6. Gravitation

6.1 Newton's law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the particles.

Thus the magnitude of the gravitational force F that two particles of masses m_1 and m_2 separated by a distance r exert on each other is given by $F \propto \frac{m_1 m_2}{r^2}$.

or $F = G\frac{m_1 m_2}{r^2}$

Also clear that $\vec{F}_{12} = -\vec{F}_{21}$. Which is Newton's third law of motion.

Here G is constant of proportionality which is called 'Universal gravitational constant'.

(i) The value of G is 6.67×10^{-11} $N\text{–}m^2\ kg^{-2}$ in S.I. and 6.67×10^{-8} $dyne\text{-}\ cm^2\text{-}g^{-2}$ in C.G.S. system.

(ii) Dimensional formula $[M^{-1}L^3T^{-2}]$.

(iii) The value of G does not depend upon the nature and size of the bodies.

(iv) It also does not depend upon the nature of the medium between the two bodies.

6.2 Properties of Gravitational Force

(1) It is always attractive in nature.

(2) It is independent of the medium between the particles.

(3) It is found true for interplanetary to inter atomic distances.

(4) It is a central force *i.e.* acts along the line joining the centres of two interacting bodies.

(5) The principle of superposition is valid.

(6) It is the weakest force in nature .

(7) It is a conservative force.

- The law of gravitation is stated for two point masses

6.3 Acceleration Due to Gravity

The force of attraction exerted by the earth on a body is called gravitational pull or gravity.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by g.

If M = mass of the earth and R = radius of the earth and g is the acceleration due to gravity, then

$$\therefore \quad g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$$

(i) Its value depends upon the mass radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet.

(ii) Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.

(iii) Dimension $[g] = [LT^{-2}]$

(iv) It's average value is taken to be 9.8 m/s^2 or 981 cm/sec^2 or 32 $feet/sec^2$, on the surface of the earth at mean sea level.

6.4 Variation in *g* Due to Shape of Earth

Earth is elliptical in shape. The equatorial radius is about 21 *km* longer than polar radius.

At equator $$g_e = \frac{GM}{R_e^2}$$

At poles $$g_p = \frac{GM}{R_p^2}$$

$\therefore \; g_{pole} > g_{equator}$

Therefore the weight of body increases as it is taken from equator to the pole.

6.5 Variation in *g* with Height

Acceleration due to gravity at height h from the surface of the earth

$$g' = \frac{GM}{(R+h)^2}$$

Also $$g' = g\left(\frac{R}{R+h}\right)^2$$

$$= g\frac{R^2}{r^2} \qquad [\text{As } r = R + h]$$

(i) If $h << R$ $$g' = g\left[1 - \frac{2h}{R}\right]$$

(ii) If $h << R$ Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} \times 100\%$

6.6 Variation in *g* With Depth

Acceleration due to gravity at depth d from the surface of the earth

$$g' = \frac{4}{3}\pi\rho G(R-d)$$

also $$g' = g\left[1 - \frac{d}{R}\right]$$

(i) The value of g decreases on going below the surface of the earth.

(ii) The acceleration due to gravity at the centre of earth becomes zero.

(iii) Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} \times 100\%$

(iv) The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

6.7 Variation in *g* Due to Rotation of Earth

If the body of mass m lying at point P, whose latitude is λ, then due to rotation of earth its apparent acceleration can be given by $g' = g - \omega^2 R \cos^2 \lambda$.

- The latitude at a point on the surface of the earth is defined as the angle, which the line joining that point to the centre of earth makes with equatorial plane. It is denoted by λ.
- For the poles $\lambda = 90°$ and for equator $\lambda = 0°$
  - (i) $g_{pole} = g$ (ii) $g_{equator} = g - \omega^2 R$
  - (iii) If ω is the angular velocity of rotation of earth for which a body at the equator will become weightless ($g' = 0$)

$$\omega = \sqrt{\frac{g}{R}}$$

or time period of rotation of earth $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

- If earth starts rotating 17 times faster then all objects on equator will become weightless.

6.8 Inertial and Gravitational Masses

(1) ***Inertial mass:*** It is the mass of the material body, which measures its inertia.

$$m_i = \frac{F}{a}$$

Hence inertial mass of a body may be measured as the ratio of the magnitude of the external force applied on it to the magnitude of acceleration produced in its motion.

(i) Gravity has no effect on inertial mass of the body.
(ii) It is proportional to the quantity of matter contained in the body.
(iii) When a body moves with velocity v, its inertial mass is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$, where m_0 = rest mass of body, c = velocity of light in vacuum,

(2) ***Gravitational Mass:*** Gravitational mass of a body may be measured as the ratio of the magnitude of the gravitational force applied on it to the magnitude of acceleration due to gravity.

Spring balance measure gravitational mass and inertial balance measure inertial mass.

6.9 Gravitational Field

The space surrounding a material body in which gravitational force of attraction can be experienced is called its gravitational field.

Gravitational field intensity: The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (test mass) placed at that point. If a test mass m at a point in a gravitational field experiences a force $\vec{F}$ then $\vec{I} = \frac{\vec{F}}{m}$.

(i) It is a vector quantity and is always directed towards the centre of gravity of body whose gravitational field is considered.

(ii) Units: $Newton/kg$ or m/s^2

(iii) Dimension: $[M^0LT^{-2}]$

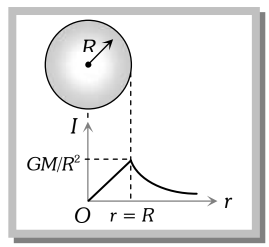
(iv) $I = \frac{GM}{r^2}$

(v) $\overrightarrow{I_{net}} = \vec{I_1} + \vec{I_2} + \vec{I_3} + \ldots\ldots$

6.10 Gravitational Field Intensity for Different Bodies

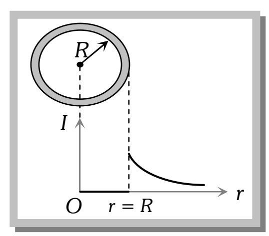
(1) ***Intensity due to uniform solid sphere***

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = \frac{GMr}{R^3}$



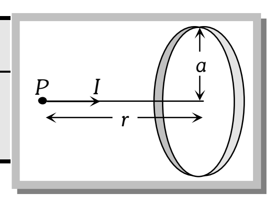
(2) ***Intensity due to spherical shell***

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = 0$



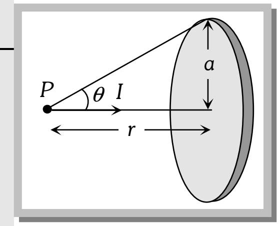
(3) ***Intensity due to uniform circular ring***

At a point on its axis	At the centre of the ring
$I = \frac{GMr}{(a^2 + r^2)^{3/2}}$	$I = 0$



(4) ***Intensity due to uniform disc***

At a point on its axis	At the centre of the disc
$I = \frac{2GMr}{a^2}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2+a^2}}\right]$ or $I = \frac{2GM}{a^2}(1-\cos\theta)$	$I = 0$



6.11 Gravitational Potential

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point *i.e.,*

$$V = -\frac{W}{m} = -\int \frac{\vec{F}.d\vec{r}}{m} = -\int \vec{I}.d\vec{r}$$

$$\therefore \quad I = -\frac{dV}{dr}$$

Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

(i) It is a scalar quantity.

(ii) Unit: *Joule / kg* or m^2/sec^2

(iii) Dimension: $[M^0L^2T^{-2}]$

(iv) If the field is produced by a point mass then

$$\therefore \quad \text{Gravitational potential } V = -\frac{GM}{r}$$

(v) ***Gravitational potential difference:*** It is defined as the work done to move a unit mass from one point to the other in the gravitational field. The gravitational potential difference in bringing unit test mass m from point A to point B under the gravitational influence of source mass M is

$$\Delta V = V_B - V_A = \frac{W_{A\to B}}{m} = -GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

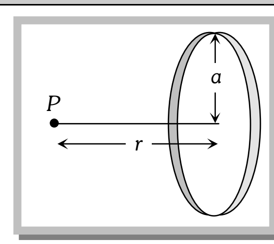
(vi) Potential due to large numbers of particle is given by scalar addition of all the potentials.

$$V = V_1 + V_2 + V_3 + ...$$

6.12 Gravitational Potential for Different Bodies

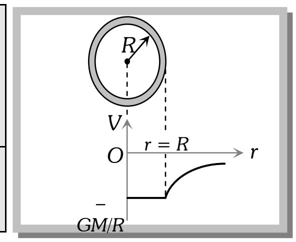
(1) ***Potential due to uniform ring***

At a point on its axis	At the centre
$V = -\frac{GM}{\sqrt{a^2+r^2}}$	$V = -\frac{GM}{a}$



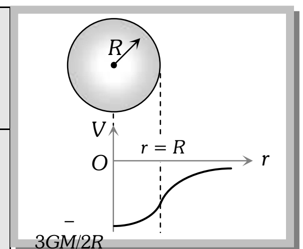
(2) ***Potential due to spherical shell***

Outside the surface $r > R$	**On the surface** $r = R$	**Inside the surface** $r < R$
$V = \frac{-GM}{r}$	$V = \frac{-GM}{R}$	$V = \frac{-GM}{R}$



(3) ***Potential due to uniform solid sphere***

Outside the surface $r > R$	**On the surface** $r = R$	**Inside the surface** $r < R$
$V = \frac{-GM}{r}$	$V_{surface} = \frac{-GM}{R}$	$V = \frac{-GM}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]$ at the centre ($r = 0$) $V_{centre} = \frac{-3}{2}\frac{GM}{R}$ $(max.)$ $V_{\text{centre}} = \frac{3}{2}V_{surface}$



6.13 Gravitational Potential Energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore \quad U = -\frac{GMm}{r}$$

(i) Potential energy is a scalar quantity.

(ii) *Unit:* Joule

(iii) *Dimension*: $[ML^2T^{-2}]$

(iv) Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

(v) If $r = \infty$ then it becomes zero (maximum)

(vi) In case of discrete distribution of masses

Gravitational potential energy $U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + \ldots\ldots\right]$

(vii) If the body of mass m is moved from a point at a distance r_1 to $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

(viii) Relation between gravitational potential energy and potential $U = mV$

(ix) Gravitational potential energy of a body at height h from the earth surface is given by

$$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h} \equiv -\frac{mgR}{1+\frac{h}{R}}$$

6.14 Work Done Against Gravity

If the body of mess m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$W = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}}$$

(i) When the distance h is not negligible and is comparable to radius of the earth, then we will use above formula.

(ii) If $h = R$ then $W = \frac{1}{2}mgR$

(iv) If h is very small as compared to radius of the earth then term h/R can be neglected

$$W = \frac{mgh}{1+h/R} = mgh$$

6.15 Escape Velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

If v_e is the required escape velocity, then

$$v_e = \sqrt{\frac{2GM}{R}} \quad \Rightarrow \quad v_e = \sqrt{2gR}$$

(i) Escape velocity is independent of the mass and direction of projection of the body.

(ii) For the earth

$$v_e = 11.2 \text{ km/sec}$$

(iii) A planet will have atmosphere if the velocity of molecule in its atmosphere is lesser than escape velocity. This is why earth has atmosphere while moon has no atmosphere

6.16 Kepler's Laws of Planetary Motion

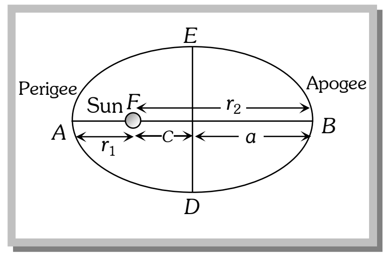
(1) ***The law of Orbits:*** Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

(2) ***The law of Area:*** The line joining the sun to the planet sweeps out equal areas in equal interval of time. *i.e.* areal velocity is constant. According to this law planet will move

slowly when it is farthest from sun and more rapidly when it is nearest to sun. It is similar to law of conservation of angular momentum.

Areal velocity $\frac{dA}{dt} = \frac{L}{2m}$

(3) ***The law of periods***: The square of period of revolution (T) of any planet around sun is directly proportional to the cube of the semi-major axis of the orbit.



$$T^2 \propto a^3 \text{ or } T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$

where a = semi-major axis

r_1 = Shortest distance of planet from sun (perigee).

r_2 = Largest distance of planet from sun (apogee).

❑ Kepler's laws are valid for satellites also.

6.17 Velocity of a Planet in Terms of Eccentricity

Speeds of planet at apogee and perigee are

$$v_a = \sqrt{\frac{2GM}{a}\left(\frac{1-e}{1+e}\right)}, \qquad v_p = \sqrt{\frac{2GM}{a}\left(\frac{1+e}{1-e}\right)}$$

❑ Angular momentum of a planet or satellite is always constant irrespective of shape of orbit.

6.18 Orbital Velocity of Satellite

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}} \qquad [r = R + h]$$

(i) Orbital velocity is independent of the mass of the orbiting body.

(ii) Orbital velocity depends on the mass of planet and radius of orbit.

(iii) Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \approx 8 \text{ km/sec}$$

6.19 Time Period of Satellite

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \qquad [\text{As } r = R + h]$$

(i) Time period is independent of the mass of orbiting body

(ii) $T^2 \propto r^3$ (Kepler's third law)

(iii) Time period of nearby satellite, $T = 2\theta\sqrt{\frac{R}{g}}$

For earth $\quad T$ = 84.6 minute ≈ 1.4 hr.

6.20 Height of Satellite

$$h = \left(\frac{T^2 g R^2}{4\pi^2}\right)^{1/3} - R$$

6.21 Geostationary Satellite

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

A geostationary satellite always stays over the same place above the earth. The orbit of a geostationary satellite is known as the parking orbit.

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane.
(ii) It sense of rotation should be same as that of earth.
(iii) Its period of revolution around the earth should be same as that of earth.
(iv) Height of geostationary satellite from the surface of earth $h = 6R = 36000$ km
(v) Orbital velocity $v = 3.08$ km/sec
(vi) Angular momentum of satellite depend on both the mass of orbiting and planet as well as the radius of orbit.

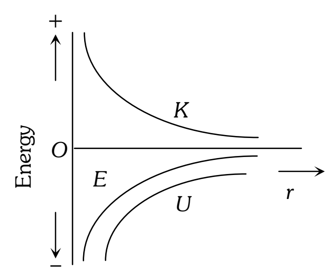
6.22 Energy of Satellite

(1) ***Potential energy:*** $U = mV = \frac{-GMm}{r} = \frac{-L^2}{mr^2}$

(2) ***Kinetic energy:*** $K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$

(3) ***Total energy:*** $E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$

(4) ***Energy graph for a satellite***



(5) ***Binding Energy:*** The energy required to remove the satellite from its orbit to infinity is called Binding Energy of the system, *i.e.*, Binding Energy (B.E.) $= -E = \frac{GMm}{2r}$

6.23 Weightlessness

The state of weightlessness (zero weight) can be observed in the following situations.

(1) When objects fall freely under gravity
(2) When a satellite revolves in its orbit around the earth
(3) When bodies are at null points in outer space. The zero gravity region is called null point.

7. Properties of Matter

7.1 Interatomic Forces

The forces between the atoms due to electrostatic interaction between the charges of the atoms are called interatomic forces.

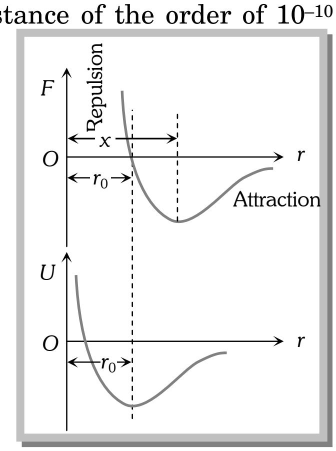
(1) When two atoms are brought close to each other to a distance of the order of 10^{-10} m, attractive interatomic force is produced between two atoms.

(2) This attractive force increases continuously with decrease in r and becomes maximum for one value of r called critical distance, represented by x (as shown in the figure).

(3) When the distance between the two atoms becomes r_0, the interatomic force will be zero. This distance r_0 is called normal or equilibrium distance.

(4) When the distance between the two atoms further decreased, the interatomic force becomes repulsive in nature and increases very rapidly.

(5) The potential energy U is related with the interatomic force F by the following relation.



$$F = \frac{-dU}{dr}$$

When the distance between the two atoms becomes r_0, the potential energy of the system of two atoms becomes minimum (*i.e.* attains maximum negative value hence the two atoms at separation r_0 will be in a state of equilibrium.

7.2 Intermolecular Forces

The forces between the molecules due to electrostatic interaction between the charges of the molecules are called intermolecular forces. These forces are also called Vander Waal forces and are quite weak as compared to inter-atomic forces.

7.3 Comparison between Interatomic and Intermolecular Forces

(i) Both the forces are electrical in origin.

(ii) Both the forces are active over short distances.

(iii) General shape of force-distance graph is similar for both the forces.

(iv) Both the forces are attractive up to certain distance between atoms/molecules and become repulsive when the distance between them become less than that value.

7.4 Solids

A solid is that state of matter in which its constituent atoms or molecules are held strongly at the position of minimum potential energy and it has a definite shape and volume.

7.5 Elastic Property of Matter

(1) ***Elasticity:*** The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

(2) ***Plasticity:*** The property of matter by virtue of which it does not regain its original shape and size after the removal of deforming force is called plasticity.

(3) ***Perfectly elastic body:*** If on the removal of deforming forces the body regain its original configuration completely it is said to be perfectly elastic.

A quartz fibre and phosphor is the nearest approach to the perfectly elastic body.

(4) ***Perfectly plastic body:*** If the body does not have any tendency to recover its original configuration, on the removal of deforming force, it is said to be perfectly plastic.

Paraffin wax, wet clay are the nearest approach to the perfectly plastic body.

Practically there is no material which is either perfectly elastic or perfectly plastic.

(5) ***Reason of elasticity:*** On applying the deforming forces, restoring forces are developed. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium position ($r = r_0$) and hence the body regains its original form.

(6) ***Elastic limit:*** The maximum deforming force upto which a body retains its property of elasticity is called elastic limit of the material of body.

Elastic limit is the property of a body whereas elasticity is the property of material of the body.

(7) ***Elastic fatigue:*** The temporary loss of elastic properties because of the action of repeated alternating deforming force is called elastic fatigue.

It is due to this reason:

(i) Bridges are declared unsafe after a long time of their use.

(ii) Spring balances show wrong readings after they have been used for a long time.

(iii) We are able to break the wire by repeated bending.

(8) ***Elastic after effect:*** The time delay in which the substance regains its original condition after the removal of deforming force is called elastic after effect. it is negligible for perfectly elastic substance, like quartz, phosphor bronze and large for glass fibre.

7.6 Stress

The internal restoring force acting per unit area of cross section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Unit: N/m^2 (S.I.), $dyne/cm^2$ (C.G.S.)

Stress developed in a body depends upon how the external forces are applied over it.

On this basis there are two types of stresses: Normal and Shear or tangential stress

(1) ***Normal stress:*** Here the force is applied normal to the surface.

It is again of two types: Longitudinal and Bulk or volume stress

(i) *Longitudinal stress*

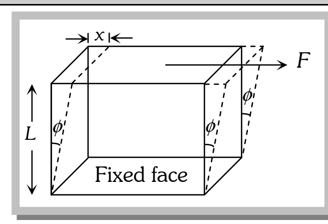
(a) Deforming force is applied parallel to the length and causes increase in length.
(b) Area taken for calculation of stress is area of cross section.
(c) Longitudinal stress produced due to increase in length of a body under a deforming force is called tensile stress.
(d) Longitudinal stress produced due to decrease in length of a body under a deforming force is called compressional stress.

(ii) *Bulk or Volume stress*
(a) It occurs in solids, liquids or gases.
(b) Deforming force is applied normal to surface at all points.
(c) It is equal to change in pressure because change in pressure is responsible for change in volume.

(2) ***Shear or tangential stress:*** It comes in picture when successive layers of solid move on each other *i.e.* when there is a relative displacement between various layers of solid.
(i) Here deforming force is applied tangential to one of the faces.
(ii) Area for calculation is the area of the face on which force is applied.
(iii) It produces change in shape, volume remaining the same.

7.7 Strain

The ratio of change in configuration to the original configuration is called strain. It has no dimensions and units. Strain are of three types:



(1) ***Linear strain:*** Linear strain = $\frac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$

Linear strain in the direction of deforming force is called longitudinal strain and in a direction perpendicular to force is called lateral strain.

(2) ***Volumetric strain:*** Volumetric strain = $\frac{\text{Change in volume}(\Delta V)}{\text{Original volume}(V)}$

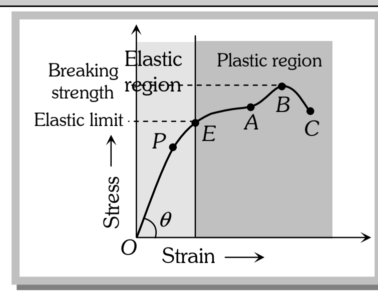
(3) ***Shearing strain:*** It is defined as angle in radians through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

$$\phi = \frac{x}{L}$$

❑ When a beam is bent both compression strain as well as an extension strain is produced.

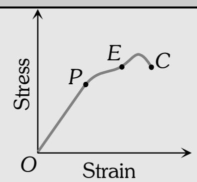
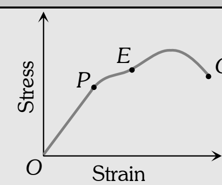
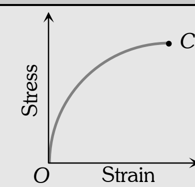
7.8 Stress-strain Curve

(1) When the strain is small (region OP) stress is proportional to strain. This is the region where the so called Hooke's law is obeyed. The point P is called limit of proportionality and slope of line OP gives the



Young's modulus Y of the material of the wire. $Y = \tan\theta$.

(2) Point E known as elastic limit or yield-point.
(3) Between EA, the strain increases much more.
(4) The region $EABC$ represents the plastic behaviour of the material of wire.
(5) Stress-strain curve for different materials.

Brittle material	**Ductile material**	**Elastomers**
		
The plastic region between E and C is small for brittle material and it will break soon after the elastic limit is crossed.	The material of the wire have a good plastic range and such materials can be easily changed into different shapes and can be drawn into thin wires.	for elastomers and strain produced is much larger than the stress applied. Such materials have no plastic range and the breaking point lies very close to elastic limit. Example rubber.

7.9 Hooke's law and Modulus of Elasticity

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain or $\dfrac{\text{stress}}{\text{strain}} = \text{constant} = E$

The constant E is called modulus of elasticity.

(1) It's value depends upon the nature of material of the body and the manner in which the body is deformed.
(2) It's value depends upon the temperature of the body.
(3) It's value is independent of the dimensions of the body.

There are three modulii of elasticity namely Young's modulus (Y), Bulk modulus (K) and modulus of rigidity (η) corresponding to three types of the strain.

7.10 Young's Modulus (Y)

It is defined as the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{Al}$$

Thermal stress: If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal. This stress is called thermal stress. Thermal stress = $Y\alpha\Delta\theta$ force produced in the body = $YA\alpha\,\Delta\theta$

7.11 Work Done in Stretching a Wire

In stretching a wire work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

$\therefore$ Energy stored in wire $U = \frac{1}{2}\frac{YAl^2}{L} = \frac{1}{2}Fl$

Energy stored in per unit volume of wire.

$$= \frac{1}{2}\times \text{stress}\times\text{strain} = \frac{1}{2}\times Y\times(\text{strain})^2 = \frac{1}{2Y}(\text{stress})^2$$

7.12 Breaking of Wire

When the wire is loaded beyond the elastic limit, then strain increases much more rapidly. The maximum stress corresponding to B (see stress-strain curve) after which the wire begin to flow and breaks, is called breaking stress or tensile strength and the force by application of which the wire breaks is called the breaking force.

(i) Breaking force depends upon the area of cross-section of the wire
(ii) Breaking stress is a constant for a given
(iii) Breaking force is independent of the length of wire.
(iv) Breaking force $\propto \pi r^2$.
(v) Length of wire if it breaks by its own weight. $L = \frac{P}{dg}$

7.13 Bulk Modulus

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus.

This is denoted by K.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}; \qquad K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

where p = increase in pressure; V = original volume; ΔV = change in volume

The reciprocal of bulk modulus is called compressibility.

C = compressibility = $\frac{1}{K} = \frac{\Delta V}{pV}$

S.I. unit of compressibility is $N^{-1}m^2$ and C.G.S. unit is $dyne^{-1}\,cm^2$.

Gases have two bulk moduli, namely isothermal elasticity E_θ and adiabatic elasticity E_ϕ.

7.14 Density of Compressed Liquid

If a liquid of density ρ, volume V and bulk modulus K is compressed, then its density increases.

$$\rho' = \rho\left[1 + \frac{\Delta P}{K}\right] = \rho[1 + C\Delta P] \qquad \left[\text{As } \frac{1}{K} = C\right]$$

7.15 Modulus of Rigidity

Within limits of proportionality, the ratio of tangential stress to the shearing strain is called modulus of rigidity of the material of the body and is denoted by η, *i.e.*

$$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi}$$

Only solids can exhibit a shearing as these have definite shape.

7.16 Poisson's Ratio

Lateral strain: The ratio of change in radius to the original radius is called lateral strain.

Longitudinal strain: The ratio of change in length to the original length is called longitudinal strain.

The ratio of lateral strain to longitudinal strain is called Poisson's ratio (σ).

i.e. $$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

7.17 Relation Between Volumetric Strain, Lateral Strain and Poisson's Ratio

$$\sigma = \frac{1}{2}\left[1 - \frac{dV}{AdL}\right] \qquad \text{[where } A = \text{cross-section of bar]}$$

(i) If a material having $\sigma = -0.5$ then
Volume = constant *i.e.,* the material is incompressible.

(ii) Theoretical value of Poisson's ratio $-1 < \sigma < 0.5$.

(iii) Practical value of Poisson's ratio $0 < \sigma < 0.5$.

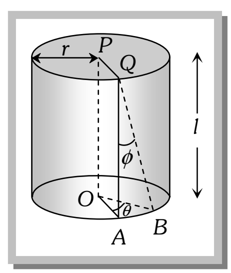
7.18 Relation between *Y*, *k*, η and σ

$$Y = 3K(1 - 2\sigma) \text{ and } Y = 2\eta(1 + \sigma); \; Y = \frac{9K\eta}{3K + \eta} \text{ and } \sigma = \frac{3K - 2\eta}{6K + 2\eta}$$

7.19 Torsion of Cylinder

If the upper end of a cylinder is clamped and a torque is applied at the lower end the cylinder gets twisted by angle θ. Simultaneously shearing strain ϕ is produced in the cylinder.

(i) The angle of twist θ is directly proportional to the distance from the fixed end of the cylinder.



(ii) The value of angle of shear ϕ is directly proportional to the radius of the cylindrical shell.

(iii) Relation between angle of twist (θ) and angle of shear (ϕ)

$$AB = r\theta = \phi l \qquad \therefore \; \phi = \frac{r\theta}{l}$$

(iv) Twisting couple per unit twist or torsional rigidity or torque required to produce unit twist.

$$C = \frac{\pi\eta r^4}{2l} \qquad \therefore \; C \propto r^4 \propto A^2$$

(v) Work done in twisting the cylinder through an angle θ is $W = \frac{1}{2}C\theta^2 = \frac{\pi\eta r^4\theta^2}{4l}$

7.20 Elastic Hysteresis

Hysteresis loop: The area of the stress-strain curve is called the hysteresis loop and it is numerically equal to the work done in loading the material and then unloading it.

7.21 Factors Affecting Elasticity

(1) ***Hammering and rolling:*** This result in increase in the elasticity of material.

(2) ***Annealing:*** Annealing results in decrease in the elasticity of material.

(3) ***Temperature:*** Elasticity decreases with rise in temperature but the elasticity of invar steel (alloy) does not change with change of temperature.

(4) ***Impurities:*** The type of effect depends upon the nature of impurities present in the material.

7.22 Important Facts about Elasticity

(1) The body which requires greater deforming force to produce a certain change in dimension is more elastic.

(2) When equal deforming force is applied on different bodies then the body which shows less deformation is more elastic.

  (i) Water is more elastic than air as volume change in water is less for same applied pressure.

  (ii) Four identical balls of different materials are dropped from the same height then after collision balls rises upto different heights.

  $h_{\text{ivory}} > h_{\text{steel}} > h_{\text{rubber}} > h_{\text{clay}}$ because $Y_{\text{ivory}} > Y_{\text{steel}} > Y_{\text{rubber}} > Y_{\text{clay}}$.

(3) For a given material there can be different moduli of elasticity depending on the type of stress and strain.

(4) $K_{\text{solid}} > K_{\text{liquid}} > K_{\text{gas}}$

(5) Elasticity of a rigid body is infinite.

7.23 Practical Applications of Elasticity

(i) The thickness of the metallic rope used in the crane is decided from the knowledge of elasticity.

(ii) Maximum height of a mountain on earth can be estimated.

(iii) A hollow shaft is stronger than a solid shaft made of same mass, length and material.

7.24 Intermolecular Force

The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic.

The intermolecular forces of attraction may be classified into two types.

Cohesive force	**Adhesive force**
The force of attraction between molecules of same substance is called the force of cohesion. This force is lesser in liquids and least in gases.	The force of attraction between the molecules of the different substances is called the force of adhesion.

7.25 Surface Tension

The property of a liquid due to which its free surface tries to have minimum surface area is called surface tension. A small liquid drop has spherical shape due to surface tension.Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid. then $T = (F/L)$.

(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

(2) It is a scalar as it has a unique direction which is not to be specified.

(3) Dimension: $[MT^{-2}]$. (Similar to force constant)

(4) Units: N/m (S.I.) and $Dyne/cm$ [C.G.S.]

7.26 Factors Affecting Surface Tension

(1) ***Temperature:*** The surface tension of liquid decreases with rise of temperature

$$T_t = T_0(1 - \alpha t)$$

where T_t, T_0 are the surface tensions at $t°C$ and 0°C respectively and α is the temperature coefficient of surface tension.

(2) ***Impurities:*** A highly soluble substance like sodium chloride when dissolved in water, increases the surface tension of water. But the sparingly soluble substances like phenol when dissolved in water, decreases the surface tension of water.

7.27 Surface Energy

The potential energy of surface molecules per unit area of the surface is called surface energy.

Unit: $Joule/m^2$ (S.I.) erg/cm^2 (C.G.S.)

Dimension: $[MT^{-2}]$

$\therefore\ W = T \times \Delta A$ [ΔA = Total increase in area of the film from both the sides]

i.e. surface tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension at constant temperature.

7.28 Work Done in Blowing a Liquid Drop or Soap Bubble

(1) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then

$W = T \times$ Increment in surface area

$W = T \times 4\pi[r_2^2 - r_1^2]$ [drop has only one free surface]

(2) In case of soap bubble

$W = T \times 8\pi[r_2^2 - r_1^2]$ [Bubble has two free surfaces]

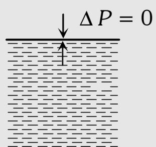
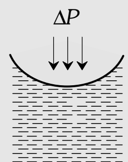
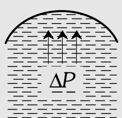
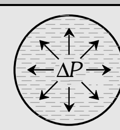
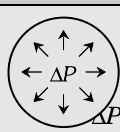
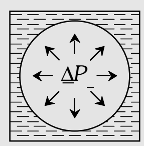
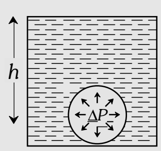
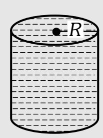
7.29 Splitting of Bigger Drop

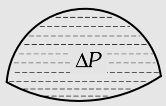
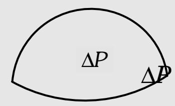
When a drop of radius R splits into n smaller drops, (each of radius r) then surface area of liquid increases. $R^3 = nr^3$

Work done = $T \times \Delta A = T$ [Total final surface area of n drops − surface area of big drop] = $T[n^4\pi r^2 - 4\pi R^2]$.

7.30 Excess Pressure

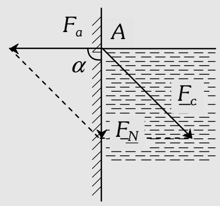
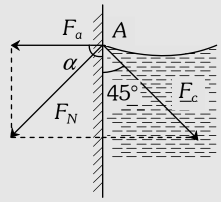
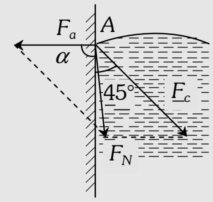
Excess pressure in different cases is given in the following table:

Plane surface	**Concave surface**
 $\Delta P = 0$	 $\Delta P = \frac{2T}{R}$
Convex surface	**Drop**
 $\Delta P = \frac{2T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble in air	**Bubble in liquid**
 $\Delta P = \frac{4T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble at depth h below the free surface of liquid of density d	**Cylindrical liquid surface**
 $\Delta P = \frac{2T}{R} + hdg$	 $\Delta P = \frac{T}{R}$

Liquid surface of unequal radii	Liquid film of unequal radii
 $\Delta P = T\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$	 $\Delta P = 2T\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$

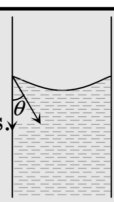
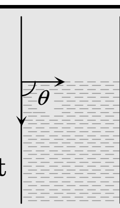
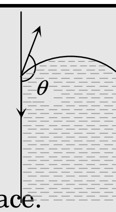
7.31 Shape of Liquid Meniscus

The curved surface of the liquid is called meniscus of the liquid.

If $F_c = \sqrt{2}Fa$ $\tan\alpha = \infty \therefore \alpha = 90^o$ *i.e.* the resultant force acts vertically downwards. Hence the liquid meniscus must be horizontal.	$F_c < \sqrt{2}Fa$ tan α = positive $\therefore$ α is acute angle *i.e.* the resultant force directed outside the liquid. Hence the liquid meniscus must be concave upward.	$F_c > \sqrt{2}Fa$ tan α = negative $\therefore$ α is obtuse angle *i.e.* the resultant force directed inside the liquid. Hence the liquid meniscus must be convex upward.
		
Example: Pure water in silver coated capillary tube.	*Example:* Water in glass capillary tube.	*Example:* Mercury in glass capillary tube.

7.32 Angle of Contact

Angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

$\theta < 90^o$; $F_a > \frac{F_c}{\sqrt{2}}$ concave meniscus. Liquid wets the solid surface. 	θ = 90°; $F_a = \frac{F_c}{\sqrt{2}}$ plane meniscus. Liquid does not wet the solid surface. 	θ > 90°; $F_a < \frac{F_c}{\sqrt{2}}$ convex meniscus. Liquid does not wet the solid surface. 

(i) Its value lies between 0^o and 180^o
θ = 0° for pure water and glass, θ = 90° for water and silver

(ii) On increasing the temperature, angle of contact decreases.

(iii) Soluble impurities increases the angle of contact.
(iv) Partially soluble impurities decreases the angle of contact.

7.33 Capillarity

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity.

The cause of capillarity is the difference in pressures on two sides curved surface of liquid.

7.34 Ascent Formula

When one end of capillary tube of radius r is immersed into a liquid of density d which wets the sides of the capillary R = radius of curvature of liquid meniscus.

T = surface tension of liquid

P = atmospheric pressure

$$\therefore \quad h = \frac{2T\cos\theta}{rdg}$$

IMPORTANT POINTS

(i) The capillary rise depends on the nature of liquid and solid both *i.e.* on T, d, θ and R.

(ii) Capillary action for various liquid-solid pair.

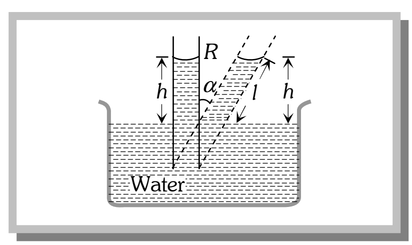
Meniscus	Angle of contact	Level
Concave	$\theta < 90^o$	Rises
Plane	$\theta = 90^o$	No rise no fall
Convex	$\theta > 90^o$	Fall

(iii) Lesser the radius of capillary greater will be the rise and vice-versa. This is called Jurin's law.

(iv) If the weight of the liquid contained in the meniscus is taken into consideration then more accurate ascent formula is given by $h = \dfrac{2T\cos\theta}{rdg} - \dfrac{r}{3}$

(v) In case of capillary of insufficient length, *i.e.*, $L < h$, the liquid will neither overflow from the upper end. The liquid after reaching the upper end will increase the radius of its meniscus without changing nature such that:

$$hr = Lr' \because L < h \qquad \therefore r' > r$$



(vi) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical, then the vertical height of liquid column remains same whereas the length of liquid column (l) in the capillary tube increases.

$$h = l\cos\alpha \text{ or } l = \frac{h}{\cos\alpha}$$

7.35 Pressure

The normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.

If F be the normal force acting on a surface of area A in contact with liquid, then pressure exerted by liquid on this surface is $P = F/A$ $P = F / A$

(1) ***Units:*** N/m^2 or Pascal (S.I.) and Dyne/cm^2 (C.G.S.)

(2) ***Dimension:*** $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

(3) ***Pressure is a tensor quantity.***

(4) ***Atmospheric pressure:*** 1 atm = 1/01 × 10^5 Pa = 1.01 bar = torr

(5) If P_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ, hydrostatic pressure P is given by $P = P_0 + h\rho g$.

(6) ***Gauge pressure:*** The pressure difference between hydrostatic pressure P and atmospheric pressure P_0 is called gauge pressure. $P - P_0 + h\rho g$

7.36 Density

In a fluid, at a point, density ρ is defined as: $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$

(1) It has dimensions [ML^{-3}] and S.I. unit kg/m^3 while C.G.S. unit g/cc with 1 g/cc = 10^{3}kg/m^3.

(2) Relative density or specific gravity which is defined as: $RD = \frac{\text{Density of body}}{\text{Density of water}}$

(3) If m_1 mass of liquid of density ρ_1 and m_2 mass of density ρ_2 are mixed, then

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)} = \frac{\sum m_i}{\sum(m_i / p_i)}$$

(4) With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease, $\rho \simeq \rho_0(1 - \gamma\Delta\theta)$

(5) With increase in pressure due to decrease in volume, density will increase.

$\rho \simeq \rho_0\left(1 + \frac{\Delta p}{B}\right)$ where B is blk modulus.

7.37 Pascal's Law

The increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

Example: Hydraulic lift, hydraulic press and hydraulic brakes.

7.38 Archimedes Principle

When a body is immersed partly or wholly in a fluid, in rest it is buoyed up with a force equal to the weight of the fluid displaced by the body. This principle is called Archimedes principle.

Apparent weight of the body of density (ρ) when immersed in a liquid of density (σ).

Apparent weight = Actual weight – Upthrust = $W - F_{up} = V\rho g - V\sigma g = V(\rho - \sigma)g = V\rho g\left(1 - \frac{\sigma}{\rho}\right)$

$$\therefore \qquad W_{APP} = W\left(1 - \frac{\sigma}{\rho}\right)$$

(1) Relative density of a body (R.D.) = $\frac{\text{Weight of body in air}}{\text{Weight in air–weight in water}} = \frac{W_1}{W_1 - W_2}$

(2) If the loss of weight of a body in water is 'a' while in liquid is 'b'

$$\therefore \qquad \frac{\sigma_L}{\sigma_W} = \frac{\text{Upthrust on body in liquid}}{\text{Upthrust on body in water}} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{a}{b} = \frac{W_{\text{air}} - W_{\text{liquid}}}{W_{\text{air}} - W_{\text{water}}}$$

7.39 Floatation

(1) ***Translatory equilibrium:*** When a body of density ρ and volume V is immersed in a liquid of density ρ, the forces acting on the body are Weight of body $W = mg = V\rho g$, acting vertically downwards through centre of gravity of the body.

Upthrust force = $V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.*, centre of buoyancy.

(i) If density of body is greater than that of liquid $\rho > \sigma$

Weight will be more than upthrust so the body will sink

(ii) If density of body is equal to that of liquid $\rho = \sigma$

Weight will be equal to upthrust so the body will float fully submerged in neutral equilibrium anywhere in the liquid.

(iii) If density of body is lesser than that of liquid $\rho < \sigma$

Weight will be less than upthrust so the body will move upwards and in equilibrium will float partially immersed in the liquid $V\rho g = V_{in}\sigma g$ where V_{in} is the volume of body in the liquid.

(2) ***Rotatory Equilibrium:*** When a floating body is slightly tilted from equilibrium position, the centre of buoyancy B shifts. The vertical line passing through the new centre of buoyancy B' and initial vertical line meet at a point M called meta-centre. If the meta-centre M is above the centre of gravity then object remains in stable equilibrium.However, if meta-centre goes below centre of gravity then object remains in stable equilibrium.

7.40 Streamline, Laminar and Turbulent Flow

(1) ***Stream line flow:*** Stream line flow of a liquid is that flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceeding element passes through that point.

The two streamlines cannot cross each other and the greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place.

(2) ***Laminar flow:*** If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which do not mix with each other, then the flow of liquid is called laminar flow.
In this flow the velocity of liquid flow is always less than the critical velocity of the liquid.

(3) ***Turbulent flow:*** When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid becomes disordered or irregular. Such a flow is called a turbulent flow.

7.41 Critical Velocity and Reynold's Number

The critical velocity is that velocity of liquid flow upto which its flow is streamlined and above which its flow becomes turbulent. Reynold's number is a pure number which determines the nature of flow of liquid through a pipe.

It is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

So by the definition of Reynolds number $N_R = \dfrac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \dfrac{v^2\rho}{\eta v / r} = \dfrac{v\rho r}{\eta}$

If the value of Reynold's number:

(i) Lies between 0 to 2000, the flow of liquid is streamline or laminar.

(ii) Lies between 2000 to 3000, the flow of liquid is unstable changing from streamline to turbulent flow.

(iii) Above 3000, the flow of liquid is definitely turbulent.

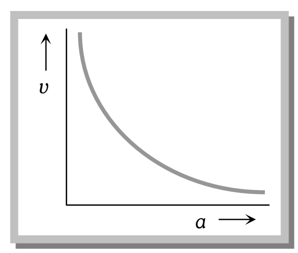
7.42 Equation of Continuity

The equation of continuity is derived from the principle of conservation of mass.

For an incompressible, streamlined and non-viscous liquid product of area of cross section of tube and velocity of liquid remains constant.

i.e. $a_1v_1 = a_2v_2$

or av = constant; or $a \propto \dfrac{1}{v}$



When water falls from a tap, the velocity of falling water under the action of gravity will increase with distance from the tap (*i.e,* $v_2 > v_1$). So in accordance with continuity equation the cross section of the water stream will decrease (*i.e.,* $A_2 < A_1$), *i.e.,* the falling stream of water becomes narrower.

7.43 Energy of a Flowing Fluid

Pressure Energy	Potential energy	Kinetic energy
It is the energy possessed by a liquid by virtue of its pressure. It is the measure of work done in pushing the liquid against pressure	It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any	It is the energy possessed by a liquid by virtue of its motion or velocity.

without imparting any velocity to it.	reference level taken as zero level.	
Pressure energy of the liquid PV	Potential energy of the liquid mgh	Kinetic energy of the liquid $\frac{1}{2}mv^2$
Pressure energy per unit mass of the liquid $\frac{P}{\rho}$	Potential energy per unit mass of the liquid gh	Kinetic energy per unit mass of the liquid $\frac{1}{2}v^2$
Pressure energy per unit volume of the liquid P	Potential energy per unit volume of the liquid ρgh	Kinetic energy per unit volume of the liquid $\frac{1}{2}\rho v^2$

7.44 Bernoulli's Theorem

According to this theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow.

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

(i) Bernoulli's theorem for unit mass of: $\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$

(ii) Dividing above equation by g we get $\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$

Here $\frac{P}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head.

7.45 Applications of Bernoulli's Theorem

(i) ***Attraction between two closely parallel moving boats***

(ii) ***Working of an aeroplane:*** 'dynamic lift' (= pressure difference × area of wing)

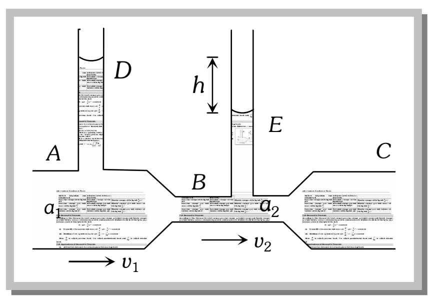
(iii) ***Action of atomiser:***

(iv) ***Blowing off roofs by wind storms***

(v) ***Magnus effect:*** When a spinning ball is thrown, it deviates from its usual path in flight. This effect is called Magnus effect.

(vi) ***Venturimeter:*** It is a device used for measuring the rate of flow of liquid through pipes.

Rate of flow of liquid $V = a_1 a_2\sqrt{\frac{2hg}{a_1^2 - a_2^2}}$



7.46 Velocity of Efflux

Velocity of efflux from a hole made at a depth h below the free surface of the liquid(of depth H) is given by $v = \sqrt{2gh}$.

Which is same as the final speed of a free falling object from rest through a distance h. This result is known as Torricelli's theorem.

(i) Time taken by the liquid to reach the base-level $t = \sqrt{\frac{2(H-h)}{g}}$

(ii) Horizontal range (x): $x = vt = \sqrt{2gh} \times \sqrt{[2(H-h)/g]} = 2\sqrt{h(H-h)}$

For maximum range $\frac{dx}{dh} = 0 \; \therefore \; h = \frac{H}{2}$

$\therefore$ Maximum range $x_{\max} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$

7.47 Viscosity and Newton's law of Viscous Force

The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion is called viscous force.

Viscous force F is proportional to the area of the plane A and the velocity gradient $\frac{dv}{dx}$ in a direction normal to the layer,

i.e., $F = -\eta A \frac{dv}{dx}$

Where η is a constant called the coefficient of viscosity. Negative sign is employed because viscous force acts in a direction opposite to the flow of liquid.

(1) Units: dyne-s-cm^{-2} or Poise (C.G.S. system);
Newton-s-m^{-2} or Poiseuille or decapoise (S.I. system)
1 Poiseuille = 1 decapoise = 10 Poise

(2) Dimension: $[ML^{-1} T^{-1}]$

(3) With increase in pressure, the viscosity of liquids (except water) increases while that of gases is independent of pressure. The viscosity of water decreases with increase in pressure.

(4) Solid friction is independent of the area of surfaces in contact and the relative velocity between them.

(5) Viscosity represents transport of momentum, while diffusion and conduction represents transport of mass and energy respectively.

(6) The viscosity of gases increases with increase of temperature.

(7) The viscosity of liquid decreases with increase of temperature.

7.48 Stoke's Law and Terminal Velocity

Stokes established that if a sphere of radius r moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

$$F = 6\pi\eta r v \text{ (Stokes law)}$$

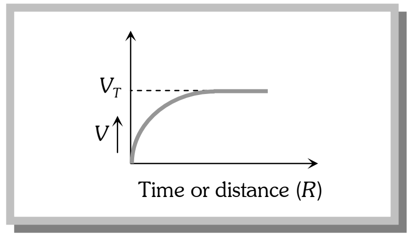
If a spherical body of radius r is dropped in a viscous fluid, it is first accelerated and then it's acceleration becomes zero and it attains a constant velocity called terminal velocity.

Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

(i) If $\rho > \sigma$ then body will attain constant velocity in downward direction.

(ii) If $\rho < \sigma$ then body will attain constant velocity in upward direction.
Example: Air bubble in a liquid and clouds in sky.

(iii) Terminal velocity graph:



7.49 Poiseuille's Formula

Poiseuille studied the stream-line flow of liquid in capillary tubes. He found that if a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l ' and radius r, then the volume of liquid coming out of the tube per second is

$$V = \frac{\pi P r^4}{8\eta l} \text{ (Poiseulle's equation)}$$

This equation also can be written as, $V = \frac{P}{R}$ where $R = \frac{8\eta l}{\pi r^4}$

R is called as liquid resistance.

(1) ***Series combination of tubes:***

(i) The volume of liquid flowing through both the tubes *i.e.* rate of flow of liquid is same.

(ii) Effective liquid resistance in series combination $R_{eff} = R_1 + R_2$

(2) ***Parallel combination of tubes:***

(i) Pressure difference across both tubes remains same.

(ii) Effective liquid resistance in parallel combination $\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$

9. Simple Harmonic Motion

9.1 Periodic Motion

A motion, which repeat itself over and over again after a regular interval of time is called a periodic motion and the fixed interval of time after which the motion is repeated is called period of the motion.

Examples: Revolution of earth around the sun (period one year).

9.2 Oscillatory or Vibratory Motion

The motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time .Oscillatory motion is also called as harmonic motion.

Example: The motion of the pendulum of a wall clock.

9.3 Harmonic and Non-harmonic Oscillation

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (*i.e.* sine or cosine function). *Example*: $y = \alpha \sin \omega t$ or $y = a \cos \omega t$.

Non-harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. *Example*: $y = a \sin \omega t + b \sin 2\omega t$.

9.4 Some Important Definitions

(1) ***Time period:*** It is the least interval of time after which the periodic motion of a body repeats itself. S.I. units of time period is second.

(2) ***Frequency:*** It is defined as the number of periodic motions executed by body per second. S.I unit of frequency is hertz (Hz).

(3) ***Angular Frequency:*** $\omega = 2\pi n$

(4) ***Displacement:*** Its deviation from the mean position.

(5) ***Phase:*** It is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

$y = a\sin\theta = a\sin(\omega t + \phi_0)$ here, $\theta = \omega t + \phi_0$ = phase of vibrating particle.

(i) *Initial phase or epoch:* It is the phase of a vibrating particle at $t = 0$.

(ii) *Same phase:* Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of π or path difference is an even multiple of $(\lambda / 2)$ or time interval is an even multiple of $(T/2)$.

(iii) *Opposite phase:* Opposite phase means the phase difference between the particle is an odd multiple of π or the path difference is an odd multiple of λ or the time interval is an odd multiple of $(T / 2)$.

(iv) *Phase difference:* If two particles performs S.H.M and their equation are $y_1 = a \sin(\omega t + \phi_1)$ and $y_2 = a \sin(\omega t + \phi_2)$ then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

9.5 Simple Harmonic Motion

Simple harmonic motion is a special type of periodic motion, in which Restoring force $\propto$ Displacement of the particle from mean position.

$$F = -kx$$

Where k is known as force constant. Its S.I. unit is Newton/meter and dimension is $[MT^{-2}]$.

9.6 Displacement in S.H.M.

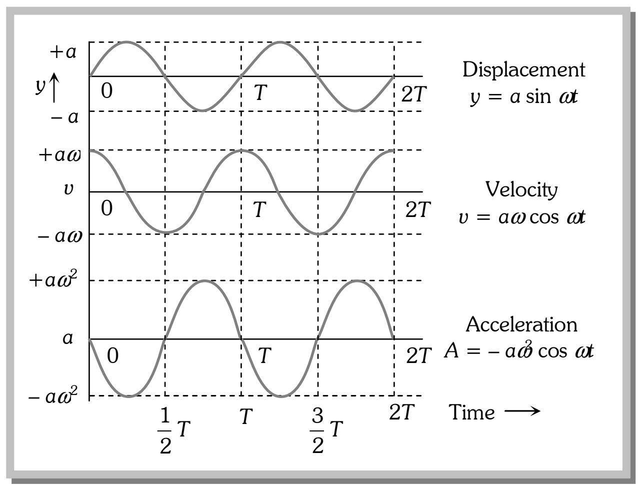
Simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference

(i) $y = a \sin \omega t$ when at $t = 0$ the vibrating particle is at mean position.

(ii) $y = a \cos \omega t$ when at $t = 0$ the the vibrating particle is at extreme position.

(iii) $y = a \sin(\omega t \pm \phi)$ when the vibrating particle is ϕ phase leading or lagging from the mean position.

9.7 Comparative Study of Displacement, Velocity and Acceleration



Displacement $y = a \sin \omega t$

Velocity $v = a\omega \cos$

$\omega t = a\, \omega \sin\left(\omega t + \dfrac{\pi}{2}\right)$

Acceleration $A = -a\omega^2 \sin$

$\omega t = a\omega^2 \sin(\omega t + \pi)$

(i) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

(ii) The velocity amplitude is ω times the displacement amplitude

(iii) The acceleration amplitude is ω^2 times the displacement amplitude

(iv) In S.H.M. the velocity is ahead of displacement by a phase angle $\pi/2$.

(v) In S.H.M. the acceleration is ahead of velocity by a phase angle $\pi/2$.

(vi) The acceleration is ahead of displacement by a phase angle of π

(vii) Various physical quantities in S.H.M. at different position:

Physical quantities	Equilibrium position (y = 0)	Extreme Position ($y = \pm a$)
Displacement $y = a \sin \omega t$	Minimum (Zero)	Maximum (a)
Velocity $v = \omega\sqrt{a^2 - y^2}$	Maximum ($a\,\omega$)	Minimum (Zero)
Acceleration $A = -\omega^2 y$	Minimum (Zero)	Maximum ($\omega^2 a$)

9.8 Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy: Potential energy and Kinetic energy

(1) ***Potential energy:*** $U = \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t$

(i) $U_{max} = \frac{1}{2} ka^2 = \frac{1}{2} m\omega^2 a^2$ when $y = \pm a$; $\omega t = \pi/2$; $t = T/4$

(ii) $U_{min} = 0$ when $y = 0$; $\omega t = 0$; $t = 0$.

(2) ***Kinetic energy:***

$$K = \frac{1}{2} ma^2\omega^2 \cos^2 \omega t \qquad K = \frac{1}{2} m\omega^2 (a^2 - y^2)$$

(i) $K_{max} = \frac{1}{2} m\omega^2 a^2$ when $y = 0$; $t = 0$; $\omega t = 0$

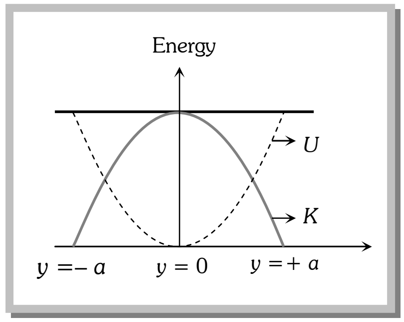
(ii) $K_{min} = 0$ when $y = a$; $t = T/4$, $\omega t = \pi/2$

(3) ***Total energy:*** Total mechanical energy = Kinetic energy + Potential energy

$$E = \frac{1}{2} m\omega^2 a^2$$

Total energy is not a position function *i.e.* it always remains constant.

(4) ***Energy position graph:***



(5) Kinetic energy and potential energy vary periodically with double the frequency of S.H.M

9.9 Time Period and Frequency of S.H.M.

Time period $(T) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ as $\omega = \sqrt{\frac{k}{m}}$

Frequency $(n) = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

In general m is called inertia factor and k is called spring factor.

Thus $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

9.10 Differential Equation of S.H.M.

For S.H.M. (linear) $m\frac{d^2y}{dt^2} + ky = 0$ [As $\omega = \sqrt{\frac{k}{m}}$]

For angular S.H.M. $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ [$\omega^2 = \frac{c}{I}$]

9.11 Simple Pendulum

Mass of the bob = m

Effective length of simple pendulum = l; $T = 2\pi\sqrt{\frac{l}{g}}$

(i) The period of simple pendulum is independent of amplitude as long as its motion is simple harmonic.

(ii) Time period of simple pendulum is also independent of mass of the bob.

(iii) If the length of the pendulum is comparable to the radius of earth then $T = 2\pi\sqrt{\frac{1}{g\left[\frac{1}{l}+\frac{1}{R}\right]}}$. If $l >> R (\to \infty) 1/l < 1/R$ so $T = 2\pi\sqrt{\frac{R}{g}} \cong 84.6$ *minutes*

(iv) The time period of simple pendulum whose point of suspension moving horizontally with acceleration a $T = 2\pi\sqrt{\frac{l}{(g^2+a^2)^{1/2}}}$ and $\theta = \tan^{-1}(a/g)$

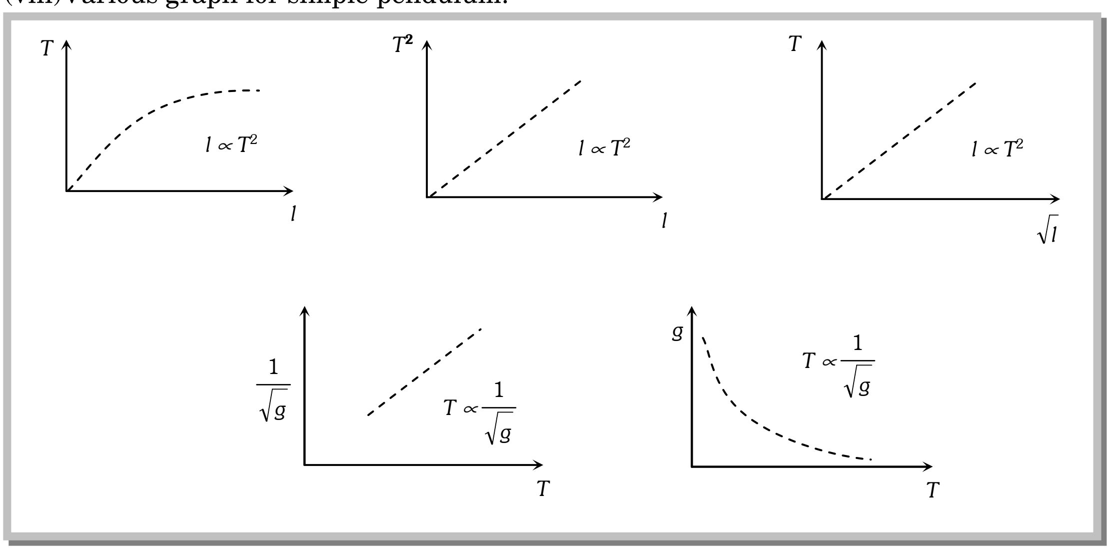
(v) Second's Pendulum: It is that simple pendulum whose time period of vibrations is two seconds.

(vi) Work done in giving an angular displacement θ to the pendulum from its mean position.

$$W = U = mgl(1-\cos\theta)$$

(vii) Kinetic energy of the bob at mean position = work done or potential energy at extreme

(viii)Various graph for simple pendulum.



9.12 Spring Pendulum

A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring constitutes a linear harmonic spring pendulum

Time period $T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad \text{Frequency } n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

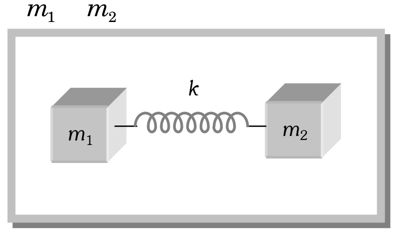
(i) Time of a spring pendulum is independent of acceleration due to gravity.

(ii) If the spring has a mass M and mass m is suspended from it, effective mass is given by $m_{eff} = m + \frac{M}{3}$

So that $$T = 2\pi\sqrt{\frac{m_{eff}}{k}}$$

(v) If two masses of mass m_1 and m_2 are connected by a spring and made to oscillate on horizontal surface, the reduced mass m_r is given by $\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$

So that $$T = 2\pi\sqrt{\frac{m_r}{k}}$$



(vi) If a spring pendulum, oscillating in a vertical plane is made to oscillate on a horizontal surface, (or on inclined plane) time period will remain unchanged.

(vii) If the stretch in a vertically loaded spring is y_0 then

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$$

` Time period does not depends on 'g' because along with g, y_0 will also change in such a way that $\frac{y_0}{g} = \frac{m}{k}$ remains constant.

(viii) Series combination: If n springs of different force constant are connected in series having force constant k_1, k_2, k_3....... respectively then $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ...$

(ix) Parallel combination: If the springs are connected in parallel then $k_{eff} = k_1 + k_2 + k_3 + ...$

(x) If the spring of force constant k is divided in to n equal parts then spring constant of each part will become nk .

(xi) The spring constant k is inversely proportional to the spring length.

As $$k \propto \frac{1}{\text{Extension}} \propto \frac{1}{\text{Length of spring}}$$

(xii) When a spring of length l is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$.

If the constant of a spring is k then Spring constant of first part $k_1 = \frac{k(n+1)}{n}$

Spring constant of second part $k_2 = (n + 1)k$ and ratio of spring constant $\frac{k_1}{k_2} = \frac{1}{n}$

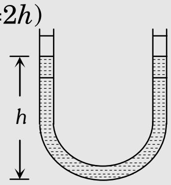
9.13 Various Formulae of S.H.M.

S.H.M. of a liquid in U tube

If a liquid of density ρ contained in a vertical U tube performs S.H.M. in its two limbs. Then time period $T = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{h}{g}}$

where L = Total length of liquid column,

h = Height of undisturbed liquid in each limb (L=2h)



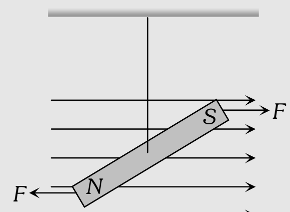
S.H.M. of a bar magnet in a magnetic field

$T = 2\pi\sqrt{\frac{I}{MB}}$

I = Moment of inertia of magnet

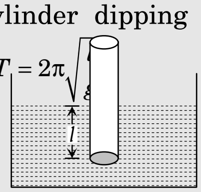
M = Magnetic moment of magnet

B = Magnetic field intensity



S.H.M. of a floating cylinder

If l is the length of cylinder dipping in liquid then time period $T = 2\pi\sqrt{\frac{l}{g}}$



S.H.M. of ball in the neck of an air chamber

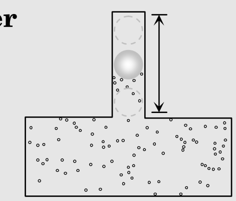
$T = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$

m = mass of the ball

V = volume of air- chamber

A = area of cross section of neck

E = Bulk modulus for Air

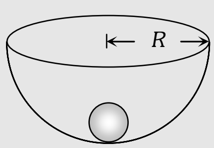


S.H.M. of a small ball rolling down in hemi-spherical bowl

$T = 2\pi\sqrt{\frac{R-r}{g}}$

R = radius of the bowl

r =radius of the ball



S.H.M. of a body suspended from a wire

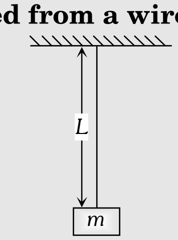
$T = 2\pi\sqrt{\frac{mL}{YA}}$

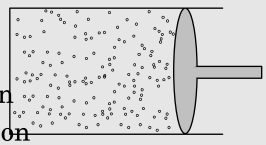
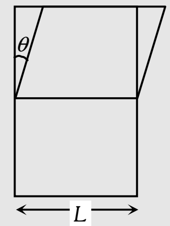
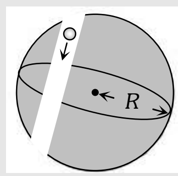
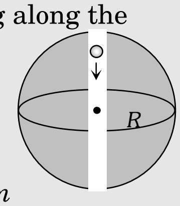
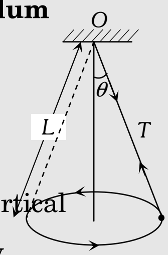
m = mass of the body

L = length of the wire

Y = young's modulus of wire

A = area of cross section of wire



S.H.M. of a piston in a cylinder $T = 2\pi\sqrt{\frac{Mh}{PA}}$  M = mass of the piston A = area of cross section h = height of cylinder P = pressure in a cylinder	**S.H.M of a cubical block** $T = 2\pi\sqrt{\frac{M}{\eta L}}$  M = mass of the block L = length of side of cube η = modulus of rigidity
S.H.M. of a body in a tunnel dug along any chord of earth  $T = 2\pi\sqrt{\frac{R}{g}}$ = 84.6 minutes	S.H.M. of body in the tunnel dug along the diameter of earth  $T = 2\pi\sqrt{\frac{R}{g}}$ T = 84.6 minutes R = radius of the earth = 6400km g = acceleration due to gravity = 9.8m/s^2 at earth's surface
S.H.M. of a conical pendulum $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$  L = length of string θ = angle of string from the vertical g = acceleration due to gravity	**S.H.M. of L-C circuit** $T = 2\pi\sqrt{LC}$ L = coefficient of self inductance C = capacity of condenser

9.14 Important Facts and Formulae

(1) When a body is suspended from two light springs separately. The time period of vertical oscillations are T_1 and T_2 respectively.

When these two springs are connected in series and the same mass m is attached at lower end and then

Time period of the system $T = \sqrt{T_1^2 + T_2^2}$

When these two springs are connected in parallel and the same mass m is attached at lower end then

Time period of the system $T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

(2) If infinite spring with force constant k, $2k$, $4k$, $8k$ .... respectively are connected in series. The effective force constant of the spring will be $k/2$.

(4) If $y_1 = a \sin \omega t$ and $y_2 = b \cos \omega t$ are two S.H.M. then by the superimposition of these two S.H.M. we get a S.H.M. as $y = A \sin(\omega t + \phi)$
where $A = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1}(b/a)$

9.15 Free, Damped, Forced and Maintained Oscillation

(1) ***Free oscillation***
  (i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations
  (ii) The amplitude, frequency and energy of oscillation remains constant
  (iii) Frequency of free oscillation is called natural frequency.

(2) ***Damped oscillation***
  (i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation
  (ii) Amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hystersis *etc*.

(3) ***Forced oscillation***
  (i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation
  (ii) Resonance: When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

(4) ***Maintained oscillation:*** The oscillation in which the loss of oscillator is compensated by the supplying energy from an external source are known as maintained oscillation.

10. Wave Motion

10.1 Wave

A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

(1) ***Necessary properties of the medium for wave propagation:***
   (i) *Elasticity:* So that particles can return to their mean position, after having been disturbed.
   (ii) *Inertia:* So that particles can store energy and overshoot their mean position.
   (iii) Minimum friction amongst the particles of the medium.
   (iv) Uniform density of the medium.

(2) ***Mechanical waves:*** The waves which require medium for their propagation are called mechanical waves.
   Example: Waves on string and spring, waves on water surface, sound waves, seismic waves.

(3) ***Non-mechanical waves:*** The waves which do not require medium for their propagation are called non- mechanical or electromagnetic waves.
   Examples: Light, heat (Infrared), radio waves, γ- rays, X-rays *etc*.

(4) ***Transverse waves:*** Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.
   (i) It travels in the form of crests and troughs.
   (ii) A crest is a portion of the medium which is raised temporarily.
   (iii) A trough is a portion of the medium which is depressed temporarily.
   (iv) Examples of transverse wave motion: Movement of string of a sitar, waves on the surface of water.
   (v) Transverse waves can not be transmitted into liquids and gases.

(5) ***Longitudinal waves:*** If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.
   (i) It travels in the form of compression and rarefaction.
   (ii) A compression (C) is a region of the medium in which particles are compressed.
   (iii) A rarefaction (R) is a region of the medium in which particles are rarefied.
   (iv) Examples sound waves travel through air in the form of longitudinal waves.
   (v) These waves can be transmitted through solids, liquids and gases.

10.2 Important Terms

(1) ***Wavelength:***
   (i) It is the length of one wave.
   (ii) Distance travelled by the wave in one time period is known as wavelength.
   λ = Distance between two consecutive crests or troughs.

(2) ***Frequency:*** Number of vibrations completed in one second.

(3) ***Time period:*** Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

(4) ***Relation between frequency and time period:*** Time period = 1/Frequency $\Rightarrow T = 1/n$

(5) ***Relation between velocity, frequency and wavelength:*** $v = n\lambda$

10.3 Sound Waves

The energy to which the human ears are sensitive is known as sound.

According to their frequencies, waves are divided into three categories:

(1) ***Audible or sound waves:*** Range 20 Hz to 20 KHz.

(2) ***Infrasonic waves:*** Frequency lie below 20 Hz.

(3) ***Ultrasonic waves:*** Frequency greater than 20 KHz.

- *Supersonic speed:* An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.
- *Mach number:* It is the ratio of velocity of source to the velocity of sound.

  Mach Number = $\dfrac{\text{Velocity of source}}{\text{Velocity of sound}}$.

10.4 Velocity of Sound (Wave motion)

(1) ***Speed of transverse wave motion:***

(i) *On a stretched string:* $v = \sqrt{\dfrac{T}{m}}$ T = Tension in the string; m = Linear density of string (mass per unit length).

(ii) *In a solid body:* $v = \sqrt{\dfrac{\eta}{\rho}}$ η = Modulus of rigidity; ρ = Density of the material.

(2) ***Speed of longitudinal wave motion:***

(i) In a solid long bar $v = \sqrt{\dfrac{Y}{\rho}}$ Y = Young's modulus; ρ = Density

(ii) In a liquid medium $v = \sqrt{\dfrac{k}{\rho}}$ k = Bulk modulus

(iii) In gases $v = \sqrt{\dfrac{k}{\rho}}$

10.5 Velocity of Sound in Elastic Medium

Velocity of sound in any medium is

$v = \sqrt{\dfrac{E}{\rho}}$ (E = Elasticity of the medium; ρ = Density of the medium)

(1) $v_{steel} > v_{water} > v_{air}$ 5000 m/s > 1500 m/s > 330 m/s

(2) ***Newton's formula:*** He assumed that propagation of sound is isothermal

$v_{air} = \sqrt{\dfrac{K}{\rho}} = \sqrt{\dfrac{P}{\rho}}$ As $K = E_\theta = P$; E_θ = Isothermal elasticity; P = Pressure.

By calculation $v_{air} = 279$ m/sec.

However the experimental value of sound in air is 332 m/sec

(3) ***Laplace correction:*** He modified that propagation of sound in air is adiabatic process.

$v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{E_\phi}{\rho}}$ (As $k = E_\phi = \gamma\rho$ = Adiabatic elasticity)

$v = 331.3\ m/s$ ($\gamma_{Air} = 1.41$)

(4) ***Effect of density:*** $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$

(5) ***Effect of pressure:*** Velocity of sound is independent of the pressure (when T = constant)

(6) ***Effect of temperature:*** $v \propto \sqrt{T\,(\text{in}\ K)}$

When the temperature change is small then $v_t = v_0(1 + \alpha t)$

Value of $\alpha = 0.608\ \frac{m/s}{^oC} = 0.61$ (Approx.)

(7) ***Effect of humidity:*** With rise in humidity velocity of sound increases.

(8) Sound of any frequency or wavelength travels through a given medium with the same velocity.

10.6 Reflection of Mechanical Waves

Medium	Longitudinal wave	Transverse wave	Change in direction	Phase change	Time change	Path change
Reflection from rigid end/denser medium	Compression as rarefaction and vice-versa	Crest as crest and Trough as trough	Reversed	π	$\frac{T}{2}$	$\frac{\lambda}{2}$
Reflection from free end/rarer medium	Compression as compression and rarefaction as rarefaction	Crest as trough and trough as crest	No change	Zero	Zero	Zero

10.7 Progressive Wave

(1) These waves propagate in the forward direction of medium with a finite velocity.

(2) Energy and momentum are transmitted in the direction of propagation of waves.

(3) In progressive waves, equal changes in pressure and density occurs at all points of medium.

(4) Various forms of progressive wave function.

(i) $y = A \sin(\omega t - kx)$

(ii) $y = A \sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$

where y = displacement
A = amplitude
ω = angular frequency
n = frequency
k = propagation constant
T = time period
λ = wave length
v = wave velocity
t = instantaneous time
x = position of particle from origin

(iii) $y = A \sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

(iv) $y = A \sin \frac{2\pi}{\lambda}(vt - x)$

(v) $y = A \sin \omega\left(t - \frac{x}{v}\right)$

(a) If the sign between t and x terms is negative the wave is propagating along positive X-axis and if the sign is positive then the wave moves in negative X-axis direction.

(b) The Argument of sin or cos function *i.e.* $(\omega t - kx)$ = Phase.

(c) The coefficient of t gives angular frequency $\omega = 2\pi n = \frac{2\pi}{T} = vk$.

(d) The coefficient of x gives propagation constant or wave number $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$.

(e) The ratio of coefficient of t to that of x gives wave or phase velocity. *i.e.* $v = \frac{\omega}{k}$.

(f) When a given wave passes from one medium to another its frequency does not change.

(g) From $v = n\lambda \Rightarrow v \propto \lambda \because n = \text{constant} \Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$.

(5) ***Some terms related to progressive waves***

(i) *Wave number ($\bar{n}$):* The number of waves present in unit length. $(\bar{n}) = \frac{1}{\lambda}$.

(ii) *Propagation constant (k):* $k = \frac{\phi}{x}$

$$k = \frac{\omega}{v} = \frac{\text{Angular velocity}}{\text{Wave velocity}} \quad \text{and} \quad k = \frac{2\pi}{\lambda} = 2\pi\bar{\lambda}$$

(iii) *Wave velocity (v):* $v = \frac{\omega}{k} = n\lambda = \frac{\omega\lambda}{2\pi} = \frac{\lambda}{T}$.

(iv) *Phase and phase difference* $\phi(x,t) = \frac{2\pi}{\lambda}(vt - x)$.

(v) Phase difference $= \frac{2\pi}{T} \times$ Time difference.

(vi) Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

$\Rightarrow$ Time difference $= \frac{T}{\lambda} \times$ Path difference.

10.8 Principle of Superposition

If $\vec{y}_1, \vec{y}_2, \vec{y}_3$ .... are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement. $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$

Important applications of superposition principle: (a) Stationary waves; (b) Beats.

10.9 Standing Waves or Stationary Waves

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

Characteristics of standing waves:

(1) The disturbance confined to a particular region

(2) There is no forward motion of the disturbance beyond this particular region.

(3) The total energy is twice the energy of each wave.

(4) Points of zero ampitude are known as nodes.

The distance between two consecutive nodes is $\frac{\lambda}{2}$

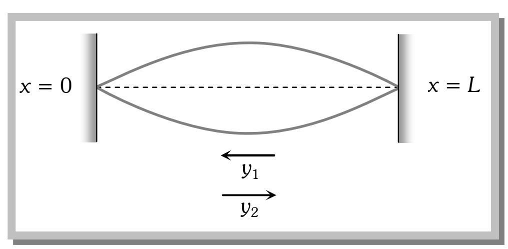
(5) Points of maximum amplitude is known as antinodes. The distance between two consecutive antinodes is also $\lambda/2$. The distance between a node and adjoining antinode is $\lambda/4$.

(6) The medium splits up into a number of segments.

(7) All the particles in one segment vibrate in the same phase. Particles in two consecutive segments differ in phase by 180°.

(8) Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

10.10 Standing Waves on a String

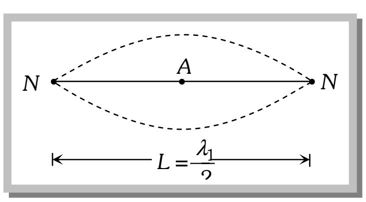


Incident wave $y_1 = a \sin \frac{2\pi}{\lambda} (vt + x)$

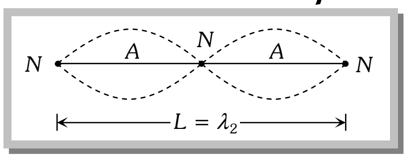
Reflected wave $y_2 = a \sin \frac{2\pi}{\lambda} [(vt - x) + \pi]$

Using superposition principle: $y = y_1 + y_2 = 2a \cos \frac{2\pi vt}{\lambda} \sin \frac{2\pi x}{\lambda}$

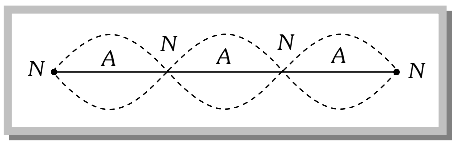
1. ***First normal mode of vibration:***



2. ***Second normal mode of vibration:***



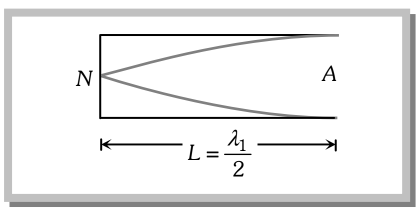
3. ***Third normal mode of vibration:***



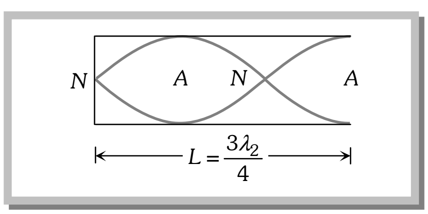
10.11 Standing Wave in a Closed Organ Pipe

Equation of standing wave y = $2a \cos \dfrac{2\pi vt}{\lambda} \sin \dfrac{2\pi x}{\lambda}$

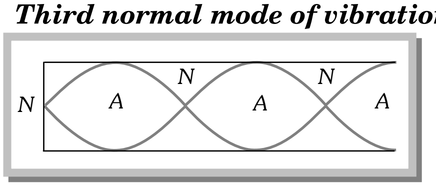
1. ***First normal mode of vibration:***



2. ***Second normal mode of vibration:***

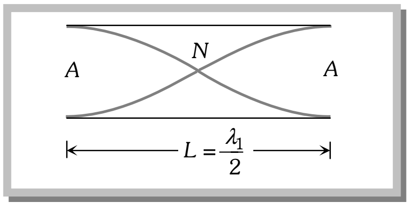


3. ***Third normal mode of vibration:***

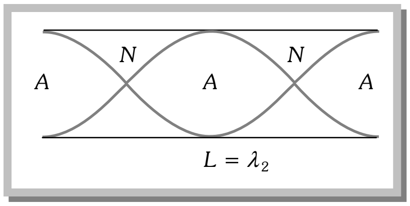


10.12 Standing Waves in Open Organ Pipes

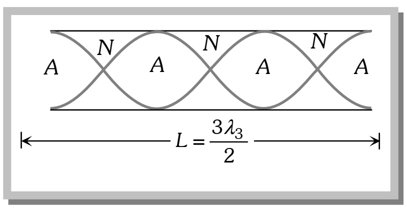
(1) ***First normal mode of vibration:***



(2) ***Second normal mode of vibration***



(3) ***Third normal mode of vibration***



10.13 Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. No.	Parameter	Stretched string	Open organ pipe	Closed organ pipe
(1)	Fundamental frequency or 1st harmonic(1st mode of vibration)	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$
(2)	Frequency of 1st overtone or 2nd harmonic(2nd mode of vibration)	$n_2 = 2n_1$	$n_2 = 2n_1$	Missing
(3)	Frequency of 2nd overtone or 3rd harmonic((3rd mode of vibration)	$n_3 = 3n_1$	$n_3 = 3n_1$	$n_3 = 3n_1$
(4)	Frequency ratio of overtones	2: 3: 4…	2: 3: 4…	3: 5: 7…

(5)	Frequency ratio of harmonics	1: 2: 3: 4…	1: 2: 3: 4…	1: 3: 5: 7…
(6)	Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary
(7)	General formula for wavelength	$\lambda = \frac{2L}{n}$ $n = 1,2,3, \ldots$	$\lambda = \frac{2L}{n}$ $n = 1,2,3, \ldots$	$\lambda = \frac{4L}{(2n-1)}$
(8)	Position of nodes	$x = 0, \frac{L}{n}, \overset{3}{\frac{L}{n}} \cdots \frac{L}{n}$	$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} \ldots \frac{(2\eta - 1)L}{2n}$	$x = 0, \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{(2n-1)} \ldots \frac{2nL}{(2n-1)}$
(9)	Position of antinodes	$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} \ldots \frac{(2\eta - 1)L}{2n}$	$x = 0, \frac{L}{n}, \overset{3}{\frac{L}{n}} \cdots \frac{L}{n}, \ldots$	$x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{2n-1} \ldots, L$

(i) Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency (n).

(ii) Overtones are the notes/sounds of frequency twice/thrice/ four times the fundamental frequency (n).

(iii) In organ pipe an antinode is not formed exactly at the open end rather it is formed a little distance away from the open end outside it. The distance of antinode from the open end of the pipe is = 0.6r (where r is radius of organ pipe). This is known as end correction.

10.14 Vibration of a String

General formula of frequency $n_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$

L = Length of string, T = Tension in the string

m = Mass per unit length (linear density), p = mode of vibration

(1) The string will be in resonance with the given body if any of its natural frequencies concides with the body.

(2) If M is the mass of the string of length L, $m = \frac{M}{L}$.

So $n = \frac{1}{2Lr}\sqrt{\frac{T}{\pi\rho}}$ (r = Radius, ρ = Density)

10.15 Beats

When two sound waves of slightly different frequencies, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. This phenomenon is called beats.

(1) ***Beat period:*** The time interval between two successive beats (*i.e.* two successive maxima of sound) is called beat period.

(2) ***Beat frequency:*** The number of beats produced per second is called beat frequency.

(3) ***Persistence of hearing:*** The impression of sound heard by our ears persist in our mind for 1/10th of a second.

So for the formation of distinct beats, frequencies of two sources of sound should be nearly equal (difference of frequencies less than 10)

(4) ***Equation of beats:*** If two waves of equal amplitudes 'a' and slightly different frequencies n_1 and n_2 travelling in a medium in the same direction then equation of beats is given by

$y = A \sin \pi(n_1 - n_2)t$ where $A = 2a \cos \pi(n_1 - n_2)t$ = Amplitude of resultant wave.

(5) ***Beat frequency:*** $n = n_1 \sim n_2$.

(6) ***Beat period:*** $T = \dfrac{1}{\text{Beat frequency}} = \dfrac{1}{n_1 \sim n_2}$

10.16 Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

Apparent frequency $n' = \dfrac{[(v+v_m)-v_L]n}{[(v+v_m)-v_S]}$

Here n = Actual frequency; v_L = Velocity of listener; v_S = Velocity of source

v_m = Velocity of medium and v = Velocity of sound wave

Sign convention: All velocities along the direction S to L are taken as positive and all velocities along the direction L to S are taken as negative. If the medium is stationary $v_m = 0$ then $n' = \left(\dfrac{v-v_L}{v-v_S}\right)n$

(1) No Doppler effect takes place($n' = n$) when relative motion between source and listener is zero.

(2) Source and listener moves at right angle to the direction of wave propagation.($n' = n$)

(i) If the velocity of source and listener is equal to or greater than the sound velocity then Doppler effect is not observed.

(ii) Doppler effect does not says about intensity of sound.

(iii) Doppler effect in sound is asymmetric but in light it is symmetric.

10.17 Some Typical Features of Doppler's Effect in Sound

(1) ***When a listener moves between two distant sound sources:*** In this case listener observed beats. Let v_L be the velocity of listener away from S_1 and towards S_2. Frequency of both the sources = n, velocity of sound = v then

$\therefore$ Beat frequency $= \dfrac{2nv_L}{v}$

(2) ***When source is revolving in a circle and listener L is on one side***

$$n_{\max} = \frac{nv}{v - v_s} \text{ and } n_{\min} = \frac{nv}{v + v_s}$$

(3) ***When listener L is moving in a circle and the source is on one side***

$$n_{\max} = \frac{(v + v_L)n}{v} \text{ and } n_{\min} = \frac{(v - v_L)n}{v}$$