When this capacitor is connected with inductor (L), free oscillations start and its natural angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$.

Here, instantaneous charge

$$Q = Q_0 \cos\omega_0 t$$

$$\therefore \; I = \frac{dQ}{dt} = -Q_0\omega_0 \sin\omega_0 t$$

At some instant of time t energy stored in the capacitor

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{Q_0^2}{2C}\cos^2\omega_0 t \qquad (\because \; V = \frac{Q}{C})$$

At the same time t energy stored in the inductor

$$U_M = \frac{1}{2}LI^2 = \frac{1}{2}LQ_0^2\omega_0^2 \sin^2(\omega_0 t) = \frac{Q_0{}^2}{2C} \sin^2\omega_0 t \qquad (\because \ \omega_0 = \frac{1}{\sqrt{LC}})$$

Summation of these two energies

$$U = U_E + U_B = \frac{Q_0^2}{2C}(\cos^2\omega_0 t + \sin^2\omega_0 t)$$

$$= \frac{Q_0^2}{2C}$$ Here, Q_0 and C are not dependent on time

$\therefore$ U = Constant.

2.10 Power and Energy Associated with L, C and R in an A.C. Circuit

According to the definition of power (P)

$$P = VI \tag{2.10.1}$$

In an A.C. circuit voltage and current both changes with time. Hence power represented according to equation (2.10.1) can be called instantaneous power for A.C. circuit. But in practice we cannot measure instantaneous power. In practice real power is defined and it is measured.

Real power = Average power for the entire period of the cycle.

For L-C-R circuit instantaneous power

P = VI

$$= V_m \cos(\omega t) \ I_m \cos(\omega t - \delta) \tag{2.10.2}$$

$$= V_m I_m \cos\omega t \ \cos(\omega t - \delta)$$

$$\text{But } \cos\omega t \ \cos(\omega t - \delta) = \frac{1}{2} \cos\delta + \frac{1}{2}\cos(2\omega t - \delta) \tag{2.10.3}$$

$$P = \frac{V_m I_m}{2}(\cos\delta + \cos(2\omega t - \delta)) \tag{2.10.4}$$

$\therefore$ According to the definition of real power (now onwards we will consider power P as real power unless specifically it is mentioned.)

$$P = \frac{V_m I_m}{2}\left[\frac{1}{T}\int_0^T \cos\delta dt + \frac{1}{T}\int_0^T \cos(2\omega t - \delta)dt\right]$$

But, $\int_0^T \cos(2\omega t - \delta)dt = 0$ and $\int_0^T \cos\delta dt = T\cos\delta$

$$\therefore \ P = \frac{V_m I_m}{2} \frac{T}{T} \cos\delta$$

$$\therefore \ P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\delta \tag{2.10.5}$$

Here, $\cos\delta$ is called power factor.

Equation (2.10.5) can also be written in the form of r.m.s. value as follows.

$$\therefore \ P = V_{rms} I_{rms} \cos\delta \tag{2.10.6}$$

Special Cases :

(1) Circuit with only Resistor : For this circuit phase difference $\delta = 0$.

$\therefore P = V_{rms} I_{rms}$

(2) A.C. Circuit with only Inductor : Phase difference between voltage and current

$\delta = \frac{\pi}{2} \quad \therefore \cos\frac{\pi}{2} = 0$

$\therefore P = 0$

Thus, power in the A.C. circuit with only inductor is zero.

When current in the inductor increases, the energy drawn from the source is stored in the form of magnetic field associated with inductor and when current decreases this stored energy is given back to the source as a result power consumed is zero. Thus with the help of the inductor current can be controlled without wasting the energy in A.C. circuit. (The choke used in tubelight which is an inductor which does this work.).

(3) Circuit with Capacitor Only : The phase difference between voltage and current for this circuit $\delta = -\frac{\pi}{2} \quad \therefore \cos\left(-\frac{\pi}{2}\right) = 0$.

Thus, also in this case power $P = 0$

In this case when charge is accumulated on the two plates of the capacitor, the energy obtained from the source is stored in the electric field produced between two plates of capacitor and when capacitor discharges, the energy stored is given back to the source. As result, power consumed is zero.

(4) For L-C-R A.C. Circuit : From the figure 2.3

$$\cos\delta = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{R}{|Z|} \quad (2.10.8)$$

On substituting this value of $\cos\delta$ in equation (2.10.6) and calculating power, it can be seen that this power is less than the power obtained in the circuit with only resistor.

When only inductor or only capacitor is there in the A.C. circuit, power consumed is zero. In this situation, the current flowing in the circuit is called watt less current.

Illustration 14 : In an L-C-R A.C. series circuit $L = 5$ H, $\omega = 100$ rads^{-1}, $R = 100\ \Omega$ and power factor is 0.5. Calculate the value of capacitance of the capacitor.

Solution : Power factor $\cos\delta = \dfrac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

taking square on both sides $\cos^2\delta = \dfrac{R^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

But, $\cos\delta = 0.5 = \frac{1}{2}$

$$\therefore \quad \frac{1}{4} = \frac{R^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\therefore \quad R^2 + (\omega L - \frac{1}{\omega C})^2 = 4R^2$$

$$\therefore \quad (\omega L - \frac{1}{\omega C})^2 = 3R^2$$

$$\therefore \quad \omega L - \frac{1}{\omega C} = \sqrt{3}\, R$$

$$\therefore \quad \omega L - \sqrt{3}\, R = \frac{1}{\omega C}$$

$$\therefore \quad C = \frac{1}{\omega}\left(\frac{1}{\omega L - \sqrt{3}\, R}\right)$$

$$= \frac{1}{100}\left(\frac{1}{100 \times 5 - \sqrt{3} \times 100}\right)$$

$$= \frac{10^{-2}}{500 - 173.2}$$

$$= \frac{10^{-2}}{326.8} = 306 \times 10^{-7}$$

$$= 30.6 \times 10^{-6} \text{ F}$$

$$= 30.6 \ \mu\text{F}$$

2.11 Transformer

Power ($P = VI$) generated in the power station is to be sent to residences and industries which are very very far from the power station through hundreds of kilometers long cable network.

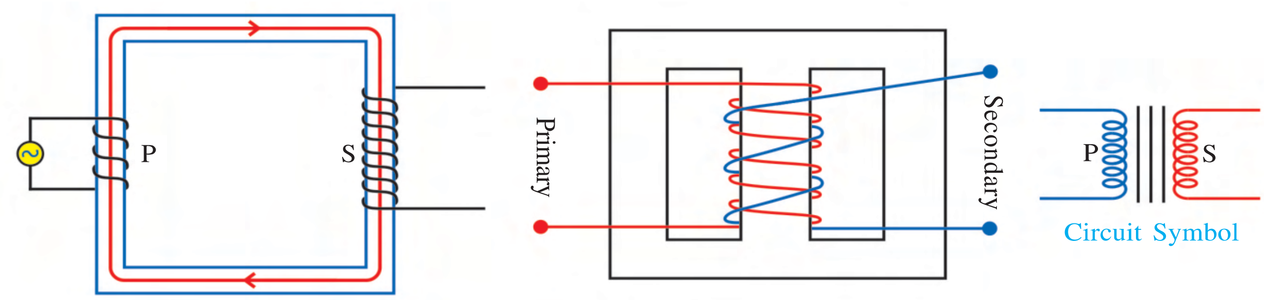
The cables have their own resistance (R). In practice the cable having zero resistance is not possible. Hence, when current (I) passes through the cable the power equals I^2R is transformed in the form of joule heat and wasted. Hence to save energy it is utmost necessary to decrease this wastage of energy. For this the current (I) should be decreased without changing the value of power ($P = VI$) before sending the current to cable network. Here, it is obvious that for the given value of power ($P = VI$), if we decrease I, the value of the voltage V has to be increased. For the safety reasons and electrical devices used in practice requires low voltage (generally 230 V or 240 V), it becomes necessary to reduce the voltage before this power is delivered to the residences or industries.

According to the above discussion, we should use a device in which without wasting power (ideally), A.C. voltage can be increased or decreased. Such a device is the transformer. The transformer with which output voltage can be increased is called step-up transformer and with which output voltage can be decreased is called step-down transformer.

It may be noted that in an ideal transformer power is not wasted. Only voltage can be increased or decreased, and correspondingly current is decreased or increased.

Principle : Transformer works on the principle of electro-magnetic induction.

Construction : Figure 2.25 shows the construction of a transformer and symbolic circuit diagram. Here, the two coils of conducting wires are wound very close to each other on a rectangular (or constituting closed loop) iron core having very high value of permeability as shown in figure 2.25. These copper coils are isolated from each other and also from core. One of these coils is called primary coil P and the other coil is called secondary coil S. Primary coil is connected with A.C. source.



(*a*) Contruction of Transformer (*b*) (*c*)

Figure 2.25

In step-up transformer, number of turns are less in primary and copper wire is thick. Whereas in secondary number of turns are more and copper wire is thin. In step-down transformer the situation is reversed. In practice the two coils are wound on the core having shape as shown in the figure 2.25 (*b*), on one another, such that the secondary coil remains on the primary. The iron core is constituted of several layers of strips of two pieces having the shape of English alphabets I and E placing them side by side such that the final shape of the core becomes as shown in the figure 2.25 (*b*). The position of two pieces of the layer I and E are interchanged in the layers coming one after another in the core to obtain the shape of the core as shown in the figure 2.25 (*b*). These layers or strips are insulated.

Due to core constructed as discussed above and winding the secondary coil on the primary coil almost all magnetic field lines due to current in the primary coil are associated with secondary coil and eddy currents can be reduced.

The magnetic flux Φ_S and Φ_P linked with secondary coil (S) and primary coil (P) are respectively proportional to their number of turns N_S and N_P.

$$\therefore \frac{\text{Magnetic flux linked with secondary coil } \Phi_S}{\text{Magnetic flux linked with primary coil } \Phi_P} = \frac{\text{Number of turns in secondary coil } N_S}{\text{Number of turns in primary coil } N_P} \qquad (2.11.1)$$

As primary is connected with A.C. source, the current passing through it is changing continuously with time (periodically) and hence magnetic flux linked with primary coil and as a result of that the magnetic flux linked with the secondary coil are changing continuously with time (periodically).

The frequency of the A.C. Voltage induced in the secondary has the frequency as that of the voltage in primary.

According to the Faraday's law

Induced emf in the primary, $\varepsilon_P = -\dfrac{d\Phi_P}{dt}$ and

induced emf in secondary, $\varepsilon_S = -\dfrac{d\Phi_S}{dt}$

Now from equation (2.11.1)

$$\Phi_S = \frac{N_S}{N_P}\Phi_P$$

$$\therefore \quad \frac{d\Phi_S}{dt} = \frac{N_S}{N_P}\frac{d\Phi_P}{dt}$$

$$\therefore \quad \varepsilon_S = \frac{N_S}{N_P}\varepsilon_P$$

$$\therefore \quad \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} = r \qquad (2.11.2)$$

Here, r is called transformation ratio.

For step-up transformer $r > 1$ and for step-down transformer $r < 1$.

It is obvious that by selecting appropriate transformation ratio, step-up transformer or step-down transformer can be prepared.

We have assumed that in transformer there is no loss of power. Hence

$$\begin{matrix}\text{Instantaneous output} \\ \text{power}\end{matrix} \begin{pmatrix}\text{i.e. instantaneous power} \\ \text{in the secondary coil}\end{pmatrix} = \begin{matrix}\text{Instantaneous input} \\ \text{power}\end{matrix} \begin{pmatrix}\text{i.e. instantaneous power} \\ \text{in the primary coil}\end{pmatrix}$$

$$\therefore \quad \varepsilon_S I_S = \varepsilon_P I_P$$

$$\therefore \quad \frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = r \qquad (2.11.3)$$

The assumption studied above is ideal. Thus, this type of transformer is called ideal transformer. In practice some power is lost in the magnetization and demagnetization of the core as well as in the formation of eddy currents on surface of the core. As a result the output power is less than the input power.

Illustration 15 : In an ideal step-up transformer input voltage is 110 V and current flowing in the secondary is 10 A. If transformation ratio is 10, calculate output voltage, current in primary and input and output power.

Solution : Transformation ratio $r = \frac{N_S}{N_P} = 10$

(1) $\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} \quad \therefore \; \varepsilon_S = \varepsilon_P \frac{N_S}{N_P} = 110(10)$

$\therefore$ Output voltage $\varepsilon_S = 1100$ V

(2) $\varepsilon_P I_P = \varepsilon_S I_S \;\Rightarrow\; I_P = \frac{\varepsilon_S}{\varepsilon_P} I_S$

$\therefore I_P = \frac{N_S}{N_P} I_S = (10)(10) = 100$ A

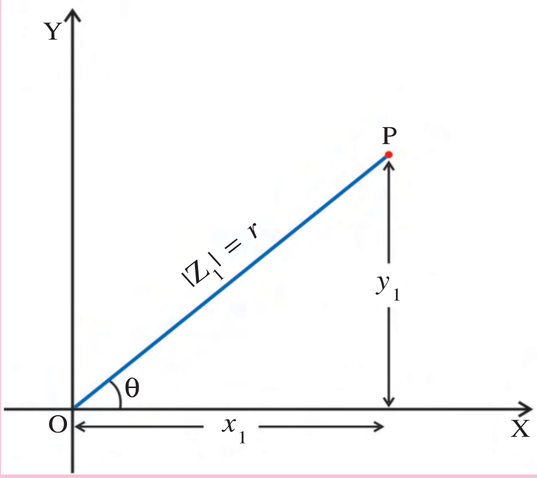
(3) Input power = Output power

$\therefore \; \varepsilon_P I_P = \varepsilon_S I_S = (1100)(10) = 11000$ W

Appendix A

Complex numbers (For Information Only) : Complex number Z is represented as $x + jy$, where $j = \sqrt{-1}$. Here x and y are real and imaginary parts of the complex number. Similarly a complex function $f(Z)$ is represented as $f(Z) = f_1(x, y) + jf_2(x, y)$. The important points related to our discussion are as follows :

(1) Any complex number can be represented by an appropriate point in a complex plane formed by the x and y variables. The number $Z_1 = x_1 + jy_1$ is represented as a point p in figure. The x co-ordinate of P gives us the real part of Z_1 and its y co-ordinate gives the imaginary part of z_1. The magnitude of the complex number is equal to r. i.e. $|Z_1| = r$.



Here, from figure,

$x_1 = r\cos\theta$ and $y_1 = r\sin\theta$

$\therefore\ Z_1 = r\cos\theta + jr\sin\theta$

$\therefore\ Z_1 = r(\cos\theta + j\sin\theta)$

since $e^{j\theta} = \cos\theta + j\sin\theta$

Hence, the complex number can also be represented as $\therefore Z = |Z|\, e^{j\theta} = re^{j\theta}$ where, $r = \sqrt{x^2+y^2}$

(2) Z* is known as the complex conjugate of the complex number Z. It is obtained by replacing j with $-j$.

$\therefore\ Z^* = x - jy$ moreover $ZZ^* = (x + jy)(x - jy) = (x^2 + y^2) = |Z|^2$

(3) We shall employ the following method to calculate the real and imaginary part of $\frac{1}{Z}$.

$$\frac{1}{Z} = \frac{Z^*}{ZZ^*} = \frac{Z^*}{|Z|^2} = \frac{(x - jy)}{(x^2 + y^2)} = \frac{x}{x^2 + y^2} - j\frac{y}{x^2 + y^2}$$

The real part of $\frac{1}{Z} = \frac{x}{x^2 + y^2}$ and the imaginary part of $\frac{1}{Z} = \frac{y}{x^2 + y^2}$.

The real part of the complex number will be represented as Re(Z) and the imaginary part as Im(Z).

Appendix B

Derivation of formula for $\Delta\omega$ (For Information Only) : When the angular frequency is equal to ω_2 we have

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2}}$$

But, $\omega_2 = \omega_0 + \frac{\Delta\omega}{2}$ (From figure 2.15)

$$\therefore \ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left\{\left(\omega_0 + \frac{\Delta\omega}{2}\right)L - \frac{1}{\left(\omega_0 + \frac{\Delta\omega}{2}\right)C}\right\}^2}} \qquad \text{(A–1)}$$

Now, substituting the value $C = \frac{1}{\omega_0^2 L}$ in the expression, $\left(\omega_0 + \frac{\Delta\omega}{2}\right)L - \frac{1}{\left(\omega_0 + \frac{\Delta\omega}{2}\right)C}$, we have,

$$\omega_0 L + \frac{(\Delta\omega)L}{2} - \frac{\omega_0^2 L}{\omega_0 + \frac{\Delta\omega}{2}}$$

$$= \frac{\omega_0^2 L + \frac{(\Delta\omega)L\omega_0}{2} + \frac{(\Delta\omega)L\omega_0}{2} + \frac{(\Delta\omega)^2 L}{4} - \omega_0^2 L}{\omega_0 + \frac{\Delta\omega}{2}}$$

$= \dfrac{(\Delta\omega)L\omega_0}{\omega_0 + \frac{\Delta\omega}{2}}$ [Here, we have ignored the term containing $(\Delta\omega)^2$]

Now, $\dfrac{(\Delta\omega)L\omega_0}{\omega_0 + \frac{\Delta\omega}{2}} = (\Delta\omega)\ L\omega_0\left(\omega_0 + \frac{\Delta\omega}{2}\right)^{-1}$

$$= (\Delta\omega)L\left(1 + \frac{\Delta\omega}{2\omega_0}\right)^{-1}$$

$$= (\Delta\omega)L\left(1 - \frac{\Delta\omega}{2\omega_0}\right)$$

$$= (\Delta\omega)L$$

(Here too, we have ignored the second and higher order terms of $(\Delta\omega)$. Substituting the above result in equation (A–1) we have,

$$\therefore \ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (L\Delta\omega)^2}} \qquad \text{(A–2)}$$

At angular frequency, ω_2 we have,

$$I_{rms} = \frac{I_{rms}\,(\max)}{\sqrt{2}} = \frac{V_{rms}}{\sqrt{2}R}$$

Substituting the above value of I_{rms} in equation (8.7.6) we have,

$$\frac{V_{rms}}{\sqrt{2}R} = \frac{V_{rms}}{\sqrt{R^2 + (L\Delta\omega)^2}}$$

$\therefore \ 2R^2 = R^2 + (L\Delta\omega)^2$

$\therefore \ R^2 = (L\Delta\omega)^2$

$\therefore \ R = L\Delta\omega \Rightarrow \Delta\omega = \dfrac{R}{L}$

SUMMARY

1. In the present chapter we have studied different A.C. circuits. We have observed equivalence between mechanical quantities and electrical quantities from the similarity between differential equation for electrical charge.

$$\frac{d^2Q}{dt^2} + \frac{L}{R}\frac{dQ}{dt} + \frac{Q}{LC} = \frac{V_m}{L}\cos\omega t$$

and the differential equation for forced escillations in mechanics

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F_0}{m}\sin\omega t.$$

For L-C-R AC. circuit the expression for complex current $i = \frac{V_m e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$ is obtained by using complex number. Above expression can be compared with Ohm's law for instantaneous values of voltage and current.

From this we note that effect of inductor and capacitor on current is similar to the effect of R on the current and can be given by $j\omega L$ and $\frac{-j}{\omega C}$ respectively.

$j\omega L$ and $\frac{-j}{\omega C}$ are known as inductive reactance of an inductor and capacitive reactance of a capacitor respectively. Their symbols are Z_L and Z_C and values are $|Z_L|$ and $|Z_C|$ and they are denoted by X_L and X_C respectively.

$\therefore\ |Z_L| = X_L = \omega L$ and $|Z_C| = X_C = \frac{1}{\omega C}$

Summation of Z_L, Z_C and R is called impedance (Z) of series circuit.

$\therefore\ Z = R + Z_L + Z_C = R + j\left(\omega L - \frac{1}{\omega C}\right).$

And its magnitude (value) $|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

We have obtained expression for complex current by solving differential equation for charge and from the real part of solution expression for current I.

$I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{V_m \cos(\omega t - \delta)}{|Z|}$ is obtained where δ is the phase difference between current and voltage which can be obtained from formula $\tan\delta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$.

2. (1) For A.C. circuit containing only inductor (L)

$Z = j\omega L = jX_L$ and $|Z| = \omega L = X_L$, $\delta = \frac{\pi}{2}$

Current $I = \frac{V_m \cos(\omega t - \frac{\pi}{2})}{\omega L} = \frac{V_m \cos(\omega t - \frac{\pi}{2})}{X_L}$

(2) For A.C. circuit containing only capacitor (C)

$$Z = \frac{-j}{\omega C} \text{ and } |Z| = \frac{1}{\omega C} = X_C,\ \delta = -\frac{\pi}{2}$$

$$\text{Current } I = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{\left(\frac{1}{\omega C}\right)} = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{X_C}$$

(3) For A.C. containing R and L in series

$$Z = R + j\omega L \ \therefore\ Z\sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X_L^2}$$

$$\delta = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ and}$$

$$\text{Current } I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (\omega L)^2}} = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (X_L)^2}}$$

(4) For A.C. circuit containing R and C in Series

$$Z = R - \frac{j}{\omega C} = R - jX_C$$

$$\therefore |Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + X_C^2}$$

$$\text{Here, } \delta = \tan^{-1}\left(\frac{1}{\omega CR}\right) \text{ and } \delta \text{ is negative}$$

$$\text{Current } I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + X_C{}^2}}$$

(4) For A.C. circuit containing L and C in series

$$Z = j\omega L - \frac{j}{\omega C} = jX_L - jX_C$$

$$\therefore |Z| = \omega L - \frac{1}{\omega C} = X_L - X_C$$

$$\text{Considering } \omega L > \frac{1}{\omega C};\ \delta = \frac{\pi}{2}$$

$$\text{and Current } I = \frac{V_m \cos(\omega t - \frac{\pi}{2})}{\omega L - \frac{1}{\omega C}} = \frac{V_m \cos(\omega t - \frac{\pi}{2})}{X_L - X_C}$$

(5) For A.C. circuit where R is in series with the parallel combination of L and C

$$Z = R - \frac{j}{\omega C - \frac{1}{\omega L}} = R - \left(\frac{j}{X_C - X_L}\right)$$

For $\omega C > \frac{1}{\omega L}$, $\delta = \tan^{-1}\left(\frac{1}{R\,(\omega C - \frac{1}{\omega L})}\right)$

$$|Z| = \sqrt{R^2 + \frac{1}{(\omega C - \frac{1}{\omega L})}} \quad \text{and}$$

$$\text{Current } I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + \frac{1}{(\omega C - \frac{1}{\omega L})^2}}}$$

3. Formula for r.m.s. values of voltage and current are

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \text{and } I_{rms} = \frac{I_m}{\sqrt{2}}$$

where V_m and I_m are the maximum values of voltage and current respectively.

4. At resonance in L-C-R circuit $\omega_0 L = \frac{1}{\omega_0 C}$ where, ω_0 is the angular frequence and current will be $I_{rms} = \frac{V_{rms}}{R}$

As $\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

Q-factor (quality factor) $= \frac{\omega_0}{\Delta\omega}$

Here $\Delta\omega$ is known as Half Power Bandwidth and $\Delta\omega = \frac{R}{L}$

$\therefore\ Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Q factor determines sharpness of the $I_{rms} \rightarrow \omega$ curve.

5. We saw that in different cases of A.C. circuit the phase difference between voltage and current can be obtained easily using phasor.

6. It is also observed from the oscillations of the charge in L–C tank circuit that when maximum charge is on the plates of capacitor the total energy is stored in the electric field produced in the capacitor and when maximum current is flowing in the inductor total energy is stored in the magnetic field produced in the inductor. Moreover, the angular frequency of the oscillations,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

7. Real power in an A.C. circuit is given by $P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\delta$ where, V_m and I_m are the maximum voltage and current respectively and δ is the phase difference between voltage and current. Here, $\cos\delta$ is known as power factor.

(i) When only resistance R is there in the circuit,

$$\delta = 0 \Rightarrow \cos\delta = 1$$

$$\therefore P = V_{rms} I_{rms}$$

(ii) When only inductor is there in the circuit,

$$\delta = \frac{\pi}{2} \Rightarrow \cos\delta = 0$$

$$\therefore P = V_{rms} I_{rms} (0) = 0$$

(iii) When only capacitor is there in the circuit,

$$\delta = -\frac{\pi}{2} \Rightarrow \cos\delta = 0$$

$$\therefore P = V_{rms} I_{rms} (0) = 0$$

Thus, in an A.C. circuit containing only inductor or only capacitor power P = 0. In this situation current flowing in the circuit is called wattless current.

8. A.C. voltage can be increased or decreased with the help of the transformer. The transformer which increases A.C. voltage is called step-up transformer and the transformer which decreases A.C. voltage is called step-down transformer.

In an ideal transformer,

Instantaneous input power $(I_P \varepsilon_P)$ = Instantaneous output power $(I_S \varepsilon_S)$

Transformer works on the principle of electromagnetic induction.

$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = r$$

Here, r is called transformation ratio.

In the transformers used in practice some part of the electrical power in the primary coil is wasted in the form of heat and some part is used in magnetzing and demagnetizing the core and producing eddy currents. Hence the output power is less than the input power.

EXERCISE

For the following statements choose the correct option from the given options :

1. In an A.C. circuit in 1 second current reduces to zero value 120 times. Hence the frequency of A.C. current is .... Hz.

(A) 50 (B) 100 (C) 60 (D) 120

2. In L–R A.C. circuit at time t current is I and time rate of change of current is $\frac{dI}{dt}$ what will be the potential difference between the two ends of the inductor ?

(A) $L\frac{dI}{dt}$ (B) $\frac{1}{L}\frac{dI}{dt}$ (C) LI (D) $\frac{L}{I}$

3. On decreasing the angular frequency of A.C. source used in L-C-R series circuit, the capacitive reactance .... and inductive reactance ....

(A) Increases, Decreases (B) Increases, Increases

(C) Decreases, Increases (D) Decreases, Decreases

4. When does the impedance of a series L-C-R AC circuit become minimum ?

(A) When the resistance is equal to zero.

(B) When the impedance is equal to zero.

(C) When the electric current is equal to zero.

(D) When the imaginary part of the impedance is equal to zero.

5. The Value of the Q factor in an L-C-R series circuit is ....

(A) dependent on the frequency of the A.C. source.

(B) dependent on the values of all the three components L, R and C.

(C) dependent only on the values of L and C.

(D) it may or may not depend on the power factor.

6. V and I are given by the following equation in an A.C. circuit :

$V = 100 \sin(100t)$ V, $I = 100 \sin\left(100t + \frac{\pi}{3}\right)$ mA. The power in the circuit is equal to .... W.

(A) 10^4 (B) 10 (C) 2.5 (C) 5.0

7. Current of $\frac{50}{\pi}$ Hz frequency is passing through an A.C. circuit having series combination of resistance $R = 100\ \Omega$ and inductor $L = 1$ H, then phase difference between voltage and current is ....

(A) 60° (B) 45° (C) 30° (D) 90°

8. If in an A.C. L–C series circuit $X_L > X_C$. Hence current ....

(A) lags behind the voltage by $\frac{\pi}{2}$ in phase (B) leads the voltage by $\frac{\pi}{2}$ in phase

(C) leads the voltage by π in phase (D) lags behind the voltage by π in phase

9. What is the r.m.s. value of the current for A.C. current $I = 100\cos(200t + 45^\circ)$ A.

(A) $50\sqrt{2}$ A (B) 100 A (C) $100\sqrt{2}$ A (D) Zero

10. Resonance frequency for L-C-R A.C. series circuit is f_0 = ...

(A) $\frac{1}{2\pi\sqrt{LC}}$ (B) $\frac{2\pi}{\sqrt{LC}}$ (C) $\frac{\sqrt{LC}}{2\pi}$ (D) $\frac{2\pi}{LC}$

11. A coil of inductance L and resistance R is connected to an A.C. source of V volt. If the angular frequency of the A.C. source is equal to ω rads^{-1}, then the current in the circuit will be ....

(A) $\frac{V}{R}$ (B) $\frac{V}{L}$ (C) $\frac{V}{R+L}$ (D) $\frac{V}{\sqrt{R^2 + \omega^2 L^2}}$

12. One inductor (of inductance L henry) is connected to an A.C. source, then the current flowing through the inductor I = ... A.

(A) $\frac{V_0}{\omega L}\sin\left(\omega t+\frac{\pi}{2}\right)$ (B) $\frac{V_0}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right)$

(C) $V_0 \omega L \sin\left(\omega t-\frac{\pi}{2}\right)$ (D) $\frac{\omega L}{V_0}\sin\left(\omega t+\frac{\pi}{2}\right)$

13. The potential difference between the two ends of the three components of L-C-R series A.C. circuit are V_L, V_C and V_R respectively. Then voltage of A.C. source is ....

(A) $V_L + V_C + V_R$ (B) $V_R + V_L - V_C$

(C) $\sqrt{V_R^2 + (V_L + V_C)^2}$ (D) $\sqrt{V_R^2 + (V_L - V_C)^2}$

14. In R–C circuit when charge on the plates of the capacitor is increasing, the energy obtained from the source is stored in ....

(A) electric field (B) magnetic field

(C) gravitational field (D) both magnetic field and gravitational field

15. In an oscillating L-C circuit the maximum charge on the capacitor is Q. What will be the charge on the plate of the capacitor, when energy stored in magnetic field and electric field are equal ?

(A) $\frac{Q}{3}$ (B) $\frac{Q}{\sqrt{2}}$ (C) Q (D) $\frac{Q}{2}$

16. For L–C–R A.C. circuit resonance frequency is 600 Hz and frequencies at half power points are 550 Hz and 650 Hz. What will be the Q-factor ?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 6 (D) 3

17. An alternating voltage given as $V = 200\sqrt{2}\sin 100t$ V is applied to a capacitor of 1μF. The current reading of the ammeter will be equal to .... mA.

(A) 100 (B) 20 (C) 40 (D) 80

18. The power in an A.C. circuit is given as $P = V_{rms} I_{rms} \cos\delta$. The power factor at the resonance frequency of a series L–C–R circuit will be ....

(A) zero (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

19. The output power in a step-up transformer is ....

(A) greater than the input power (B) equal to the input power

(C) maintained even during the power cut (D) less than the input power

20. In an L-C oscillator circuit having a completely charged capacitor, with the passage of time ....

(A) The electric current increases gradually.

(B) The energy of the circuit continuously increases.

(C) The energy of the circuit continuously decreases.

(D) There is a continuous absorption of the electromagnetic wave.

21. The equivalent inductance of two inductors is 2.4 H when connected in paralled and 10 H when connected in series, then the individual inductance is

(A) 6 H, 4 H (B) 5 H, 5 H (C) 7 H, 3 H (D) 8 H, 2 H

22. Which device is used to increase or decrease A.C. voltage ?

(A) Oscillator (B) Voltmeter (C) Transformer (D) Rectifier

23. For step-down transformer value of transformation ratio is ....

(A) $r > 1$ (B) $r < 1$ (C) $r = 1$ (D) $r = 0$

24. If for an ideal step-up transformer current in primary is I_P and current in secondary is I_S, their respective voltages are V_P and V_S, then

(A) $I_S V_S = I_P V_P$ (B) $I_S V_S > I_P V_P$ (C) $I_S V_S < I_P V_P$ (D) $I_S V_P = I_P V_S$

25. In an A.C. circuit current is 2 A and voltage is 220 V and power is 40 W power factor is ....

(A) 0.9 (B) 0.09 (C) 1.8 (D) 0.18

ANSWERS

1. (C)	2. (A)	3. (A)	4. (D)	5. (B)	6. (C)
7. (B)	8. (A)	9. (A)	10. (A)	11. (D)	12. (B)
13. (A)	14. (A)	15. (B)	16. (C)	17. (B)	18. (B)
19. (D)	20. (C)	21. (A)	22. (C)	23. (B)	24. (A)
25. (B)					

EXERCISE

Answer the following questions in brief :

1. Which value of the A.C. voltage can be measured by an A.C. voltmeter ?
2. Give units of $\frac{1}{\omega L}$ and ωC.
3. At resonance what is the phase difference between voltage and current in an A.C. L-C-R series circuit ?
4. State the condition for resonance in an A.C., L-C-R series circuit.
5. Which quantity enables us to know sharpness of resonance ?
6. Define half power band width.
7. On which factors does Q-factor depend ?
8. Define real power.
9. Give relation between output power and input power for an ideal transformer.
10. Give relation between real power and maximum power.
11. What does Q-factor give measure of ?
12. What is emitted by an oscillating charge ?
13. On which principle does a transformer work ?
14. What is done to reduce the effects of eddy currents in a transformer ?
15. What is meant by transformation ratio ?
16. What is meant by an ideal transformer ?

17. Give maximum value of the energy associated with an inductor in L-C oscillator ?
18. Give maximum value of the energy associated with a capacitor in an L-C oscillator ?
19. What is meant by r.m.s. value ?

Answer the following questions :

1. Explain alternating current using circuit containing only resistance R and drawing graph of current and time ($I \rightarrow t$).
2. L, C and R are connected in series to an A.C. voltage $V = V_m \cos\omega t$. Obtain the differential equation for the charge.
3. Write the differential equation for current in an A.C. L–C–R series circuit in complex form and derive expression for complex current.
4. Write the formula for impedance of A.C., L–C–R series circuit and represent it in complex plane. Hence obtain the equation of magnitude of Z and phase difference.
5. Obtain the expression for current in an A.C. circuit containing only inductor. (Draw necessary figure and graph.)
6. Obtain the expression for current in an A.C. circuit containing only capacitor (Draw necessary figure and graph.)
7. Obtain the expression for the current in an A.C. circuit containing resistor and an inductor in series (Draw necessary figure and graph.)
8. Obtain the expression for the current in an A.C. circuit containing resistor and the capacitor in series (Draw necessary figure and graph.)
9. Obtain the expression for the current in an A.C. circuit containing an inductor and a capacitor in series (Draw necessary figure and graph.)
10. Obtain the expression for resonance frequency and rms current at resonance in an A.C. L–C–R series circuit using expression for rms current I_{rms}.
11. Draw the graph of $I_{rms} \rightarrow \omega$ for an A.C., L–C–R series circuit and hence explain Q-factor.
12. Using the expressions for charge and current for L–C oscillator, explain L–C oscillations.
13. Derive expression $P = V_{rms} I_{rms} \cos\delta$ for an A.C. circuit.
14. Using $P = V_{rms} I_{rms} \cos\delta$ discuss the special cases for power consumed in an A.C. circuit.
15. Explain necessity of transformer for power transmission and distribution.

Solve the following examples :

1. Find the necessary inductance, if 110 V, 10 W ratting bulb is to be used with 220 V A.C. source having frequency 50 Hz. [**Ans. :** L = 6.67 H]
2. L = 8.1 mH, C = 12.5 μF and R = 100 Ω are connected in series with A.C. source of 230V and frequency 500 Hz. Calculate voltage across the two ends of resistance. [**Ans. :** 230 V]
3. In medium wave broadcast a radio can be tuned in the frequency range 800 kHz to 1200 kHz. In L–C circuit of this radio effective inductance is 200 μH, what should be the range of the variable capacitor ? [**Ans. :** 88 pF to 198 pF]
4. An inductor of 0.5 H and 200 Ω resistor are connected in series with A.C. source of 230 V and frequency 50 Hz, then calculate : (1) Maximum current in the inductor (2) Phase difference between current and voltage and time difference (time lag) [**Ans. :** I_{max} = 1.28 A, 38°, 8' and 2.1 ms]

5. In an ideal transformer input A.C. voltage is 220 V. Current in secondary coil is 2.5 A. If the ratio of number of turns in primary coil and secondary coil is 1 : 10, find

(1) Output voltage

(2) Current in primary coil

(3) Input and output power. [**Ans. :** 2200 V, 25 A, 5500 W]

6. In an A.C. circuit L and R connected in series. Maximum A.C. voltage is 220 volts. If reactance of inductor is 60 Ω and resistance is 80 Ω, find power and power factor.

[**Ans. :** 193.6 W, 0.8]

7. Prove that the average value of an A.C. voltage source is given by $V = V_m \sin\omega t$ is equal to $\frac{2V_m}{\pi}$ for half period of its cycle.

8. The value of the A.C. voltage of a generator is V = 0 volts at time t = 0. At time $t = \frac{1}{100\pi}$ seconds, the voltage V = 2 volts. The voltage keeps on increasing upto 100 volts. After that it starts decreasing. Determine the frequency of the voltage source.

[**Ans. :** 1 Hz]

9. An A.C. circuit contains only an inductor. The frequency of voltage source is 159.2 Hz. And V_m = 100 V. The inductance of the inductor is L = 1 H. Obtain the expression for the current flowing in the circuit. Consider voltage to be $V = V_m \cos\omega t$.

[**Ans. :** $I = 0.1\cos\left(1000\,t - \frac{\pi}{2}\right)$ A]

10. In an A.C. circuit maximum voltage and maximum currents are 220 V and 4.4 A respectively. Calculate power and power factor in the circuit. (Here X_C = 30 Ω and R = 40 Ω)

[**Ans. :** 387.2 W, 0.8]

11. In an A.C. circuit R and L are connected in series with source of maximum 220 V and frequency 50 Hz. If L = 1 H and R = 100 Ω. Find the maximum current in inductor and phase difference between voltage and current.

[**Ans. :** 0.668 A, 72°, 20′]

12. A.C. Current is given by following formula $I = I_1 \sin\omega t + I_2 \cos\omega t$. Show that rms value of this current given by $I_{rms} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$.

13. The tank circuit of L-C oscillator contains a capacitor of 30 μF and an inductor of 27 mH. Find the natural angular frequency of oscillations. [**Ans. :** 1.111×10^3 rads^{-1}]

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